An introduction to timed systems

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Outline

Time!

Context: verification of critical systems

Time

- naturally appears in real systems (for ex. protocols, embedded systems)
- appears in properties (for ex. bounded response time)

  "Will the airbag oben within 5ms after the car crashes?"

$\leadsto$ Need of models and specification languages integrating timing aspects

Adding timing informations

- **Untimed case:** sequence of observable events
  a: send message b: receive message

$$a\ b\ a\ b\ a\ b\ a\ b\ a\ b\ \cdots = (a\ b)^\omega$$

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$$(a, d_1)\ (b, d_2)\ (a, d_3)\ (b, d_4)\ (a, d_5)\ (b, d_6)\ \cdots$$

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  - **Discrete-time semantics:** dates are *e.g.* taken in $\mathbb{N}$
    Ex: $(a, 1)(b, 3)(c, 4)(a, 6)$

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  - **Discrete-time semantics:** dates are *e.g.* taken in $\mathbb{N}$
    Ex: $(a, 1)(b, 3)(c, 4)(a, 6)$
  - **Dense-time semantics:** dates are *e.g.* taken in $\mathbb{Q}_+$, or in $\mathbb{R}_+$
    Ex: $(a, 1.28).(b, 3.1).(c, 3.98)(a, 6.13)$

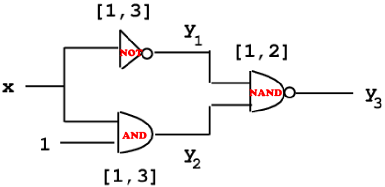
A case for dense-time

Time domain: discrete (*e.g.* $\mathbb{N}$) or dense (*e.g.* $\mathbb{Q}_+$ or $\mathbb{R}_+$)

- A compositionality problem with discrete time
- Dense-time is a more general model than discrete time
- But, can we not always discretize?

A digital circuit [Alu91]

Discussion in the context of reachability problems for asynchronous digital circuits [BS91]



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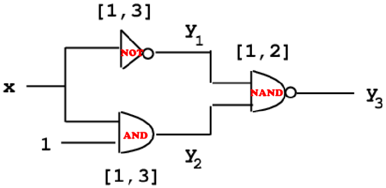

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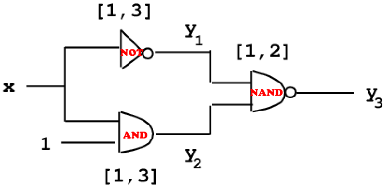

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The input x changes to 1. The corresponding stable state is y=[011]

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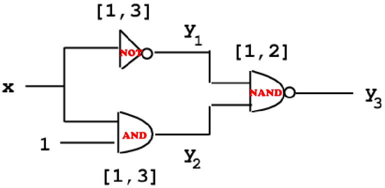

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However, many possible behaviours, e.g.

$$[101] \xrightarrow[1.2]{y_2} [111] \xrightarrow[2.5]{y_3} [110] \xrightarrow[2.8]{y_1} [010] \xrightarrow[4.5]{y_3} [011]$$

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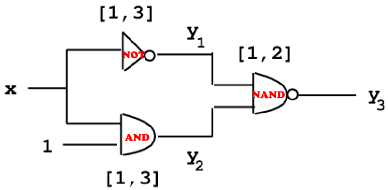

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Reachable configurations: $\{[101], [111], [110], [010], [011], [001]\}$

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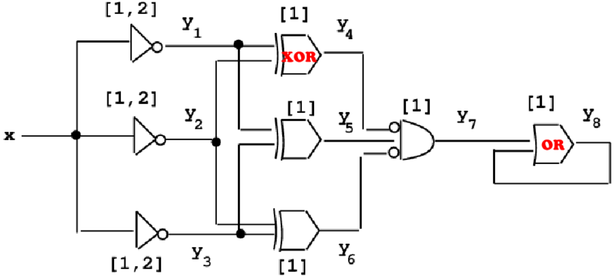

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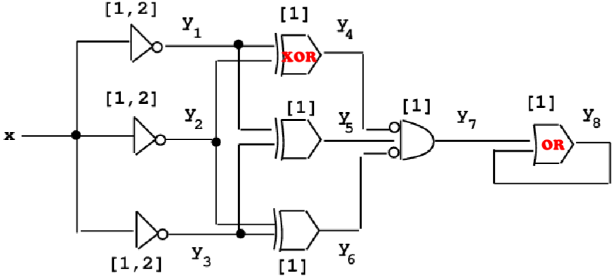

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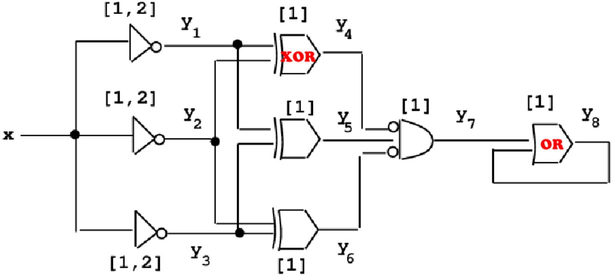

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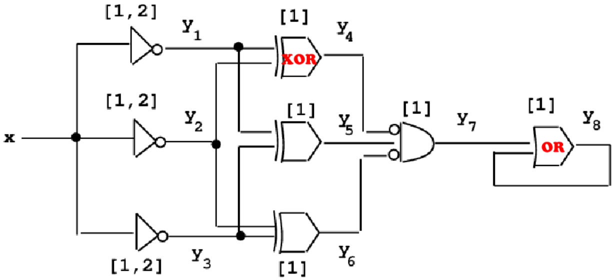

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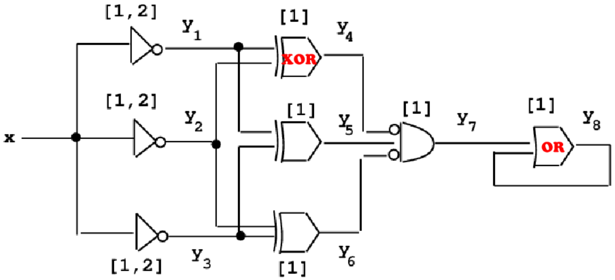

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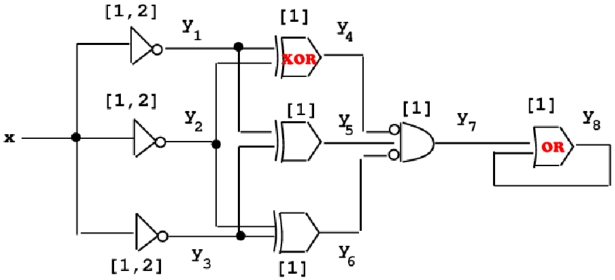

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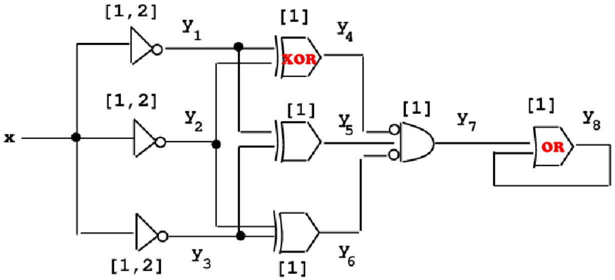

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Theorem [BS91]

For every $k \geq 1$, there exists a digital circuit such that the reachability set of states in dense-time is strictly larger than the one in discrete time (with granularity $\frac{1}{k}$).

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Hence, we better consider a dense-time domain!

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Outline

A plethora ot models...

- ... for real-time systems:
    - timed circuits,
    - time(d) Petri nets,
    - timed process algebra,
    - timed automata,
    - ...
- ... and for real-time properties:
    - timed observers,
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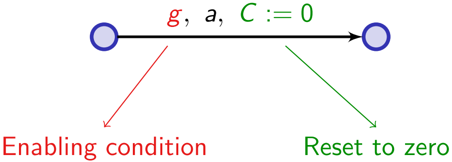
Timed automata [AD90]

- A finite control structure + variables (clocks)
- A transition is of the form:



- An enabling condition (or guard) is:

$$g \quad ::= \quad x \sim c \;\mid\; x - y \sim c \;\mid\; g \wedge g$$

where $\sim \in \{<, \leq, =, \geq, >\}$

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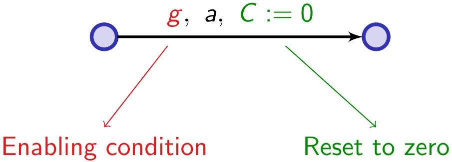

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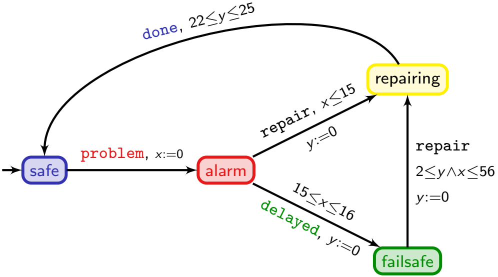

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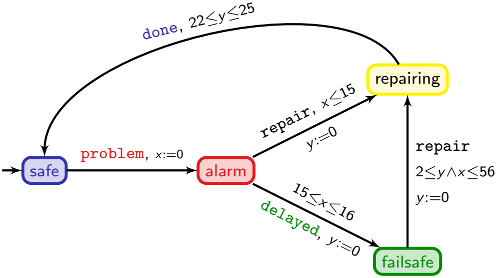

	safe
x	0
y	0

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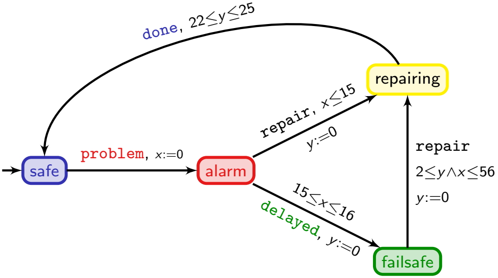

	safe	$\xrightarrow{23}$	safe
x	0		23
y	0		23

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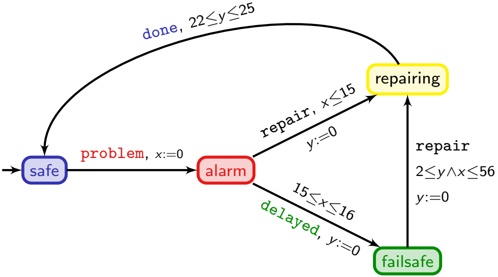

	safe	$\xrightarrow{23}$	safe	$\xrightarrow{\texttt{problem}}$	alarm
x	0		23		0
y	0		23		23

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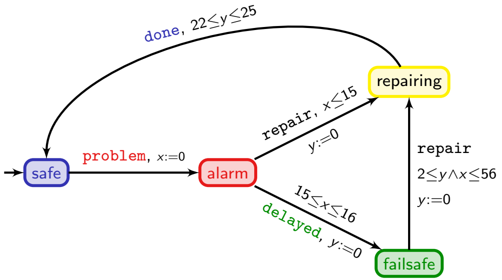

	safe	$\xrightarrow{23}$	safe	$\xrightarrow{\texttt{problem}}$	alarm	$\xrightarrow{15.6}$	alarm
x	0		23		0		15.6
y	0		23		23		38.6

