

***What matters more for image matching  
and the comparison of descriptors:  
invariance and causality requirements or  
repeatability criteria?  
(a case study with SIFT, SURF, SIFER)***

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Saarbrücken, September 2013

# References

Paper on SIFT:

[1] D. Lowe, “Distinctive image features from scale-invariant keypoints,” *International Journal of Computer Vision*, vol. 60, pp. 91–110, 2004.

Paper on SURF:

[2] H. Bay, A. Ess, T. Tuytelaars, and L. Van Gool, “Speeded-up robust features (surf),” *Computer vision and image understanding*, vol. 110, no. 3, pp. 346–359, 2008.

Paper on SIFER:

[3] P. Mainali, G. Lafruit, Q. Yang, B. Geelen, L. Gool, and R. Lauwereins, “Sifer: Scale-invariant feature detector with error resilience,” *International Journal of Computer Vision*, pp. 1–26, 2013.

Discussion on scale invariance:

[4] J.-M. Morel and G. Yu, “Is sift scale invariant?,” *Inverse Problems and Imaging*, vol. 5, no. 1, pp. 115–136, 2011.

Review of detectors:

[5] K. Mikolajczyk and C. Schmid, “Scale & affine invariant interest point detectors,” *International Journal of Computer Vision*, vol. 60, no. 1, pp. 63–86, 2004.

Comparison methodology:

[6] K. Mikolajczyk, T. Tuytelaars, C. Schmid, A. Zisserman, J. Matas, F. Schafalitzky, T. Kadir, and L. V. Gool, “A comparison of affine region detectors,” *International Journal of Computer Vision*, vol. 65, no. 1-2, pp. 43–72, 2005.

Modified keypoint detector comparison methodology (overlap)

[7] S. Ehsan, N. Kanwal, A. F. Clark, and K. D. McDonald-Maier, “Measuring the coverage of interest point detectors,” in *Image Analysis and Recognition*, pp. 253–261, 2011.

Modified keypoint comparison methodology (precision)

[8] K. Cordes, B. Rosenhahn, and J. Ostermann, “Increasing the accuracy of feature evaluation benchmarks using differential evolution,” in *Differential Evolution (SDE), 2011 IEEE Symposium on*, pp. 1–8, 2011.

# Summary

- SIFT, SURF, SIFER, their invariances properties
- The repeatability criteria
- Possible bias in the performance measure.
- A suggested correction

# SIFT, SURF, SIFER

share a general “scale space” framework:

Detection	Extract the 3D extrema from $v(\sigma, x, y)$ , a multi-scale detector of the image $u(x, y)$ .
Description	Extract an image patch around each keypoint $(\sigma, x, y)$ to compute the feature vector.

**SIFT / SURF / SIFER: extrema of the multiscale detector yield the key points or points of interest  $(x,y)$  with an associated scale  $\sigma$**

	<p>Multi-scale detectors</p> $v(\sigma, x, y) = K_\sigma * u(x, y)$
SIFT	$K_\sigma(x, y) = G_{k\sigma}(x, y) - G_\sigma(x, y)$
SURF	<p>box filters used to approximate <math>\sigma^4 \det(\mathcal{H}(G_\sigma))</math></p>
SIFER	$K_\sigma(x, y) = 2\pi\sigma^2 G_\sigma(x, y) \left( \cos\left(\frac{cx}{\sigma}\right) + \cos\left(\frac{cy}{\sigma}\right) \right)$

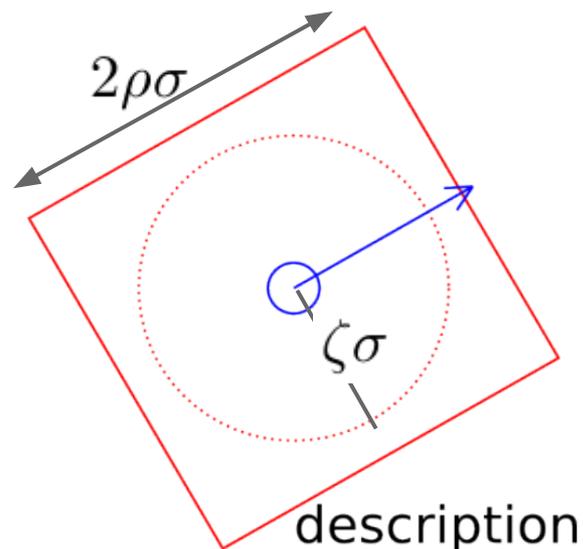
$$G_\sigma(x, y) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}}$$

# SIFT / SURF / SIFER

For each detection in scale-space  $(\sigma, x, y)$

- Assign a principal orientation  $\theta$ .  
(or several orientations in the case of SIFT)
- Extract a truncated Gaussian window centered on  $(x, y)$  and aligned with the orientation  $\theta$ .  
Its standard deviation is  $(\zeta\sigma)$ .  
Its width is  $(2\rho\sigma)$

	$\rho$	$\zeta$
SIFT	6	6
SURF	10	3.3
SIFER	6	6



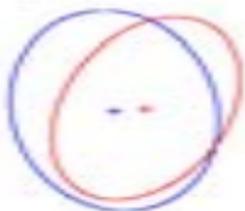
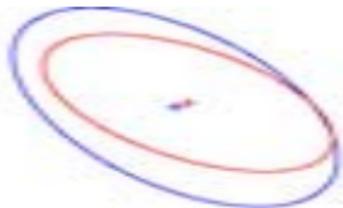
**The classic repeatability criteria: a transform is simulated on a benchmark image, and the detector is applied before and after transform. Then keypoints are compared:**

Depending on the adopted criteria, two detections  $(\sigma_a, \mathbf{x}_a)$  and  $(\sigma_b, \mathbf{x}_b)$  are one repeated detection if

$$1 - \frac{R_{\mu_a} \cap R_{(H^T \mu_b H)}}{R_{\mu_a} \cup R_{(H^T \mu_b H)}} \leq \text{overlap error}_{\max} \quad \left| \quad \left| 1 - s(H)^2 \frac{\min(\sigma_a^2, \sigma_b^2)}{\max(\sigma_a^2, \sigma_b^2)} \right| \leq \text{overlap error}_{\max} \right.$$

$\mu_a, \mu_b$  the characteristic ellipses  $(\mathbf{x}^T \mu \mathbf{x})$ .  $s(H)$  the measured scale factor between  $u_a(\mathbf{x})$  and  $u_b(\mathbf{x})$ .

