EFX Exists for Three Agents

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Discrete Fair Division

Given

- Set $[n]$ of n agents.
- Set M of m **indivisible** goods, e.g., a car, a house, a toothbrush.
- Valuations $v_i \colon 2^M \to \mathbb{R}_{\geq 0}$ for every agent i.
  $v_i(\emptyset) = 0$ and $v_i(A) \leq v_i(B)$ for $A \subseteq B$.

Find: A fair partition $X = \langle X_1, X_2, \ldots, X_n \rangle$ of M.

Problem is ubiquitous: Split an estate, divorce settlements, splitting rent, ...

Today: Fairness = Envy-Freeness

X is fair iff for all pairs i and j we have $v_i(X_j) \leq v_i(X_i)$, i.e., every agent likes their own bundle at least as much as any other bundle.

This is too much to ask for: Consider two agents having positive valuation towards a single good.

Relaxation: Envy-Freeness upto One Good (EF1) [Budish'11]

X is fair iff for all i and j, $v_i(X_j \setminus \{g\}) \leq v_i(X_i)$ for **some** $g \in X_j$.

An EF1-allocation always exists.

Hypothetical dialogue after an inheritance settlement: Brother, I envy you because you are getting a house, a TV set, and a toothbrush.

This is OK, because my envy disappears if I discount the house.

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Does an EFX-Allocation Always Exist?

For two agents, there is always an EFX-allocation (Plaut/Roughgarden).

For three agents and additive valuations, there is always an EFX-allocation (Chaudhury/Garg/M, EC ’20 and JACM ’23)

For three agents, two general valuations, and one additive valuation, there is always an EFX-allocation (Akrami/Alon/Chaudhury/Garg/M/Metha, EC ’23)

For three agents and three general valuations and four or more agents and additive valuations, the question is open.

Since four months, my computer is searching for a counter example for four agents, 17 goods, and general valuations. So far, it checked more than 7 mio instances and did not find one.

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