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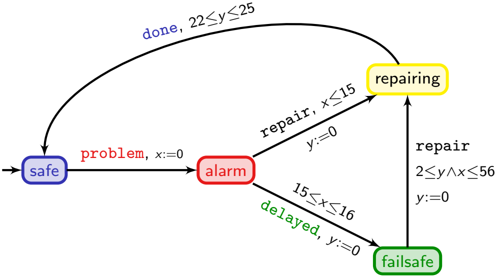

	safe	$\xrightarrow{23}$	safe	$\xrightarrow{\texttt{problem}}$	alarm	$\xrightarrow{15.6}$	alarm	$\xrightarrow{\texttt{delayed}}$	failsafe		
x	0		23		0		15.6		15.6		···
y	0		23		23		38.6		0		

	failsafe
···	15.6
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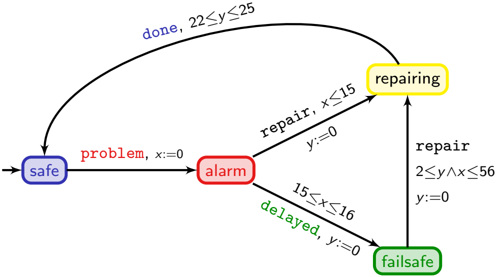

	safe	$\xrightarrow{23}$	safe	$\xrightarrow{\texttt{problem}}$	alarm	$\xrightarrow{15.6}$	alarm	$\xrightarrow{\texttt{delayed}}$	failsafe	
x	0		23		0		15.6		15.6	⋯
y	0		23		23		38.6		0	

	failsafe	$\xrightarrow{2.3}$	failsafe
⋯	15.6		17.9
	0		2.3

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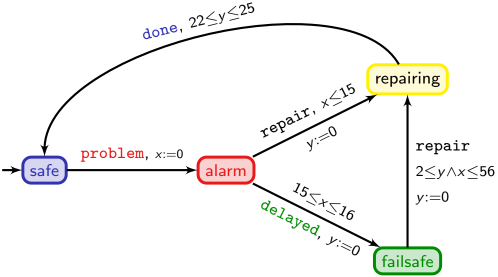

	safe	$\xrightarrow{23}$	safe	$\xrightarrow{\texttt{problem}}$	alarm	$\xrightarrow{15.6}$	alarm	$\xrightarrow{\texttt{delayed}}$	failsafe	
x	0		23		0		15.6		15.6	⋯
y	0		23		23		38.6		0	

	failsafe	$\xrightarrow{2.3}$	failsafe	$\xrightarrow{\texttt{repair}}$	repairing
⋯	15.6		17.9		17.9
	0		2.3		0

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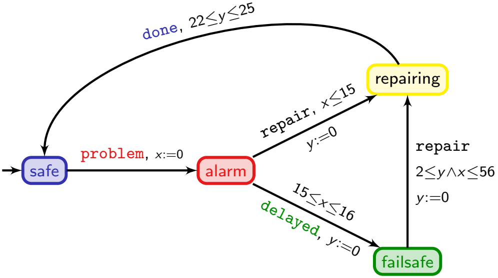

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x	0		23		0		15.6		15.6	⋯
y	0		23		23		38.6		0	

	failsafe	$\xrightarrow{2.3}$	failsafe	$\xrightarrow{\texttt{repair}}$	repairing	$\xrightarrow{22.1}$	repairing
⋯	15.6		17.9		17.9		40
	0		2.3		0		22.1

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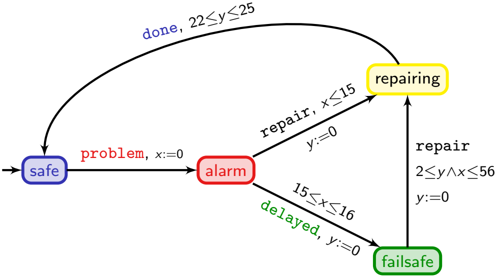

	safe	$\xrightarrow{23}$	safe	$\xrightarrow{\texttt{problem}}$	alarm	$\xrightarrow{15.6}$	alarm	$\xrightarrow{\texttt{delayed}}$	failsafe	
x	0		23		0		15.6		15.6	$\cdots$
y	0		23		23		38.6		0	

	failsafe	$\xrightarrow{2.3}$	failsafe	$\xrightarrow{\texttt{repair}}$	repairing	$\xrightarrow{22.1}$	repairing	$\xrightarrow{\texttt{done}}$	safe
$\cdots$	15.6		17.9		17.9		40		40
	0		2.3		0		22.1		22.1

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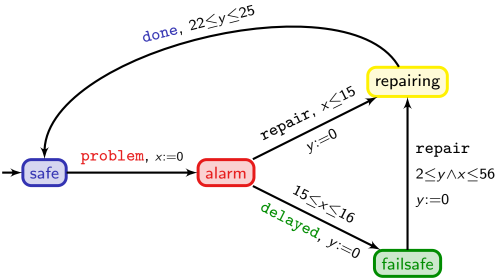

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⋯	15.6		17.9		17.9		40		40
	0		2.3		0		22.1		22.1

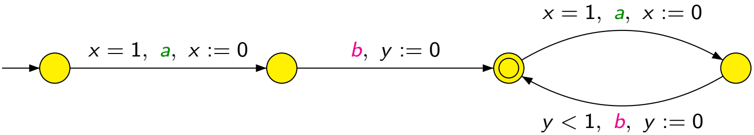
This run read the timed word
$(\texttt{problem}, 23)(\texttt{delayed}, 38.6)(\texttt{repair}, 40.9), (\texttt{done}, 63)$.

Timed automata semantics

- $\mathcal{A} = (\Sigma, L, X, \longrightarrow)$ is a TA
- **Configurations:** $(\ell, v) \in L \times T^X$ where T is the time domain

  v is called the (clock) valuation
- **Timed transition system:**
  - **action transition**: $(\ell, v) \xrightarrow{a} (\ell', v')$ if $\exists \ell \xrightarrow{g,a,r} \ell' \in \mathcal{A}$ s.t. $\begin{cases} v \models g \\ v' = v[r \leftarrow 0] \end{cases}$
  - **delay transition**: $(\ell, v) \xrightarrow{\delta(d)} (\ell, v + d)$ if $d \in T$

Discrete *vs* dense-time semantics



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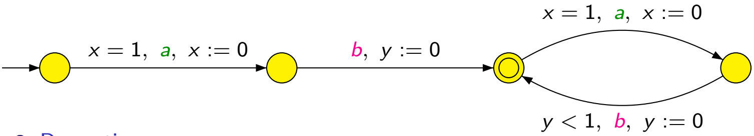

- Dense-time:
  $L_{dense} = \{((ab)^\omega, \tau) \mid \forall i,\ \tau_{2i-1} = i \text{ and } \tau_{2i} - \tau_{2i-1} > \tau_{2i+2} - \tau_{2i+1}\}$

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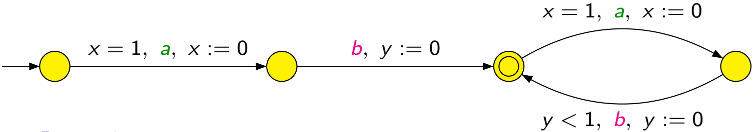
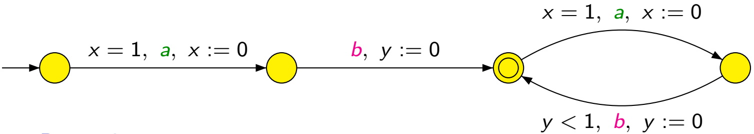


- Dense-time:
  $L_{dense} = \{((ab)^\omega, \tau) \mid \forall i,\ \tau_{2i-1} = i \text{ and } \tau_{2i} - \tau_{2i-1} > \tau_{2i+2} - \tau_{2i+1}\}$
- Discrete-time: $L_{discrete} = \emptyset$

Discrete *vs* dense-time semantics



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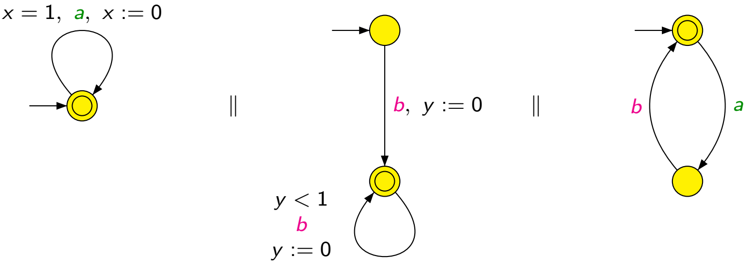
However, it does result from the following parallel composition:

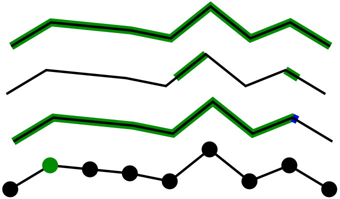


Classical verification problems

- reachability of a control state
- $\mathcal{S} \sim \mathcal{S}'$: bisimulation, etc...
- $L(\mathcal{S}) \subseteq L(\mathcal{S}')$: language inclusion
- $\mathcal{S} \models \varphi$ for some formula φ: model-checking
- $\mathcal{S} \parallel A_T$ + reachability: testing automata
- ...

Classical temporal logics

Path formulas:



$\mathbf{G}\,\varphi$ — "Always"

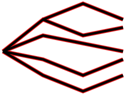
$\mathbf{F}\,\varphi$ — "Eventually"

$\varphi\,\mathbf{U}\,\varphi'$ — "Until"

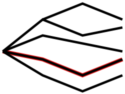
$\mathbf{X}\,\varphi$ — "Next"

State formulas:

$\mathbf{A}\psi$



$\mathbf{E}\psi$



$\rightsquigarrow$ LTL: Linear Temporal Logic [Pnu77],
CTL: Computation Tree Logic [EC82]

[Pnu77] Pnueli. The temporal logic of programs *(FoCS'77)*.
[EC82] Emerson, Clarke. Using branching time temporal logic to synthesize synchronization skeletons *(Science of Computer Programming 1982)*.

Adding time to temporal logics

Classical temporal logics allow us to express that

"any problem is followed by an alarm"

[ACD90] Alur, Courcoubetis, Dill. Model-checking for real-time systems *(LICS'90)*.
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$\rightsquigarrow$ TCTL: Timed CTL **[ACD90,ACD93,HNSY94]**

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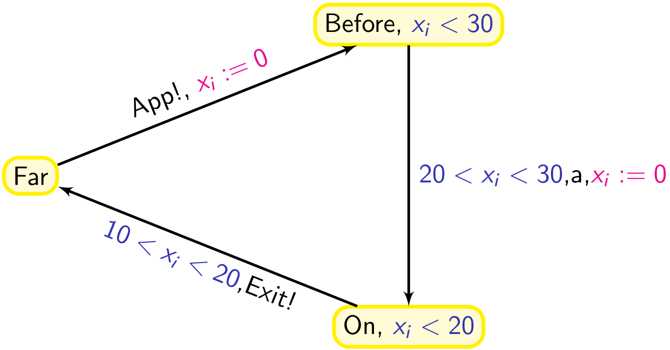
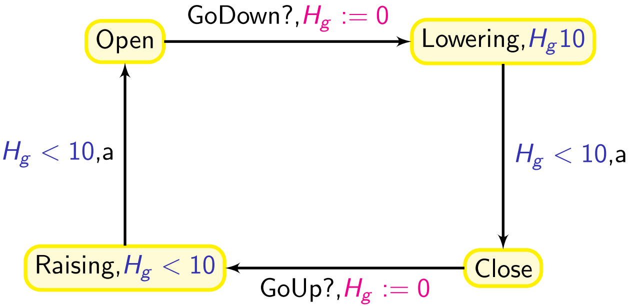
The train crossing example (1)

Train$_i$ with $i = 1, 2, \ldots$



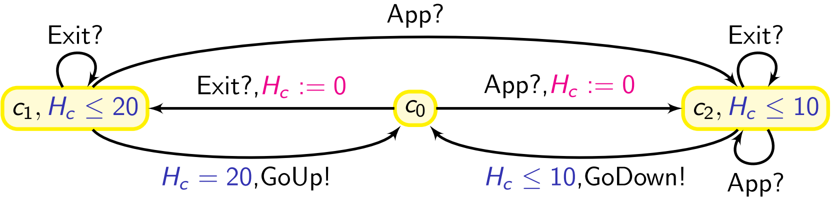
The train crossing example (2)

The gate:



The train crossing example (3)

The controller:



The train crossing example (4)

We use the synchronization function f:

Train_1	Train_2	Gate	Controller	
App!	.	.	App?	App
.	App!	.	App?	App
Exit!	.	.	Exit?	Exit
.	Exit!	.	Exit?	Exit
a	.	.	.	a
.	a	.	.	a
.	.	a	.	a
.	.	GoUp?	GoUp!	GoUp
.	.	GoDown?	GoDown!	GoDown

to define the parallel composition $(\text{Train}_1 \parallel \text{Train}_2 \parallel \text{Gate} \parallel \text{Controller})$

NB: the parallel composition does not add expressive power!