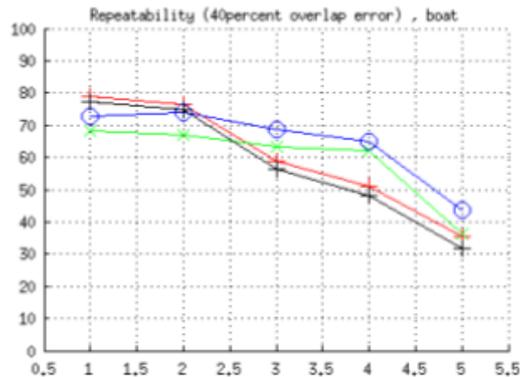
OR



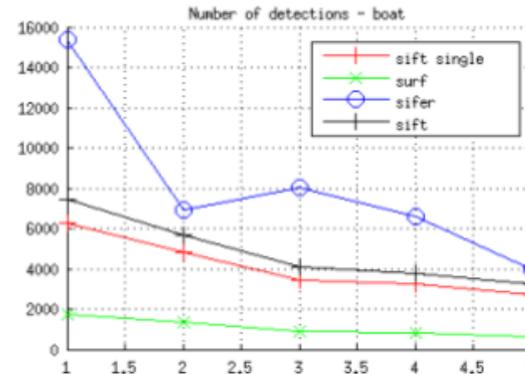
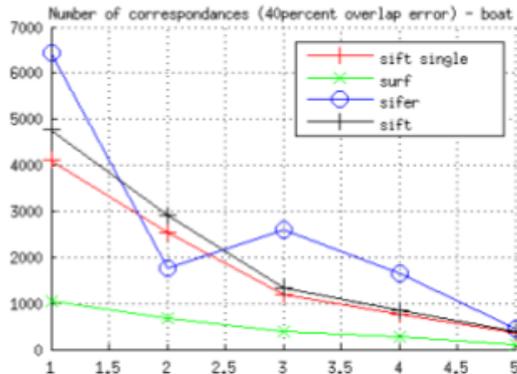
# Classic detection/repeatability results

## SIFT, SURF, SIFER

perturbation: rotation and scale



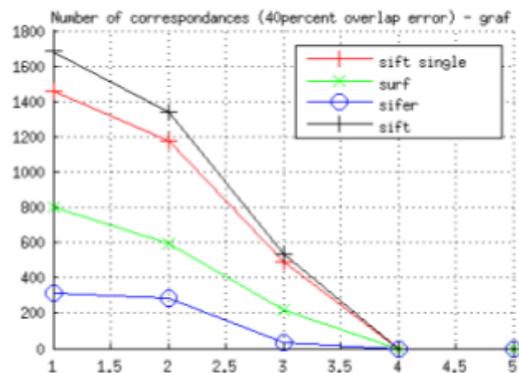
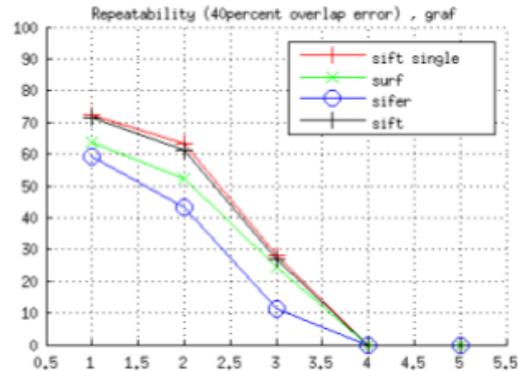
Problem: SIFER is NOT rotation or scale invariant and nevertheless beats two theoretically scale and rotation invariant detectors !



# Classic detection/repeatability results

## SIFT, SURF, SIFER

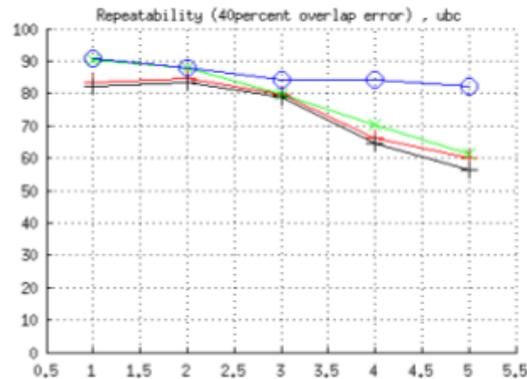
perturbation: tilt



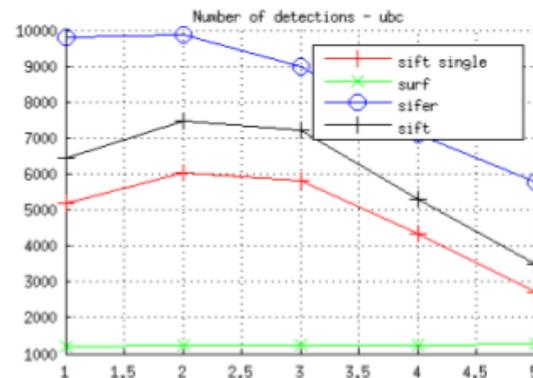
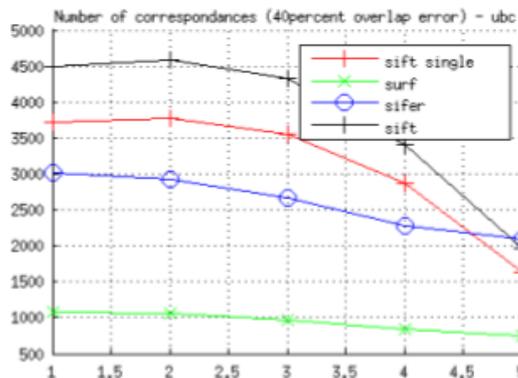
# Classic detection/repeatability results

## SIFT, SURF, SIFER

perturbation: JPG compression



Apparent conclusion: SIFER is more repeatable and has more detections than SIFT or SURF. It is therefore better.