The train crossing example (5)

Some properties one could check:

- Is the gate closed when a train crosses the road?

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$$\neg\mathbf{EF}\,(\text{gate.Close} \wedge \mathbf{E}(\text{gate.Close}\ \mathbf{U}_{>5\text{ min}}\ \neg\text{gate.Close}))$$

Another example: A Fischer protocol

A mutual exclusion protocol with a shared variable *id* [AL94].

[AL94] Abadí, Lamport. An old-fashioned recipe for real time *(ACM Transactions on Programming Languages and Systems)*.

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Process i:

a : await ($id = 0$);
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c : await ($id = i$);
d : enter critical section.

$\rightsquigarrow$ a max. delay k_1 between a and b
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$\leadsto$ See the demo with the tool Uppaal
(can be downloaded freely on http://www.uppaal.com/)

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Outline

Verification

Emptiness problem: is the language accepted by a timed automaton empty?

- basic reachability/safety properties (final states)
- basic liveness properties (ω-regular conditions)

[AD90] Alur, Dill. Automata for modeling real-time systems *(ICALP'90)*.
[AD94] Alur, Dill. A theory of timed automata *(Theoretical Computer Science)*.

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Emptiness problem: is the language accepted by a timed automaton empty?

- Problem: the set of configurations is infinite
  $\leadsto$ classical methods for finite-state systems cannot be applied

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The emptiness problem for timed automata is decidable and PSPACE-complete.

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Method: construct a finite abstraction

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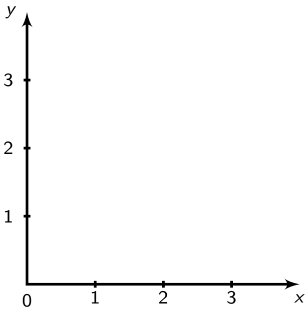
The region abstraction



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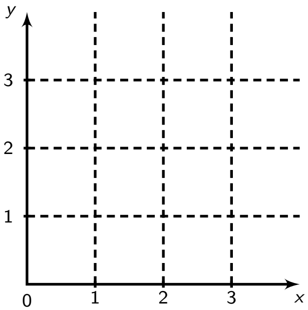

- "compatibility" between regions and constraints

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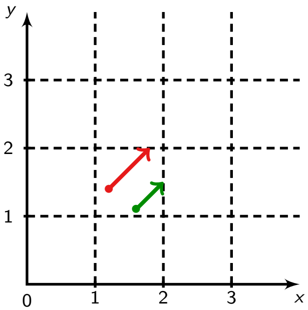

- “compatibility” between regions and constraints
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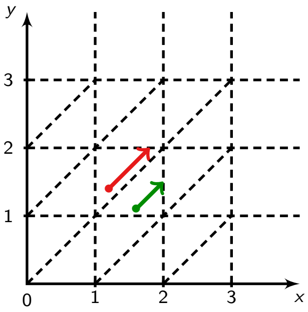

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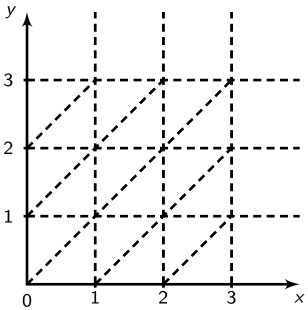

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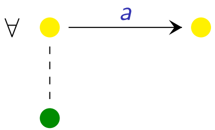
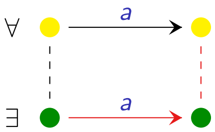
$\rightsquigarrow$ an equivalence of finite index
a time-abstract bisimulation

Time-abstract bisimulation



Time-abstract bisimulation



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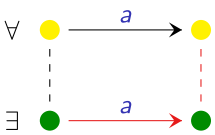

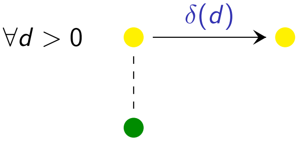

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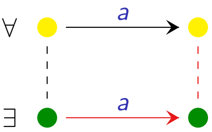
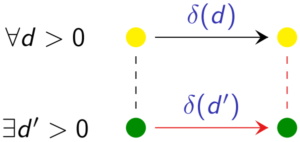
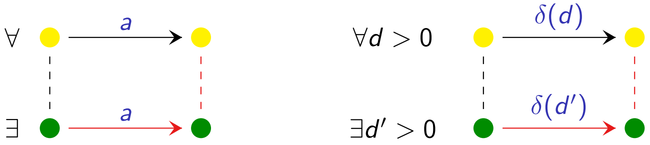


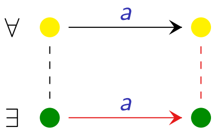


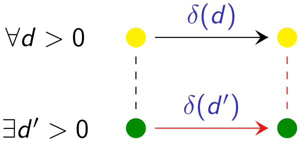
Time-abstract bisimulation



$$(\ell_0, v_0) \xrightarrow{a_1, t_1} (\ell_1, v_1) \xrightarrow{a_2, t_2} (\ell_2, v_2) \xrightarrow{a_3, t_3} \ldots$$

Time-abstract bisimulation





$$
\begin{tikzcd}
(\ell_0, v_0) \arrow[r, "a_1, t_1"] \arrow[d] & (\ell_1, v_1) \arrow[r, "a_2, t_2"] \arrow[d] & (\ell_2, v_2) \arrow[r, "a_3, t_3"] \arrow[d] & \ldots \\
(\ell_0, R_0) \arrow[r, "a_1"] & (\ell_1, R_1) \arrow[r, "a_2"] & (\ell_2, R_2) \arrow[r, "a_3"] & \ldots
\end{tikzcd}
$$

with $v_i \in R_i$ for all i.

Time-abstract bisimulation

$$\begin{tikzcd} \forall & \bullet \arrow[r, "a"] \arrow[d, dash, dashed] & \bullet \arrow[d, dash, dashed] \\ \exists & \bullet \arrow[r, "a"] & \bullet \end{tikzcd}
\qquad
\begin{tikzcd} \forall d > 0 & \bullet \arrow[r, "\delta(d)"] \arrow[d, dash, dashed] & \bullet \arrow[d, dash, dashed] \\ \exists d' > 0 & \bullet \arrow[r, "\delta(d')"] & \bullet \end{tikzcd}$$

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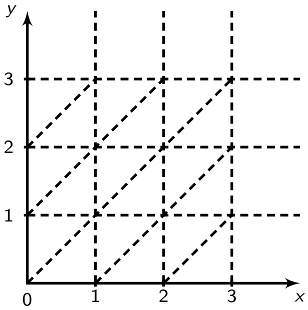

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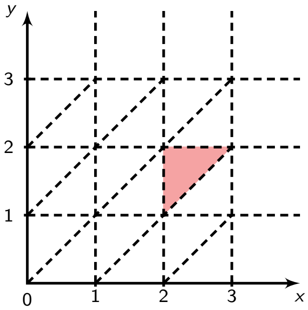
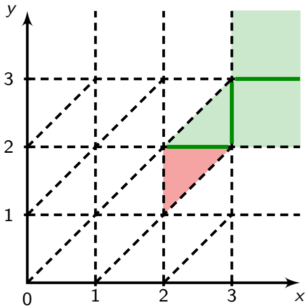


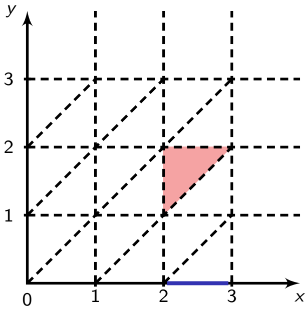
$$\begin{cases} 2<x<3 \\ 1<y<2 \\ \{x\}<\{y\} \end{cases}$$

The region abstraction



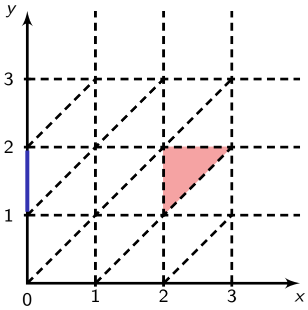
time successors

The region abstraction



reset of clock y

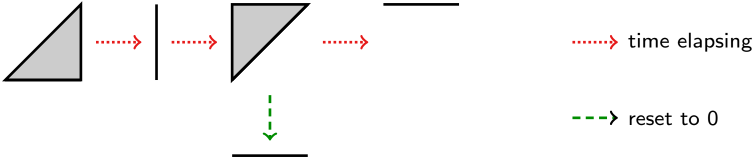
The region abstraction



reset of clock x

The region graph

A finite graph representing time elapsing and reset of clocks:



Region automaton $\equiv$ finite bisimulation quotient

timed automaton $\bigotimes$ region graph

Region automaton $\equiv$ finite bisimulation quotient

timed automaton $\bigotimes$ region graph

$\ell \xrightarrow{g,a,C:=0} \ell'$ is transformed into:

$(\ell, R) \xrightarrow{a} (\ell', R')$ if there exists $R'' \in \mathsf{Succ}_t^*(R)$ s.t.

- $R'' \subseteq g$
- $[C \leftarrow 0]R'' \subseteq R'$

Region automaton $\equiv$ finite bisimulation quotient

timed automaton $\bigotimes$ region graph

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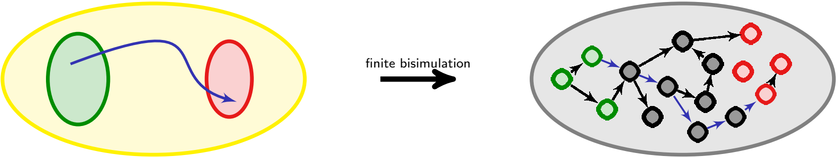
$\mathcal{L}$**(reg. aut.) = UNTIME(**$\mathcal{L}$**(timed aut.))**

where $\mathrm{UNTIME}((a_1, t_1)(a_2, t_2)\dots) = a_1 a_2 \dots$

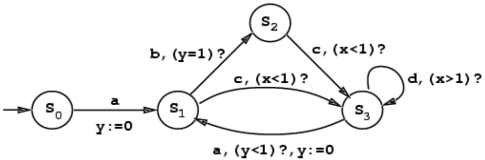


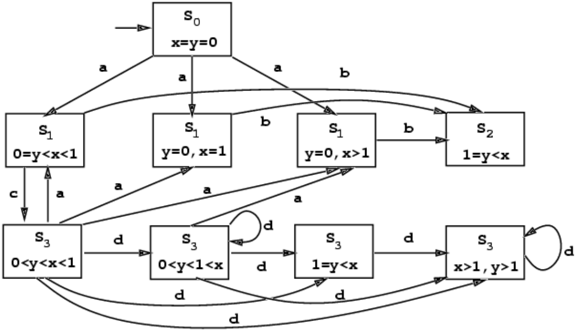
timed automaton

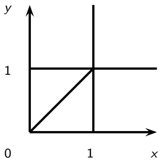
large (but finite) automaton
(region automaton)

$$\mathcal{L}(\textbf{reg. aut.}) = \textbf{UNTIME}(\mathcal{L}(\textbf{timed aut.}))$$

An example [AD94]







PSPACE membership

The size of the region graph is in $\mathcal{O}(|X|!.2^{|X|})$

- **One configuration:** a discrete location + a region

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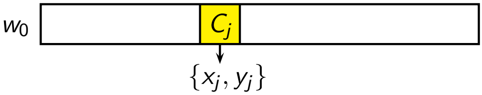
$\rightsquigarrow$ in NPSPACE, thus in PSPACE

PSPACE-hardness

$$\left.\begin{array}{c}\mathcal{M}\ \text{LBTM}\\ w_0 \in \{a,b\}^*\end{array}\right\} \rightsquigarrow \begin{array}{l}A_{\mathcal{M},w_0}\ \text{s.t. } \mathcal{M} \text{ accepts } w_0 \text{ iff the final state}\\ \text{of } A_{\mathcal{M},w_0} \text{ is reachable}\end{array}$$



C_j contains an "a" if $x_j = y_j$
C_j contains a "b" if $x_j < y_j$

(these conditions are invariant by time elapsing)

LBTM: linearly bounded Turing machine (a witness for PSPACE-complete problems)

PSPACE-hardness (cont.)

If $q \xrightarrow{\alpha,\alpha',\delta} q'$ is a transition of $\mathcal{M}$, then for each position i of the tape, we have a transition

$$(q,i) \xrightarrow{g,r:=0} (q',i')$$

where:

- g is $x_i = y_i$ (resp. $x_i < y_i$) if $\alpha = a$ (resp. $\alpha = b$)
- $r = \{x_i, y_i\}$ (resp. $r = \{x_i\}$) if $\alpha' = a$ (resp. $\alpha' = b$)
- $i' = i + 1$ (resp. $i' = i - 1$) if δ is right and $i < n$ (resp. left)

Enforcing time elapsing: on each transition, add the condition $t = 1$ and clock t is reset.

Initialization: init $\xrightarrow{t=1, r_0:=0}$ $(q_0, 1)$ where $r_0 = \{x_i \mid w_0[i] = b\} \cup \{t\}$

Termination: $(q_f, i) \longrightarrow$ end

The case of single-clock timed automata

Exercise [LMS04]

Think of the special case of single-clock timed automata. Can we do better than PSPACE?

[LMS04] Laroussinie, Markey, Schnoebelen. Model checking timed automata with one or two clocks *(CONCUR'04)*.

Consequence of region automata construction

Region automata:

correct finite (and exponential) abstraction for checking reachability/Büchi-like properties.

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correct finite (and exponential) abstraction for checking reachability/Büchi-like properties.

However...

everything can not be reduced to finite automata...

A model not far from undecidability

Some bad news...

- Language universality is undecidable [AD90]
- Language inclusion is undecidable [AD90]
- Complementability is undecidable [Tri03,Fin06]
- ...

[Tri03] Tripakis. Folk theorems on the determinization and minimization of timed automata *(FORMATS'03)*.
[Fin06] Finkel. Undecidable problems about timed automata *(FORMATS'06)*.
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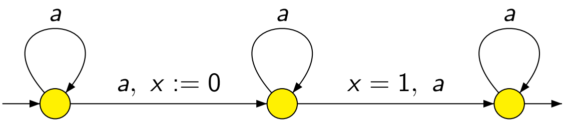

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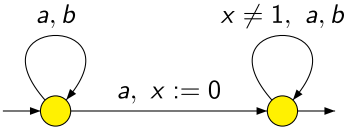

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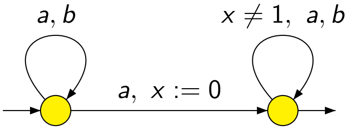

UNTIME $\left(\overline{L} \cap \{(a^* b^*, \tau) \mid \text{all } a's \text{ happen before 1 and no two } a's \text{ simultaneously}\}\right)$ is not regular **(exercise!)**

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[AM04] Alur, Madhusudan. Decision problems for timed automata: A survey *(SFM-04:RT))*.

The two-counter machine

Definition

A two-counter machine is a finite set of instructions over two counters (c and d):

- Incrementation:
  $\texttt{(p):}\quad c := c+1;\ \texttt{goto (q)}$
- Decrementation:
  $\texttt{(p):}\quad \texttt{if } c > 0 \texttt{ then } c := c-1;\ \texttt{goto (q) else goto (r)}$

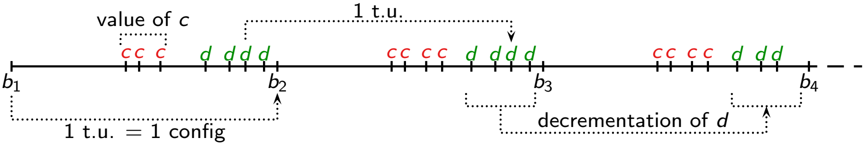
Theorem **[Minsky 67]**

The halting problem for two counter machines is undecidable.

Undecidability of universality

Theorem [AD90]

Universality of timed automata is undecidable.

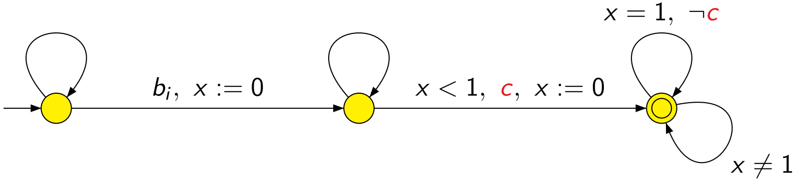


- one configuration is encoded in one time unit
- number of c's: value of counter c
- number of d's: value of counter d
- one time unit between two corresponding c's (resp. d's)

$\rightsquigarrow$ **We encode "non-behaviours" of a two-counter machine**

Example

Module to check that if instruction i does not decrease counter c, then all actions c appearing less than 1 t.u. after b_i has to be followed by an other c 1 t.u. later.



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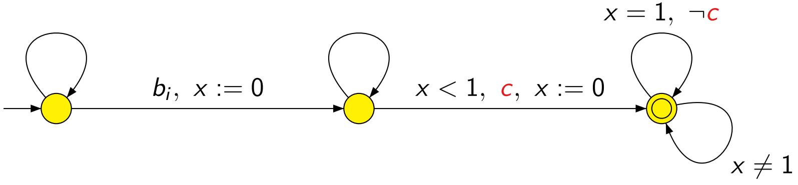

The union of all small modules is not universal
iff
The two-counter machine has a recurring computation

Partial conclusion

- This idea of a finite bisimulation quotient has been applied to many “timed” or “hybrid” systems:
  - various extensions of timed automata
    [Bérard,Diekert,Gastin,Petit 1998] [Choffrut,Goldwurm 2000]
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  - $\cdots$
- Note however that it might be hard to prove there is a finite bisimulation quotient!

Outline

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A general model: hybrid systems

[Hen96]

What is a hybrid system?

a discrete control (the mode of the system)
$+$ a continuous evolution within a mode (given by variables)

[Hen96] Henzinger. The theory of hybrid automata *(LICS'96)*.

A general model: hybrid systems

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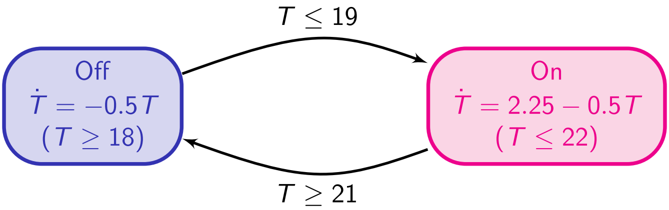
Example (The thermostat)

A simple thermostat, where T (the temperature) depends on the time:



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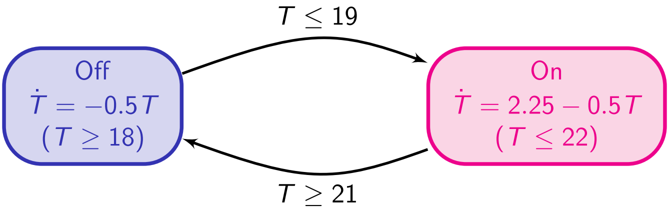

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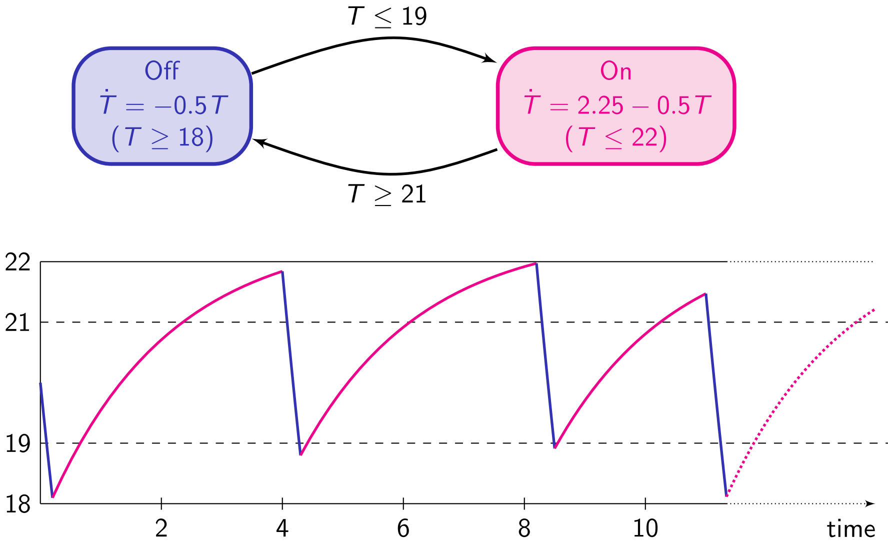
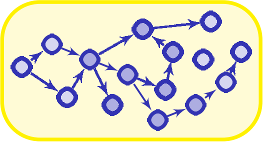


Ok...



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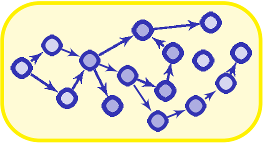

Easy...

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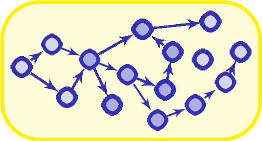

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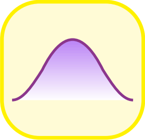

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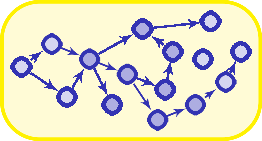

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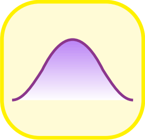
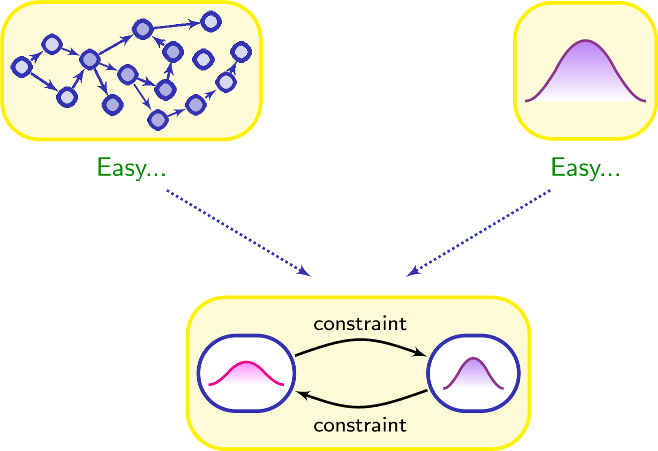


Easy...

Ok... but?



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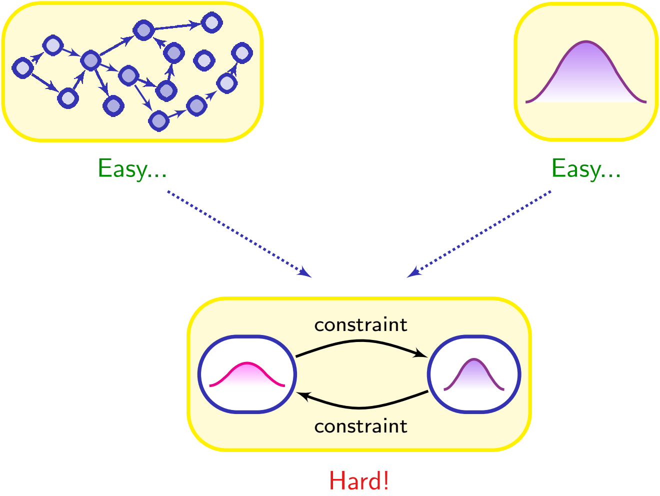

What about decidability?