# Detections maps



SIFT



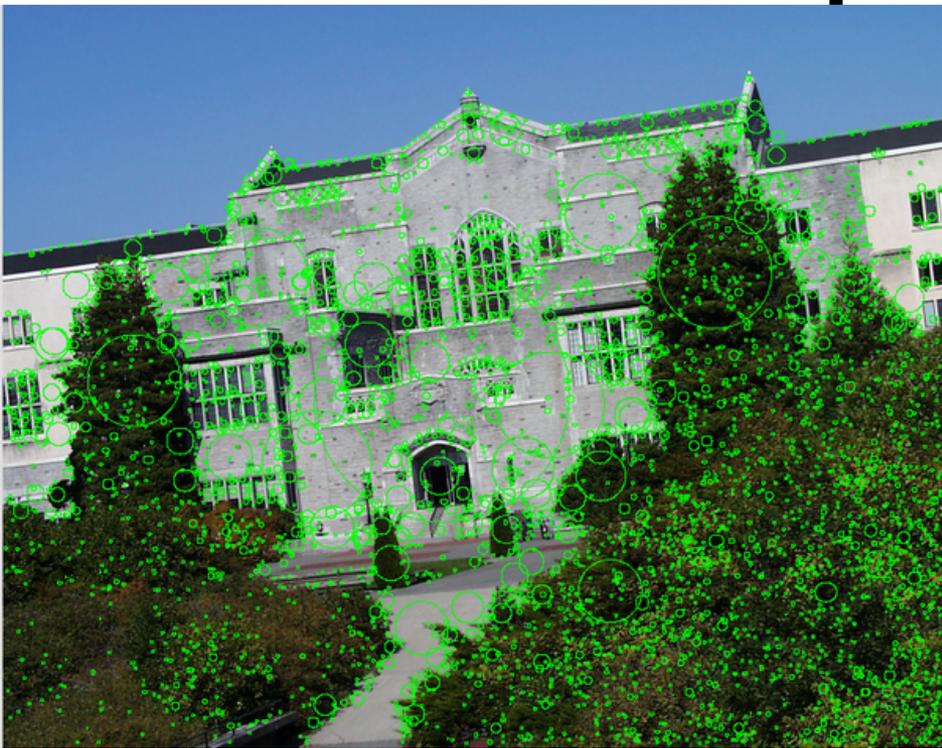
SURF



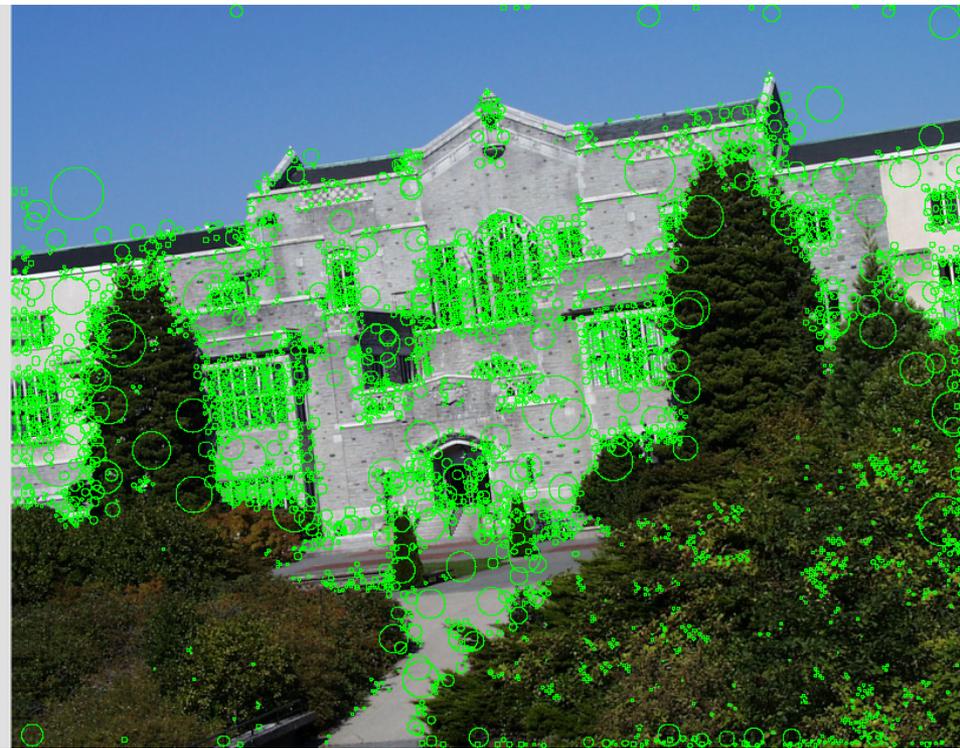
SIFER

The SIFT descriptors are more spread out than the SURF descriptors

# Detections maps



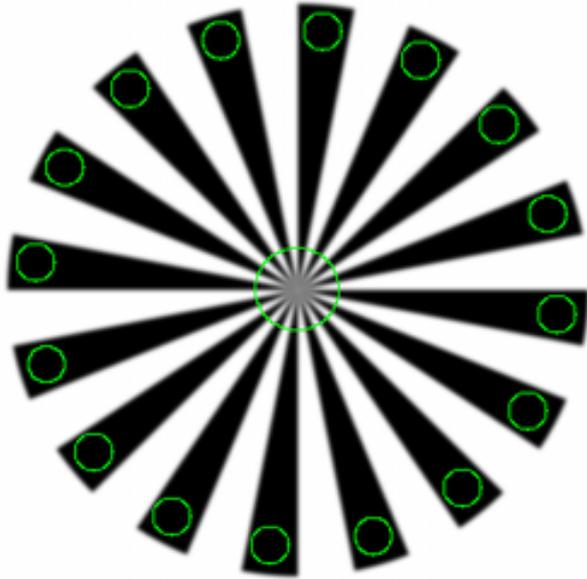
SIFT



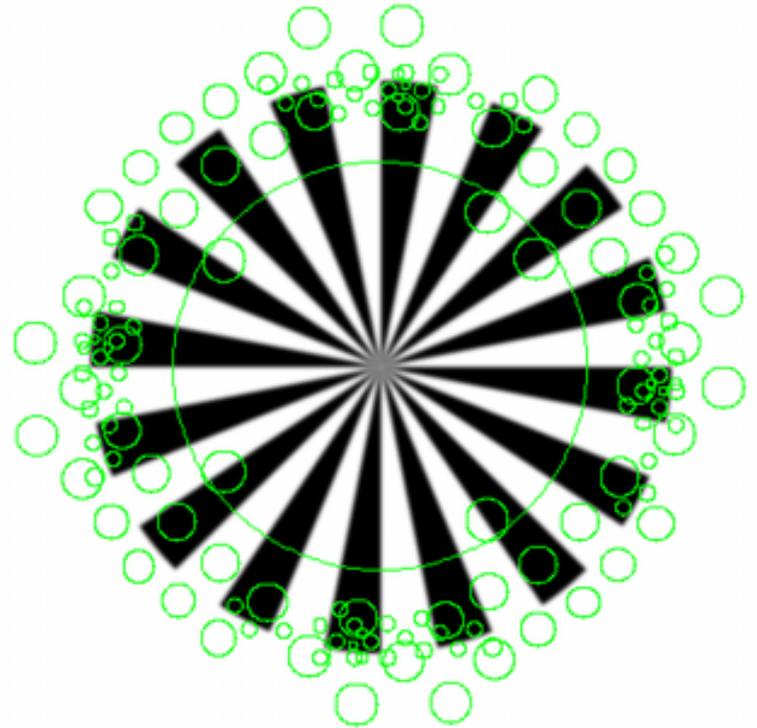
SIFER

The SIFER descriptors are more cluttered than the SIFT descriptors

# Detections maps (siemens star)



SIFT

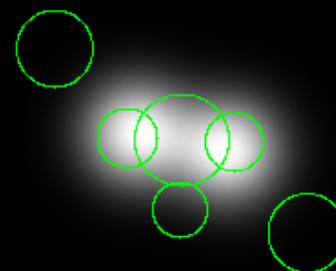


SIFER

# Detections maps (two Gaussian blobs)

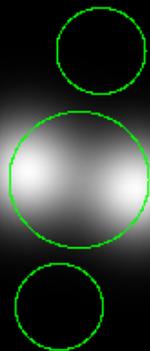


SIFT

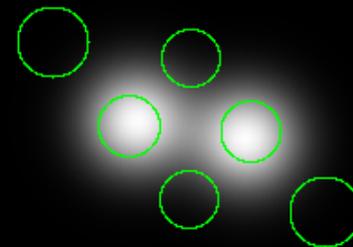


SIFER

# Detections maps (two Gaussian blobs)

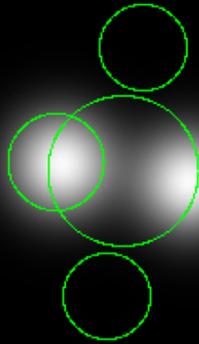


SIFT

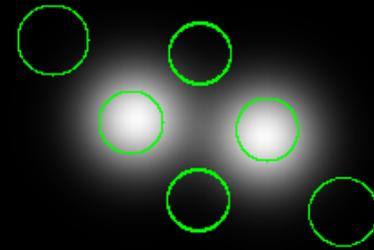


SIFER

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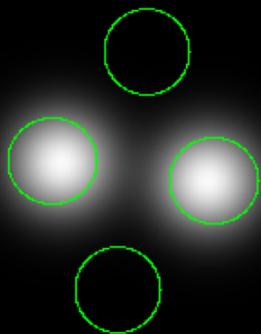


SIFT  
SIFT

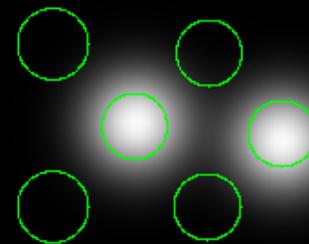


SIFER  
SIFER

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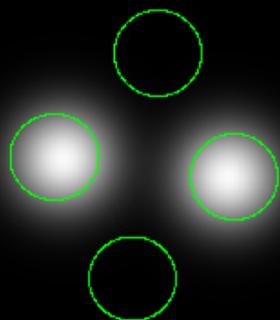


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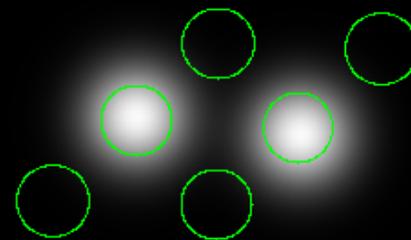


SIFER

# Detections maps (two Gaussian blobs)

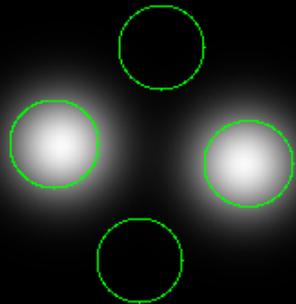


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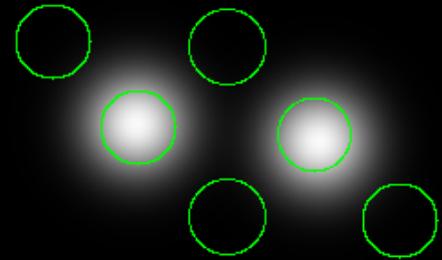


SIFER

# Detections maps (two Gaussian blobs)

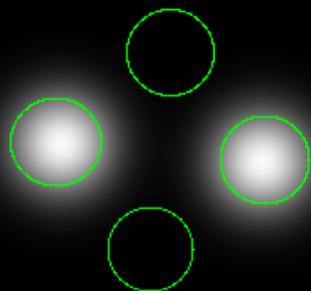


SIFT  
SIFT



SIFER  
SIFER

# Detections maps (two Gaussian blobs)

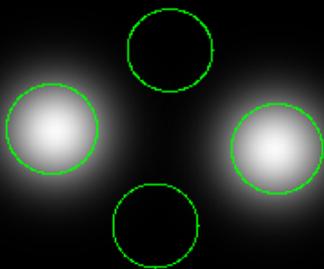


SIFT

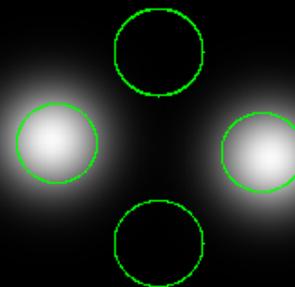


SIFER

# Detections maps (two Gaussian blobs)

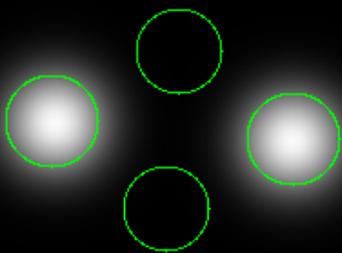


SIFT

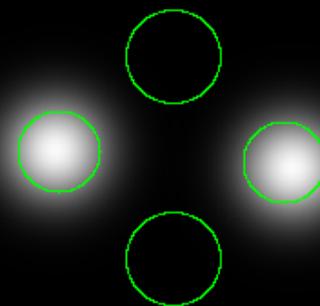


SIFER

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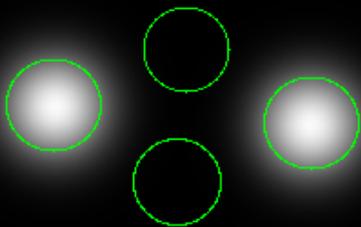