$\rightsquigarrow$ almost everything is undecidable

Negative results [HKPV95]

- The class of hybrid systems with clocks and only one variable having possibly two slopes $k_1 \neq k_2$ is undecidable.
- The class of *stopwatch* automata is undecidable.

[HKPV95] Henzinger, Kopke, Puri, Varaiya. What's decidable about hybrid automata? *(STOC'95)*.

Outline

Role of diagonal constraints

$$\boxed{x - y \sim c \quad \text{and} \quad x \sim c}$$

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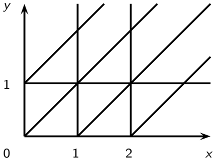
- **Decidability:** yes, using the region abstraction



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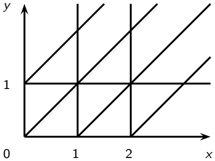
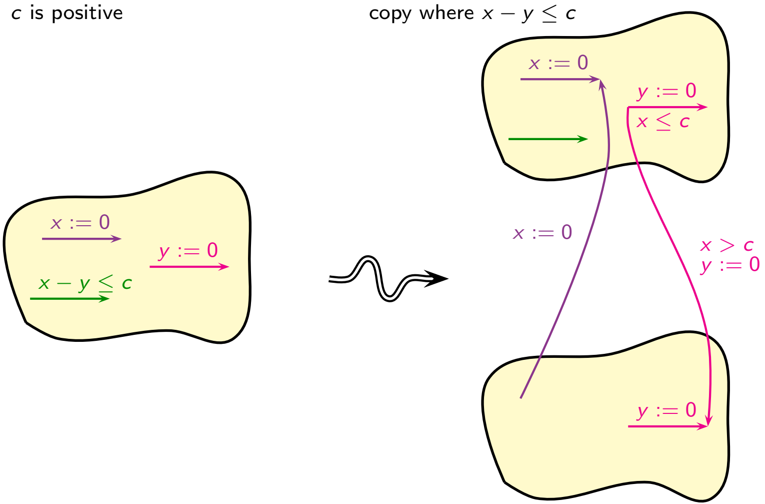


- **Expressiveness:** no additional expressive power

Role of diagonal constraints (cont.)



copy where $x - y > c$

$\rightsquigarrow$ proof in [BDGP98]

[BDGP98] Bérard, Diekert, Gastin, Petit. Characterization of the expressive power of silent transitions in timed automata *(Fundamenta Informaticae)*.

Role of diagonal constraints (cont.)

Exercise [BC05]

Consider, for every positive integer n, the timed language:

$$\mathcal{L}_n = \{(a, t_1) \dots (a, t_{2^n}) \mid 0 < t_1 < \cdots < t_{2^n} < 1\}$$

1. Construct a timed automaton with diagonal constraints which recognizes $\mathcal{L}_n$. What is the size of this automaton?
2. Idem without diagonal constraints. Can you do better?
3. Conclude.

[BC05] Bouyer, Chevalier. On conciseness of extensions of timed automata *(Journal of Automata, Languages and Combinatorics)*.

Adding silent actions

$$\xrightarrow{g,\ \varepsilon,\ C := 0}$$

[BDGP98]

Adding silent actions

$$\xrightarrow{g, \varepsilon, C := 0}$$

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- **Decidability:** yes

(actions have no influence on region automaton construction)

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$$\xrightarrow{g,\, \varepsilon,\, C := 0}$$

[BDGP98]

- **Decidability:** yes

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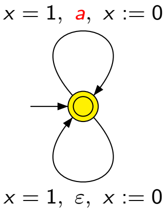
- **Expressiveness:** strictly more expressive!



Adding additive constraints

$\boxed{x + y \sim c \quad \text{and} \quad x \sim c}$ **[BD00]**

[BD00] Bérard, Dufourd. Timed automata and additive clock constraints *(Information Proecessing Letters)*.

Adding additive constraints

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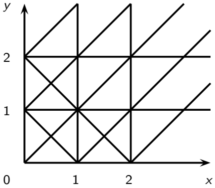

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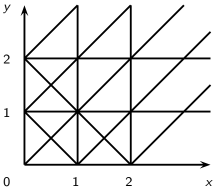
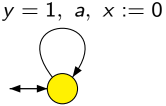


  - for four clocks (or more), undecidable!

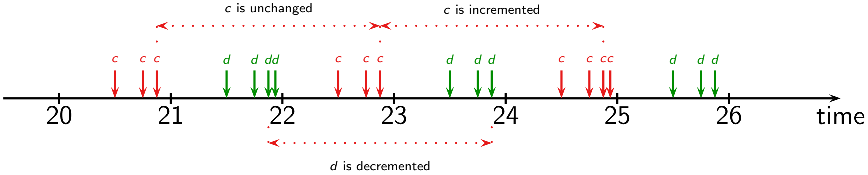
- **Expressiveness:** more expressive! (even using two clocks)

$$\{(a^n, t_1 \dots t_n) \mid n \geq 1 \text{ and } t_i = 1 - \frac{1}{2^i}\}$$



[BD00] Bérard, Dufourd. Timed automata and additive clock constraints *(Information Proecessing Letters)*.

Undecidability proof



$\rightsquigarrow$ simulation of
- decrementation of a counter
- incrementation of a counter

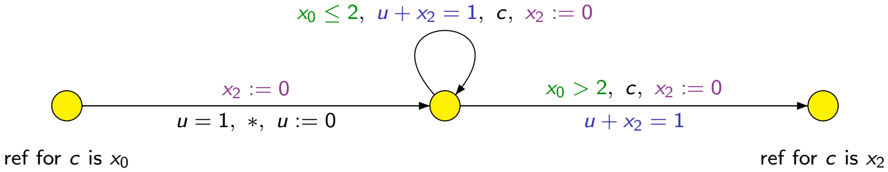
We will use 4 clocks:

- u, "tic" clock (each time unit)
- x_0, x_1, x_2: reference clocks for the two counters

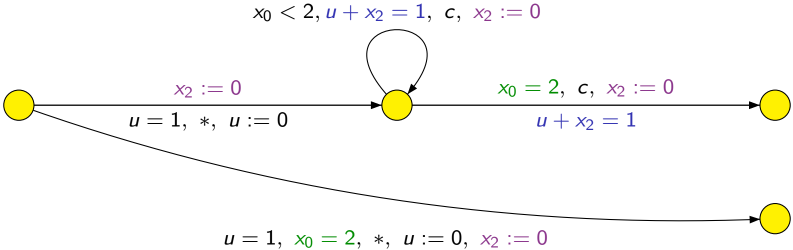
"x_i reference for c" $\equiv$ "the last time x_i has been reset is the last time action c has been performed"

Undecidability proof (cont.)

- **Incrementation of counter c:**

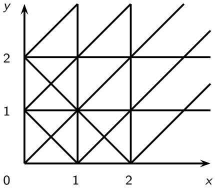


- **Decrementation of counter c:**



Adding constraints of the form $x + y \sim c$

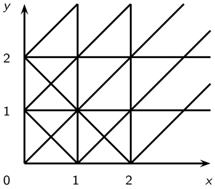
- Two clocks: decidable using the abstraction



- Four clocks (or more): undecidable!

Adding constraints of the form $x + y \sim c$

- Two clocks: decidable using the abstraction



- Three clocks: **open question**

- Four clocks (or more): undecidable!

Adding new operations on clocks

Several types of updates: $x := y + c$, $x :< c$, $x :> c$, etc...

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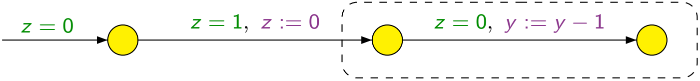
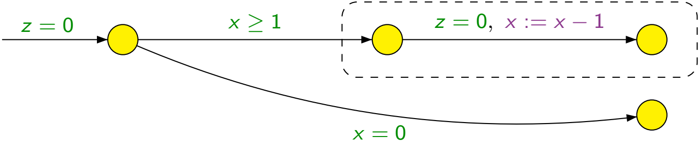
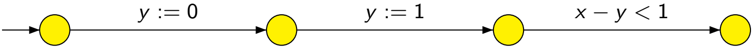
- Only decrementation also leads to undecidability
  - **Incrementation of counter x**



  - **Decrementation of counter x**



Decidability



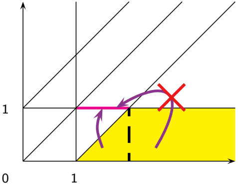


image by $y := 1$

$\rightsquigarrow$ the bisimulation property is not met

The classical region automaton construction is not correct.

Decidability (cont.)

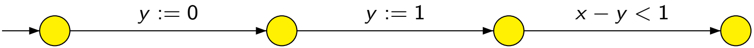
$$\begin{array}{lcl} \mathcal{A} & \rightsquigarrow & \text{Diophantine linear inequations system} \\ & \rightsquigarrow & \text{is there a solution?} \\ & \rightsquigarrow & \text{if yes, belongs to a decidable class} \end{array}$$

Examples:

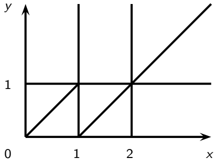
- constraint $x \sim c$ — $c \leq \max_x$
- constraint $x - y \sim c$ — $c \leq \max_{x,y}$
- update $x :\sim y + c$ — $\max_x \leq \max_y + c$
  and for each clock z, $\max_{x,z} \geq \max_{y,z} + c$, $\max_{z,x} \geq \max_{z,y} - c$
- update $x :< c$ — $c \leq \max_x$
  and for each clock z, $\max_z \geq c + \max_{z,x}$

The constants $(\max_x)$ and $(\max_{x,y})$ define a set of regions.

Decidability (cont.)



$$\left\{\begin{array}{l} \max_y \geq 0 \\ \max_x \geq 0 + \max_{x,y} \\ \max_y \geq 1 \\ \max_x \geq 1 + \max_{x,y} \\ \max_{x,y} \geq 1 \end{array}\right. \qquad \text{implies} \qquad \left\{\begin{array}{l} \max_x = 2 \\ \max_y = 1 \\ \max_{x,y} = 1 \\ \max_{y,x} = -1 \end{array}\right.$$

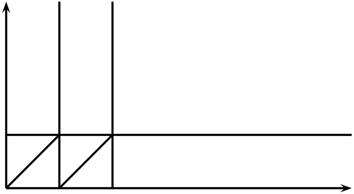


The bisimulation property is met.

What's wrong when undecidable?

Decrementation $x := x - 1$

$$\max_x \leq \max_x - 1$$



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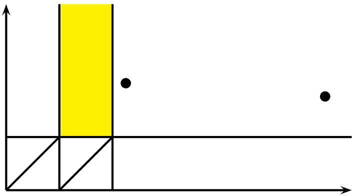

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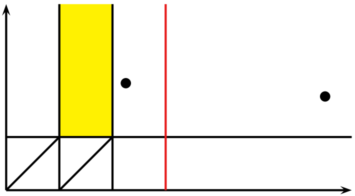

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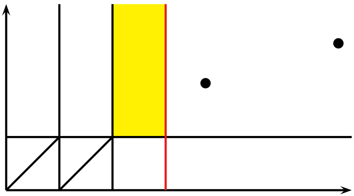

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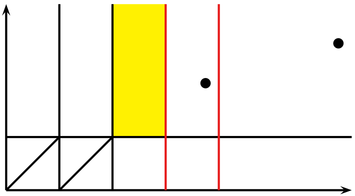

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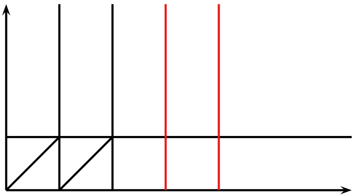

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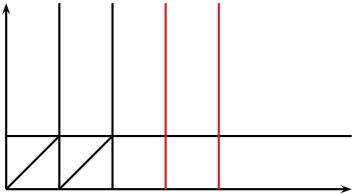

etc...

Decidability (cont.)