SIFT

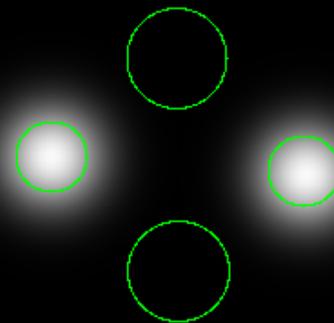


SIFER

# Detections maps (two Gaussian blobs)



SIFT

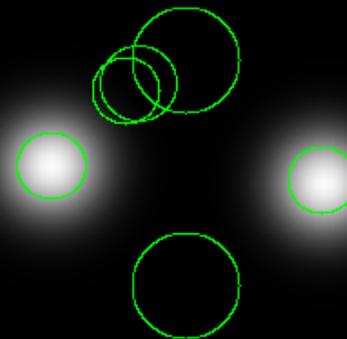


SIFER

# Detections maps (two Gaussian blobs)



SIFT

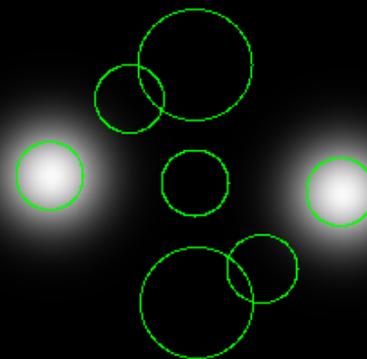


SIFER

# Detections maps (two Gaussian blobs)



SIFT

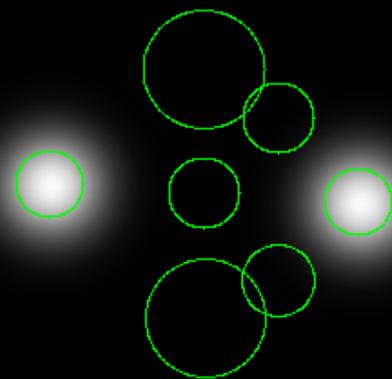


SIFER

# Detections maps (two blobs)

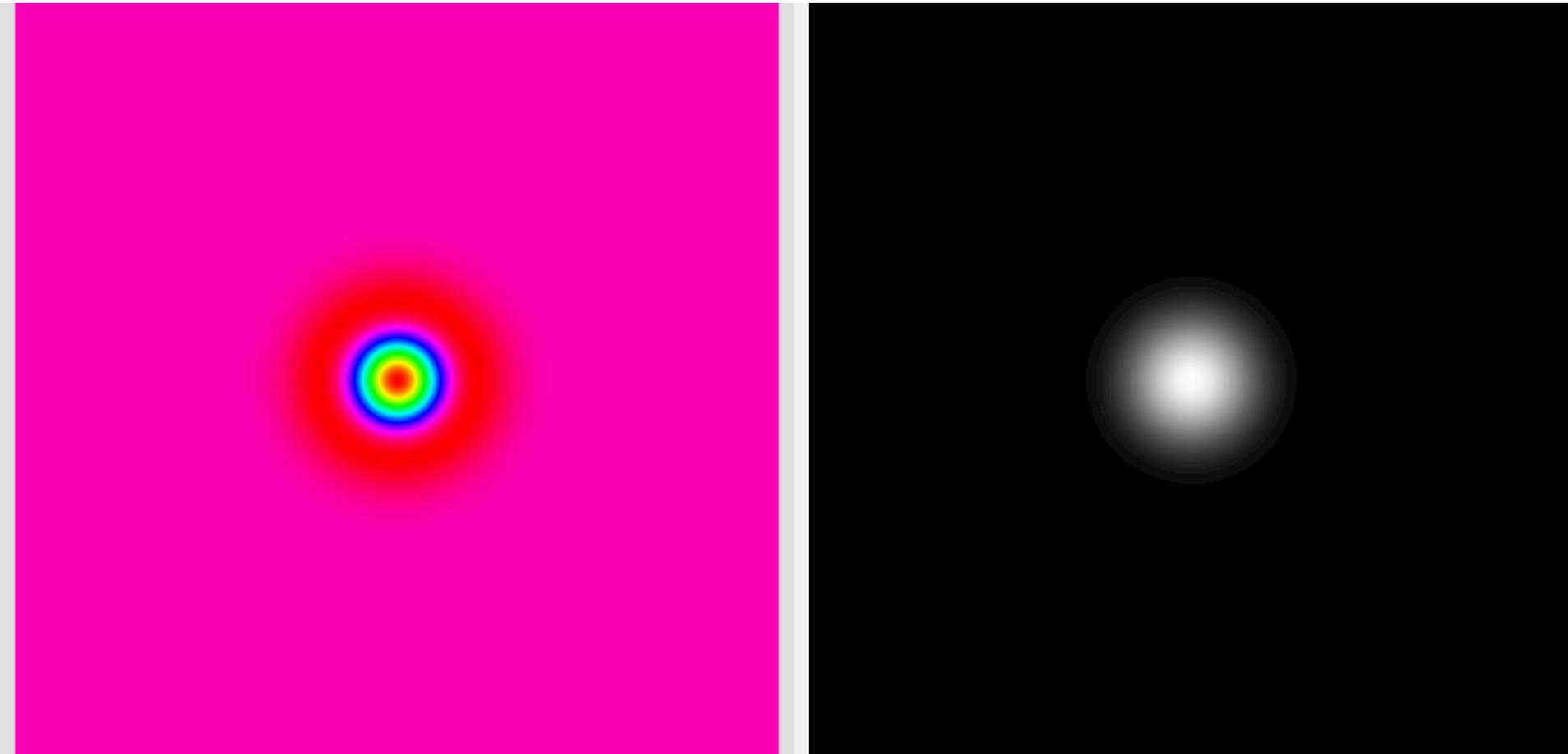


SIFT



SIFER

# Multi-scale response (Gaussian blob)

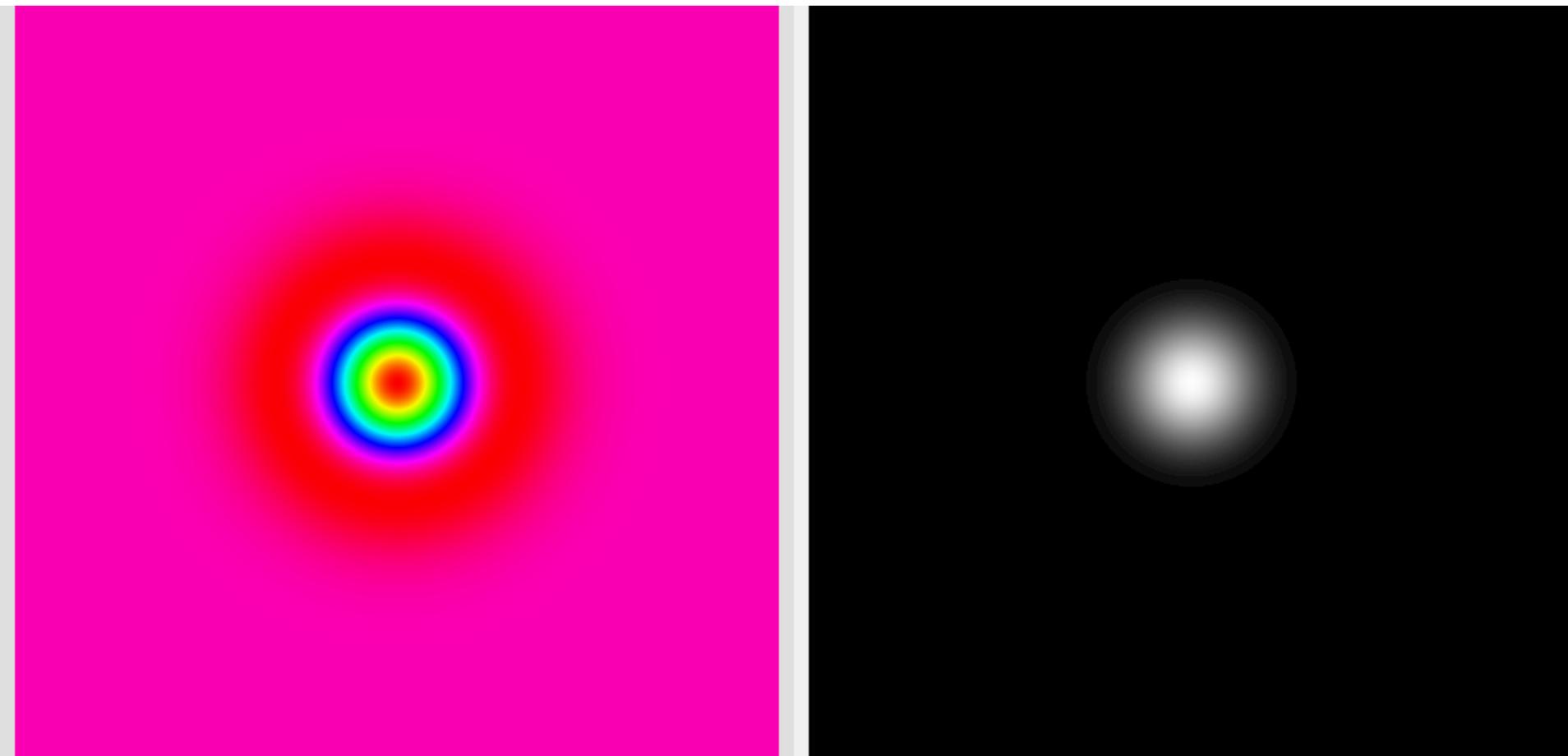


$$K_{\sigma}(x, y) = G_{k\sigma}(x, y) - G_{\sigma}(x, y)$$

$u(\mathbf{x})$

SIFT

# Multi-scale response (Gaussian blob)

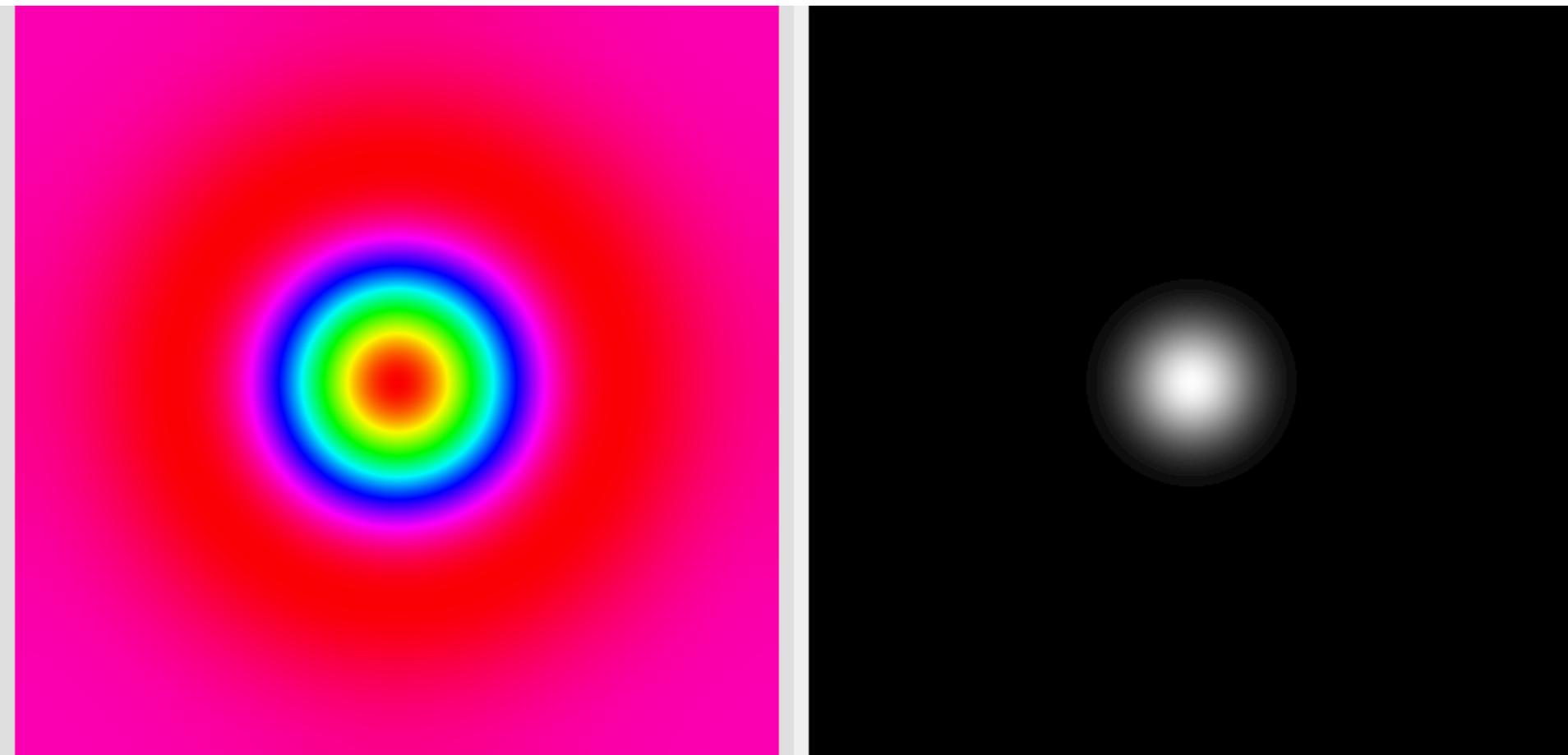


$$K_{\sigma}(x, y) = G_{k\sigma}(x, y) - G_{\sigma}(x, y)$$

$u(\mathbf{x})$

SIFT

# Multi-scale response (Gaussian blob)



$$K_\sigma(x, y) = G_{k\sigma}(x, y) - G_\sigma(x, y)$$