<table>
<tr><th></th><th>Diagonal-free constraints</th><th>General constraints</th></tr>
<tr><td>$x := c,\ x := y$</td><td rowspan="3">PSPACE-complete</td><td>PSPACE-complete</td></tr>
<tr><td>$x := x + 1$</td><td rowspan="3">Undecidable</td></tr>
<tr><td>$x := y + c$</td></tr>
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<tr><td>$y + c <: x :< y + d$</td></tr>
<tr><td>$y + c <: x :< z + d$</td><td>Undecidable</td></tr>
</table>

[BDFP00]

[BDFP00] Bouyer, Dufourd, Fleury, Petit. Updatable timed automata *(Theoretical Computer Science)*.

Outline

The example of alternating timed automata

Alternating timed automata $\equiv$ ATA

[LW05,OW05]

[LW05] Lasota, Walukiewicz. Alternating timed automata *(FoSSaCS'05)*.
[OW05] Ouaknine, Worrell. On the decidability of Metric Temporal Logic *(LICS'05)*.

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Example

"No two a's are separated by 1 unit of time"

$$\begin{cases} \ell_0, a, \mathit{true} & \mapsto \quad \ell_0 \wedge (x := 0, \ell_1) \\ \ell_1, a, x \neq 1 & \mapsto \quad \ell_1 \\ \ell_1, a, x = 1 & \mapsto \quad \ell_2 \\ \ell_2, a, \mathit{true} & \mapsto \quad \ell_2 \end{cases}$$

$$\begin{cases} \ell_0 \text{ initial state} \\ \ell_0, \ell_1 \text{ final states} \\ \ell_2 \text{ losing state} \end{cases}$$

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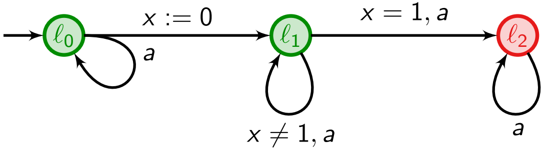

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Theorem

- Emptiness of ATA is undecidable.
- Emptiness of one-clock ATA is decidable, but non-primitive recursive.
- Emptiness for Büchi properties of one-clock ATA is undecidable.
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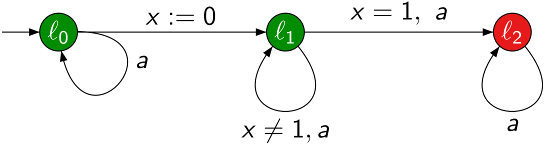
Lower bound: simulation of a lossy channel system... [Sch02]

[Sch02] Schnoebelen. Verifying lossy channel systems has nonprimitive recursive complexity *(Information Processing Letters)*.

Example



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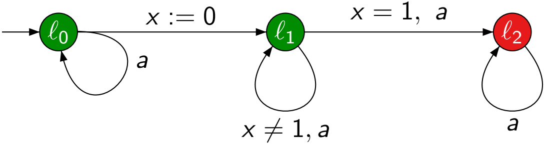
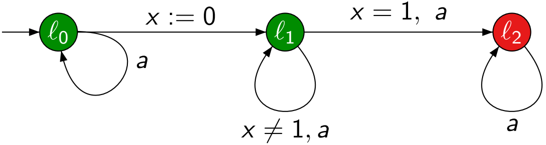


Execution over timed word $(a, .3)(a, .8)(a, 1.4)(a, 1.8)(a, 2)$

Example



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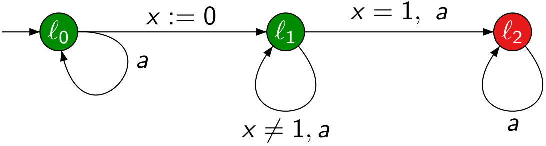
$$\{\ (\ell_0, 0)\ \}$$

Example



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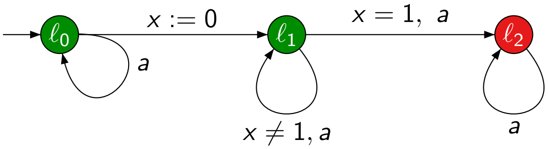
$$\{\ \boxed{(\ell_0, 0)}\ \}$$
$$\downarrow$$
$$\{\ \boxed{(\ell_0, .3)}, \boxed{(\ell_1, 0)}\ \}$$

Example



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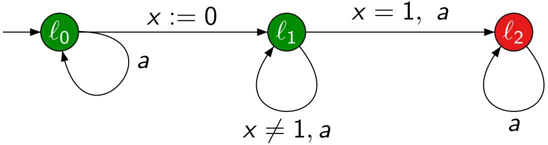
$$\downarrow$$
$$\{\boxed{(\ell_0, .3)}, \boxed{(\ell_1, 0)}\}$$
$$\downarrow$$
$$\{\boxed{(\ell_0, .8)}, \boxed{(\ell_1, 0)}, \boxed{(\ell_1, .5)}\}$$

Example



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$$\downarrow$$
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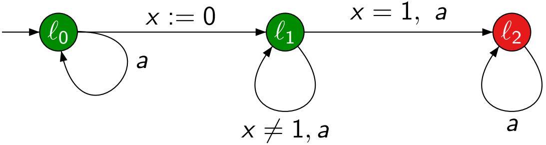
$$\downarrow$$
$$\{ \boxed{(\ell_0, 1.4)}, \boxed{(\ell_1, 0)}, \boxed{(\ell_1, .6)}, \boxed{(\ell_1, 1.1)} \}$$

Example



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$$\downarrow$$
$$\{(\ell_0, .3), (\ell_1, 0)\}$$
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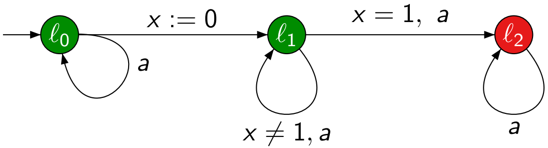

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$$\downarrow$$
$$\{(\ell_0, 2), (\ell_1, 0), (\ell_1, .2), (\ell_1, .6), (\ell_2, 1.2), (\ell_1, 1.7)\}$$

An abstraction

A configuration = a finite set of pairs (ℓ, x)

$$(\ell, 0) \quad (\ell, 0.3) \quad (\ell, 1.2) \quad (\ell, 2.3) \quad (\ell', 0.4) \quad (\ell', 1) \quad (\ell', 0.8)$$

An abstraction

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$(\ell, 0)$ $(\ell, 0.3)$ $(\ell, 1.2)$ $(\ell, 2.3)$ $(\ell', 0.4)$ $(\ell', 1)$ $(\ell', 0.8)$

$\{(\ell, 0), (\ell', 1)\}$

0.0

An abstraction

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$(\ell, 0)$ $(\ell, 0.3)$ $(\ell, 1.2)$ $(\ell, 2.3)$ $(\ell', 0.4)$ $(\ell', 1)$ $(\ell', 0.8)$

$\{(\ell, 0), (\ell', 1)\}$ $\{(\ell, 1)\}$

0.0 0.2

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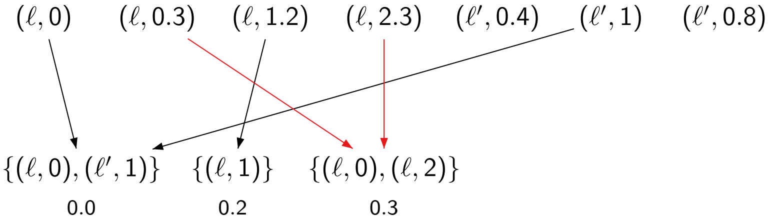

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$$\underset{0.0}{\{(\ell,0),(\ell',1)\}} \quad \underset{0.2}{\{(\ell,1)\}} \quad \underset{0.3}{\{(\ell,0),(\ell,2)\}} \quad \underset{0.4}{\{(\ell',0)\}}$$

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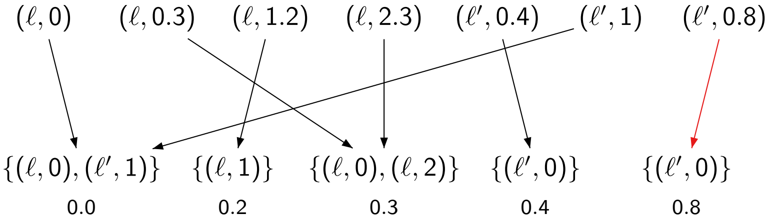

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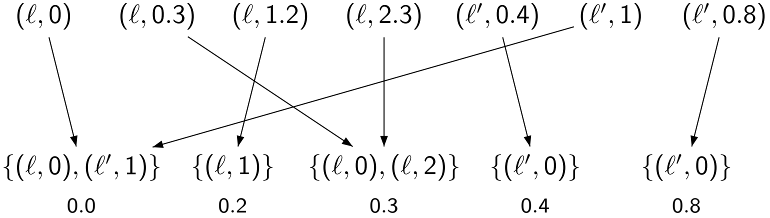

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$\{(\ell,0),(\ell',1)\}$	$\{(\ell,1)\}$	$\{(\ell,0),(\ell,2)\}$	$\{(\ell',0)\}$	$\{(\ell',0)\}$
0.0	0.2	0.3	0.4	0.8

Abstracted into: $\quad \{(\ell,0),(\ell',1)\} \cdot \{(\ell,1)\} \cdot \{(\ell,0),(\ell,2)\} \cdot \{(\ell',0)\} \cdot \{(\ell',0)\}$

Abstract transition system

$\{(\ell, 0), (\ell', 1)\} \cdot \{(\ell, 1)\} \cdot \{(\ell, 0), (\ell, 2)\} \cdot \{(\ell', 0)\} \cdot \{(\ell', 0)\}$

Abstract transition system

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Time successors:

Abstract transition system

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Transition $\ell \xrightarrow{x>2,x:=0} \ell''$**:**

Abstract transition system

$\{(\ell,0),(\ell',1)\} \cdot \{(\ell,1)\} \cdot \{(\ell,0),(\ell,2)\} \cdot \{(\ell',0)\} \cdot \{(\ell',0)\}$

Time successors:

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Correctness?

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The previous abstraction is (almost) a time-abstract bisimulation.

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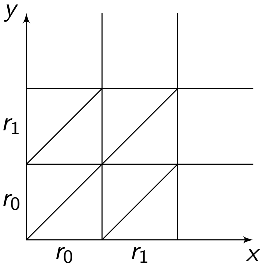
Alternative

The abstract transition system can be simulated by a kind of FIFO channel machine.

A digression on timed automata



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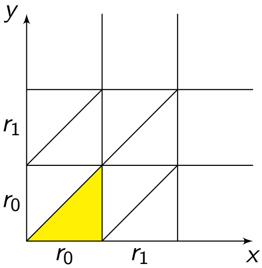

$x, y \in r_0$, $\{y\} < \{x\}$

$\boxed{(y, r_0)} \cdot \boxed{(x, r_0)}$

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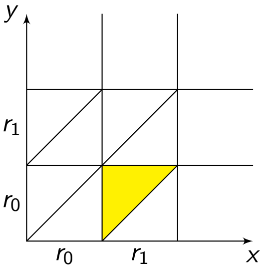

$x \in r_1,\ y \in r_0,\ \{x\} < \{y\}$

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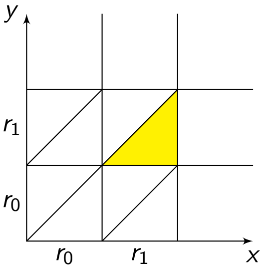

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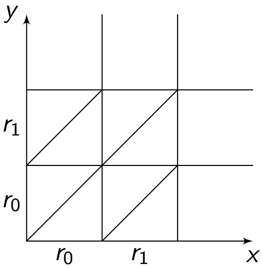

The classical region automaton can be simulated by a channel machine (with a single bounded channel).

Partial conclusion

Similar technics apply to:

- networks of single-clock timed automata **[Abdulla,Jonsson 1998]**

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Outline

1. Introduction
2. The timed automaton model
3. Timed automata, decidability issues
4. How far can we extend the model and preserve decidability?
   - Hybrid systems
   - Smaller extensions of timed automata
   - An alternative way of proving decidability
5. Timed automata in practice
6. Conclusion

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- instead, symbolic computations are performed

What do we need?