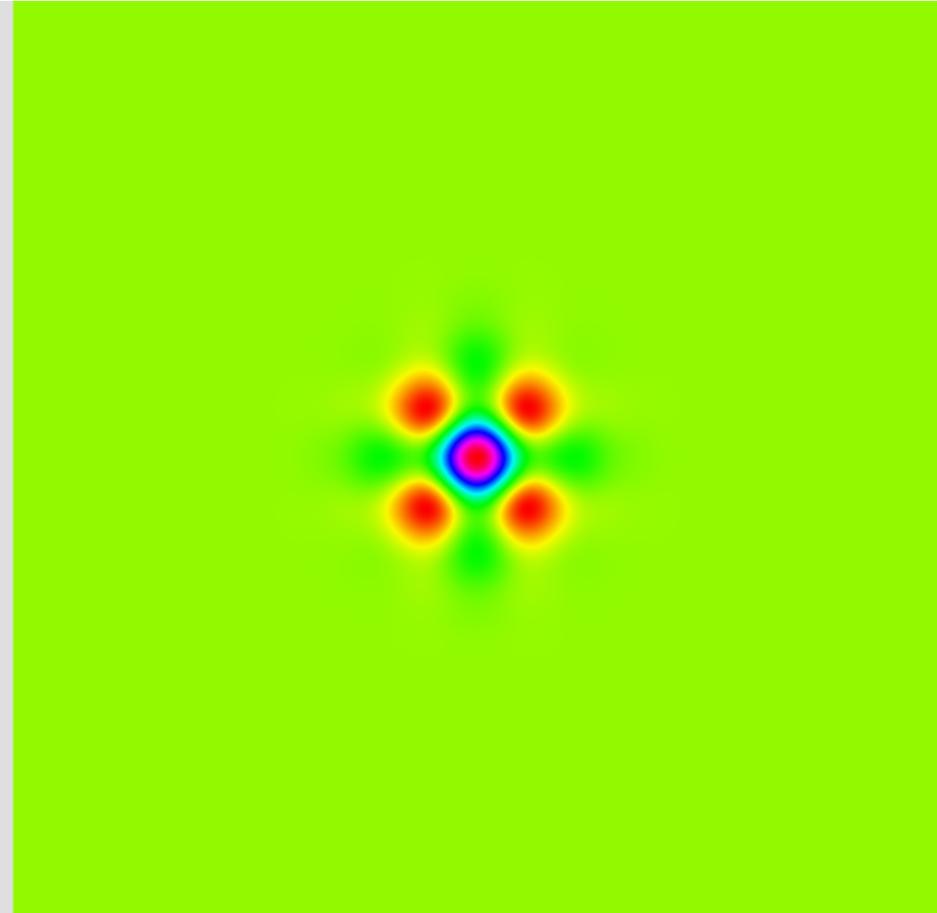
$u(\mathbf{x})$

SIFT

# Multi-scale response



Anisotropic response

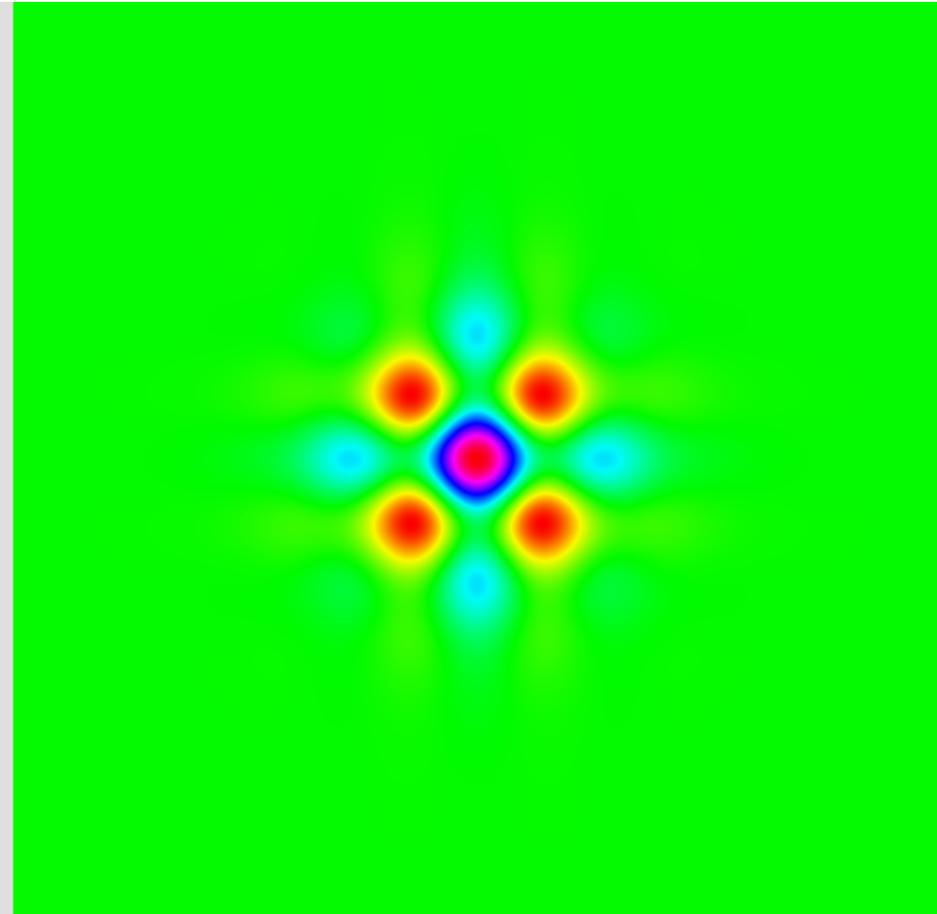


$$K_{\sigma}(x, y) = 2\pi\sigma^2 G_{\sigma}(x, y) \left( \cos\left(\frac{cx}{\sigma}\right) + \cos\left(\frac{cy}{\sigma}\right) \right)$$

# Multi-scale response (Gaussian blob)



Anisotropic response

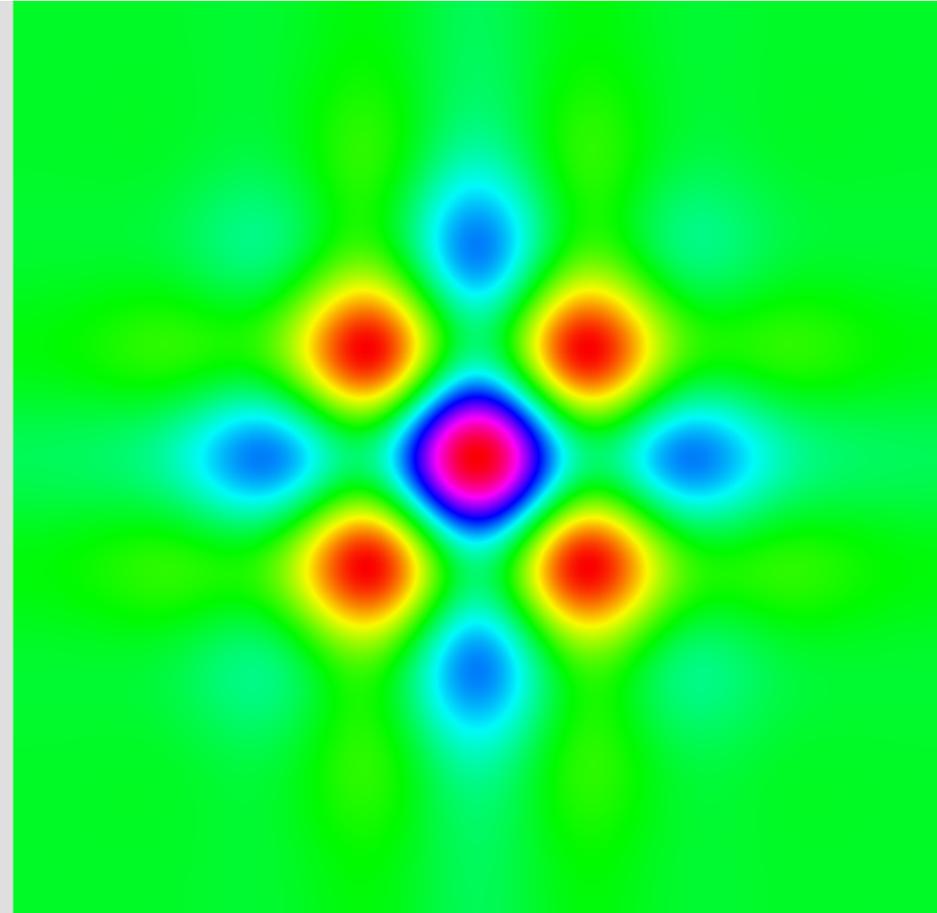