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Finite representation of infinite sets of configurations

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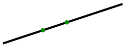

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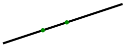

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    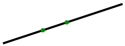

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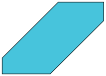

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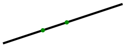

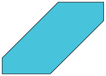

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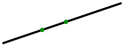
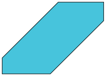




- Need of abstractions, heuristics, etc...

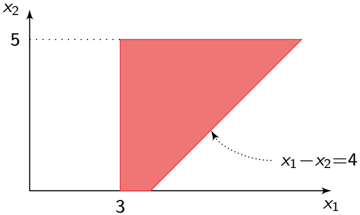
An example of computation with HyTech

```
command:  /usr/local/bin/hytech gas_burner
=================================================================
HyTech:  symbolic model checker for embedded systems
Version 1.04f (last modified 1/24/02) from v1.04a of 12/6/96
For more info:
    email:  hytech@eecs.berkeley.edu
    http://www.eecs.berkeley.edu/~tah/HyTech
Warning:  Input has changed from version 1.00(a).  Use -i for more info
=================================================================

Backward computation
Number of iterations required for reachability:  6
System satisfies non-leaking duration property

Location:  not_leaking
x >= 0 & t >= 3 & y <= 20t & y >= 0
| x + 20t >= y + 11 & y <= 20t + 19 & t >= 2 & x >= 0 & y >= 0
| y >= 0 & t >= 1 & x + 20t >= y + 22 & y <= 20t + 8 & x >= 0
| y >= 0 & x + 20t >= y + 33 & 20t >= y + 3 & x >= 0
Location:  leaking
19x + y <= 20t + 19 & y >= x + 59 & x <= 1 & x >= 0
| t >= x + 2 & x <= 1 & y >= 0 & 19x + y <= 20t + 19 & x >= 0
| t >= x + 1 & x <= 1 & y >= 0 & 19x + y <= 20t + 8 & x >= 0
| 20t >= 19x + y + 3 & y >= 0 & x <= 1 & x >= 0

=================================================================
Max memory used = 0 pages = 0 bytes = 0.00 MB
Time spent = 0.02u + 0.00s = 0.02 sec total

=================================================================
```

Zones: A symbolic representation for timed systems

Example of a zone and its DBM representation

$$Z \;=\; (x_1 \geq 3) \;\wedge\; (x_2 \leq 5) \;\wedge\; (x_1 - x_2 \leq 4)$$



$$\begin{array}{cc} & \begin{array}{ccc} x_0 & x_1 & x_2 \end{array} \\ \begin{array}{c} x_0 \\ x_1 \\ x_2 \end{array} & \begin{pmatrix} \infty & -3 & \infty \\ \infty & \infty & 4 \\ 5 & \infty & \infty \end{pmatrix} \end{array}$$

DBM: Difference Bound Matrice [BM83,Dill89]

[BM83] Berthomieu, Menasche. An enumerative approach for analyzing time Petri nets *World Comupter Congress*.
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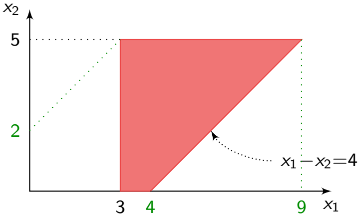

$$\begin{array}{c} \\ x_0 \\ x_1 \\ x_2 \end{array} \begin{array}{c} \begin{array}{ccc} x_0 & x_1 & x_2 \end{array} \\ \begin{pmatrix} 0 & -3 & 0 \\ 9 & 0 & 4 \\ 5 & 2 & 0 \end{pmatrix} \end{array}$$

"normal form"

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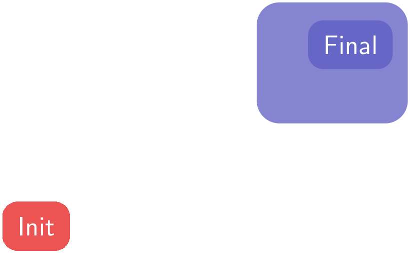

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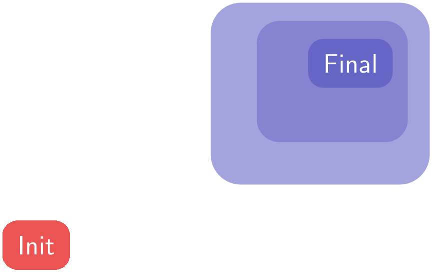

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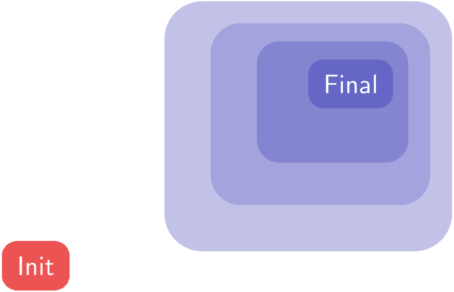
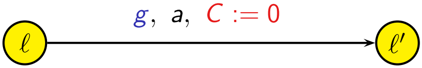


Backward computation

Note on the backward analysis of TA



$$\overleftarrow{[C \leftarrow 0]^{-1}(Z \cap (C = 0)) \cap g} \qquad Z$$

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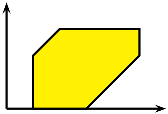
$$\ell \xrightarrow{g,\ a,\ C := 0} \ell'$$

$\overleftarrow{[C \leftarrow 0]^{-1}(Z \cap (C = 0)) \cap g}$ $\qquad$ Z



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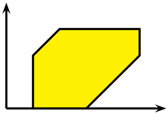

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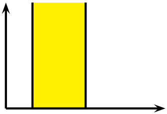

$[C \leftarrow 0]^{-1}(Z \cap (C = 0))$

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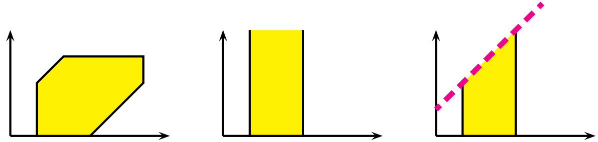

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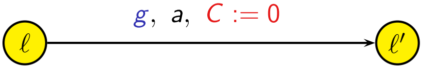

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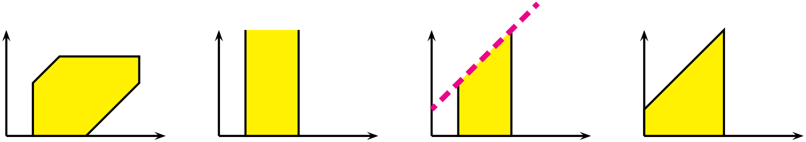

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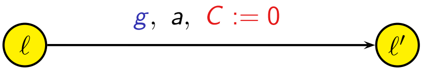

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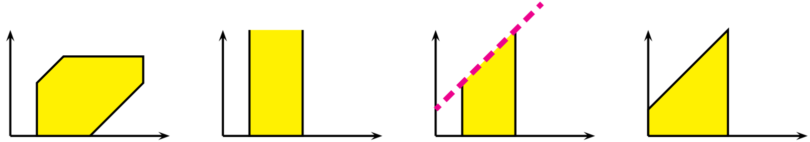

Z $\qquad$ $[C \leftarrow 0]^{-1}(Z \cap (C = 0))$ $\qquad$ $\overleftarrow{[C \leftarrow 0]^{-1}(Z \cap (C = 0)) \cap g}$

☺ the backward computation always terminates!

☺☺ ... and it is correct!!!

Note on the backward analysis (cont.)

If $\mathcal{A}$ is a timed automaton, we construct its corresponding set of regions.

Because of the bisimulation property, we get that:

"Every set of valuations which is computed along the backward computation is a finite union of regions"

Note on the backward analysis (cont.)

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Let R be a region. Assume:

- $v \in \overleftarrow{R}$ (for ex. $v + t \in R$)
- $v' \equiv_{\text{reg.}} v$

There exists t' s.t. $v' + t' \equiv_{\text{reg.}} v + t$, which implies that $v' + t' \in R$ and thus $v' \in \overleftarrow{R}$.

Note on the backward analysis (cont.)

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But, the backward computation is not so nice, when also dealing with integer variables...

$$i := j.k + \ell.m$$

Forward computation

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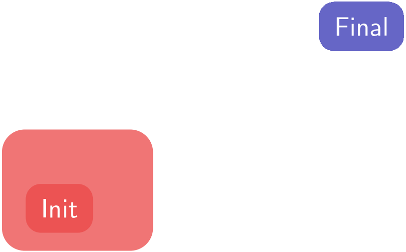

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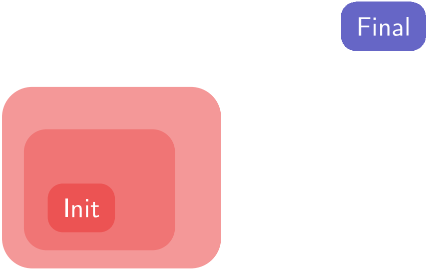

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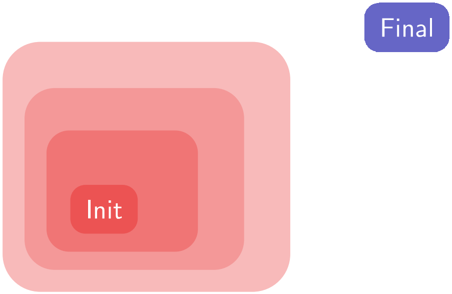

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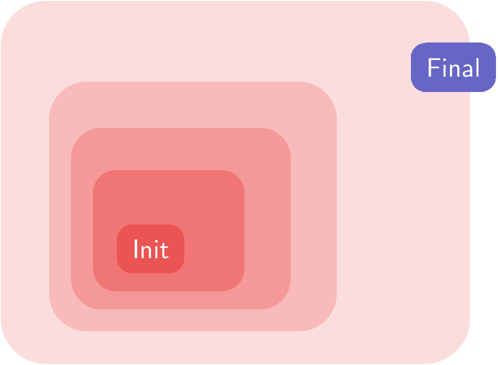

Forward analysis of timed automata

$$\ell \xrightarrow{\;g,\ a,\ C := 0\;} \ell'$$

zones $\quad Z \quad\quad [C \leftarrow 0](\overrightarrow{Z} \cap g)$

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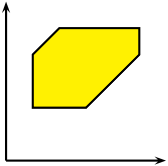

Z

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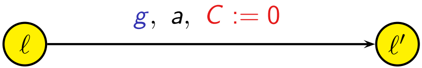

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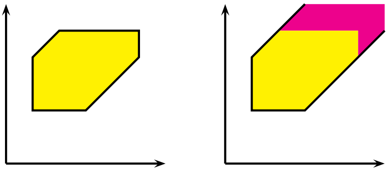

$Z \qquad \overrightarrow{Z}$

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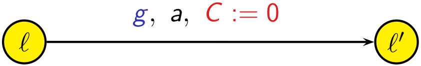

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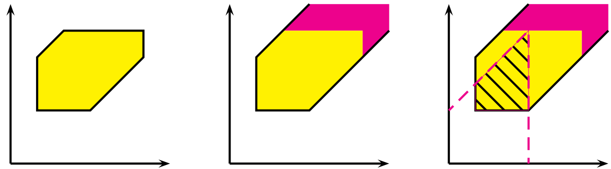

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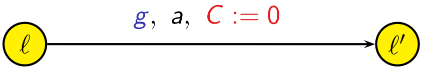

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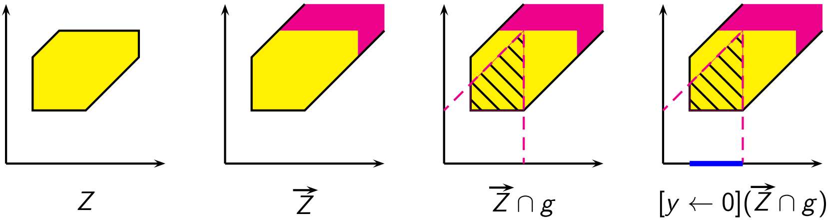

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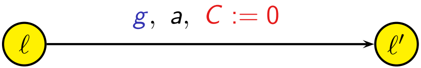

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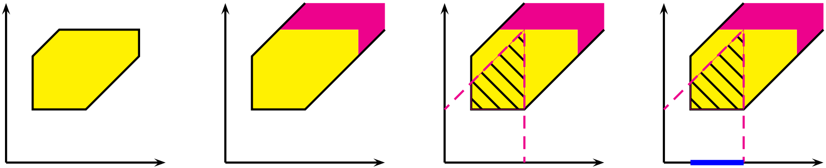
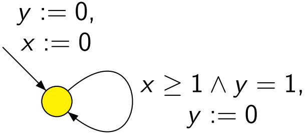
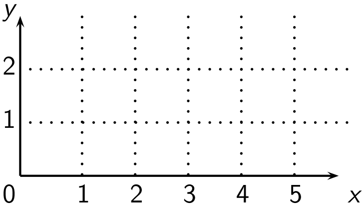


$Z \qquad \overrightarrow{Z} \qquad \overrightarrow{Z} \cap g \qquad [y \leftarrow 0](\overrightarrow{Z} \cap g)$

☹ the forward computation may not terminate...

Non termination of the forward analysis





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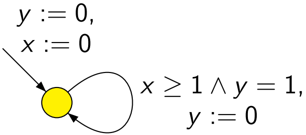

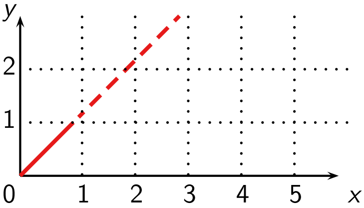

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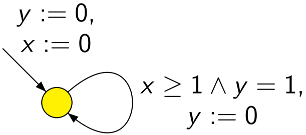

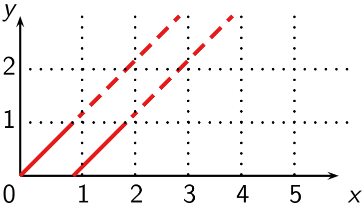

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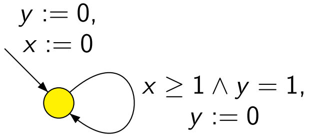

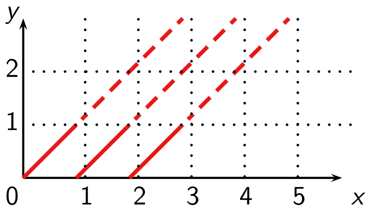

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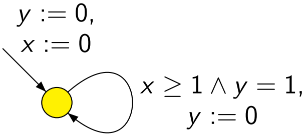

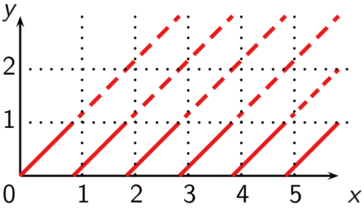

$\rightsquigarrow$ an infinite number of steps...

"Solutions" to this problem

(f.ex. in **[Larsen,Pettersson,Yi 1997]** or in **[Daws,Tripakis 1998]**)

- **inclusion checking**: if $Z \subseteq Z'$ and Z' already considered, then we don't need to consider Z

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- **activity**: eliminate redundant clocks **[Daws,Yovine 1996]**

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$$q \xrightarrow{g,a,C:=0} q' \quad \text{implies} \quad \mathsf{Act}(q) = \mathsf{clocks}(g) \cup (\mathsf{Act}(q') \setminus C)$$

...

"Solutions" to this problem (cont.)

- **convex-hull approximation**: if Z and Z' are computed then we overapproximate using "$Z \sqcup Z'$".

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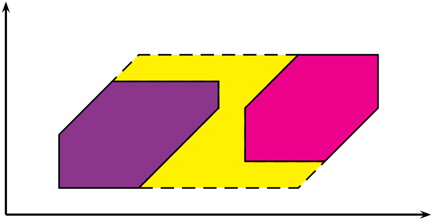

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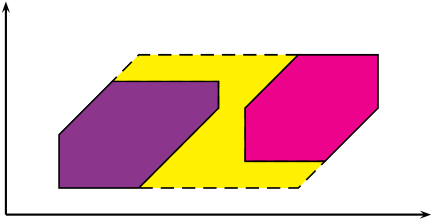
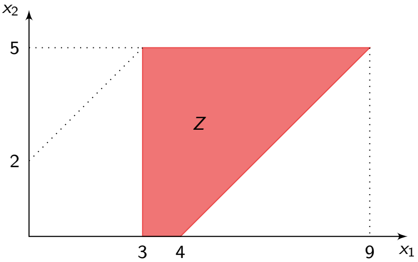


- **extrapolation**, an abstraction operator on zones

An abstraction: the extrapolation operator

$\mathsf{Approx}_2(Z)$: "the smallest zone containing Z that is defined only with constants no more than 2"



$$\begin{pmatrix} 0 & -3 & 0 \\ 9 & 0 & 4 \\ 5 & 2 & 0 \end{pmatrix}$$

$\leadsto$ The extrapolation operator ensures termination of the computation!

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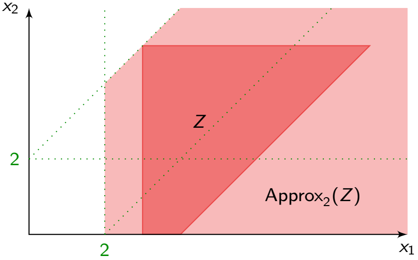

$$\begin{pmatrix} 0 & -3 & 0 \\ 9 & 0 & 4 \\ 5 & 2 & 0 \end{pmatrix} \xRightarrow{\mathsf{Approx}_2} \begin{pmatrix} 0 & -2 & 0 \\ \infty & 0 & \infty \\ \infty & 2 & 0 \end{pmatrix}$$

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Classical algorithm, focus on correctness

Challenge

Choose a **good** constant for the extrapolation so that the forward computation is correct. Classical algorithm, the choice goes to the maximal constant.

[Bou03] Bouyer. Untameable timed automata! *(STACS'03)*.
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The classical algorithm is correct for diagonal-free timed automata.

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This theorem does not extend to timed automata using diagonal clock constraints... [Bou03,Bou04]

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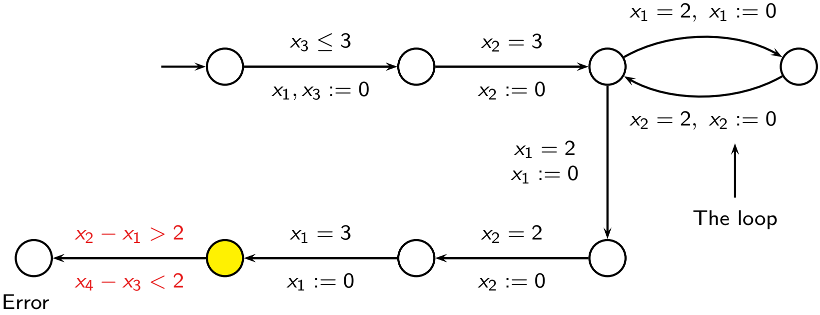
A problematic automaton



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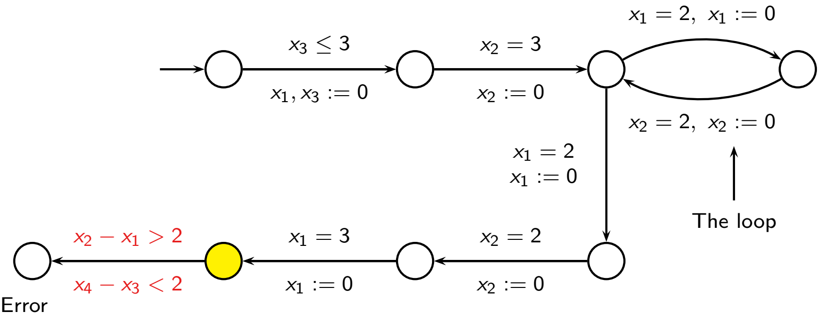

$$\begin{cases} v(x_1) = 0 \\ v(x_2) = d \\ v(x_3) = 2\alpha + 5 \\ v(x_4) = 2\alpha + 5 + d \end{cases}$$

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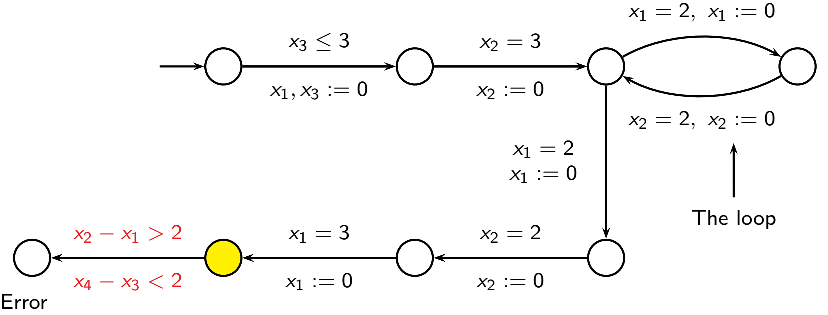

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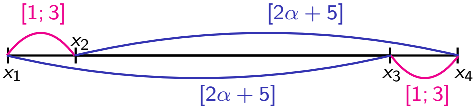
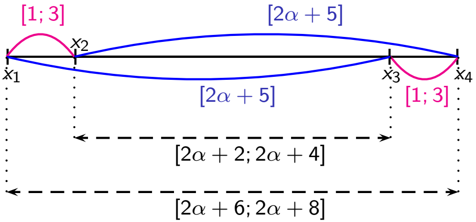


The problematic zone



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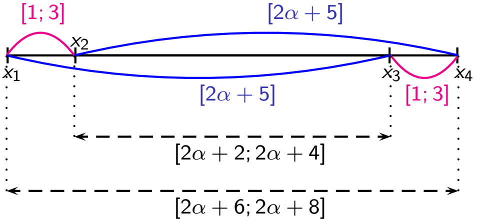

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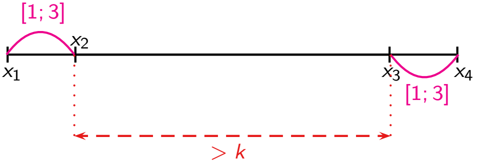

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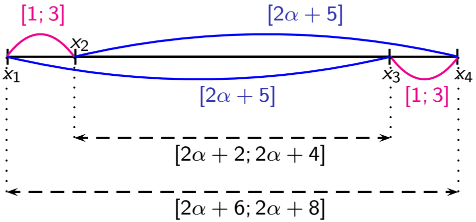

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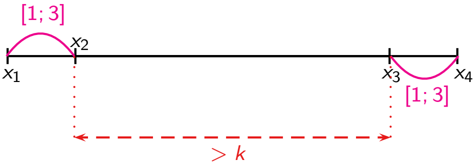

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Hence, any choice of constant is erroneous!

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For the previous automaton,

no abstraction operator can satisfy all these criteria!

Why that?

Assume there is a "nice" operator Abs.

The set $\{M \text{ DBM representing a zone } \mathrm{Abs}(Z)\}$ is finite.

$\rightsquigarrow$ k the max. constant defining one of the previous DBMs

We get that, for every zone Z,

$$Z \subseteq \mathrm{Extra}_k(Z) \subseteq \mathrm{Abs}(Z)$$

Problem!

Open questions:
- which conditions can be made weaker?
- find a clever termination criterium?
- use an other data structure than zones/DBMs?

Improving the classical algorithm

- the extrapolation operator can be made coarser:
  - local extrapolation constants [BBFL03];
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$\rightsquigarrow$ the tool Uppaal is under development since 1995...

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Outline

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- Several technics for proving decidability of real-time systems
  - finite time-abstract bisimulation
  - well-quasi-order on the time-abstract transition system
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- quantitative model-checking,
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