$$K_{\sigma}(x, y) = 2\pi\sigma^2 G_{\sigma}(x, y) \left( \cos\left(\frac{cx}{\sigma}\right) + \cos\left(\frac{cy}{\sigma}\right) \right)$$

# Multi-scale response (Gaussian blob)



Anisotropic response



$$K_{\sigma}(x, y) = 2\pi\sigma^2 G_{\sigma}(x, y) \left( \cos\left(\frac{cx}{\sigma}\right) + \cos\left(\frac{cy}{\sigma}\right) \right)$$

# Multi-scale response

(siemens star)



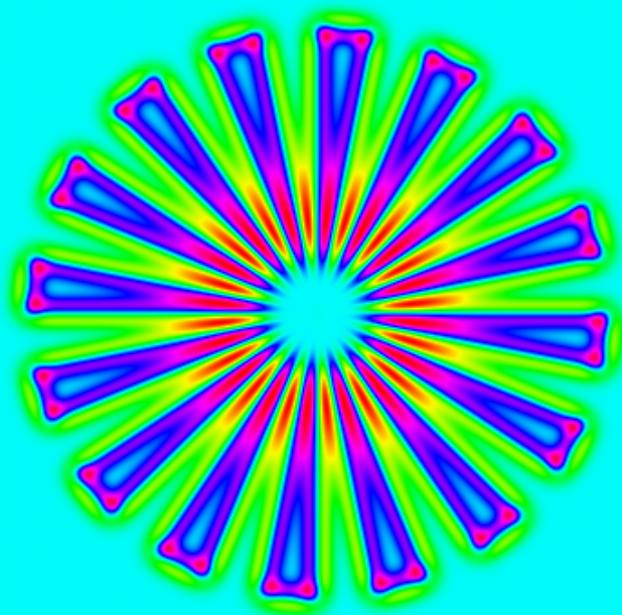
SIFT



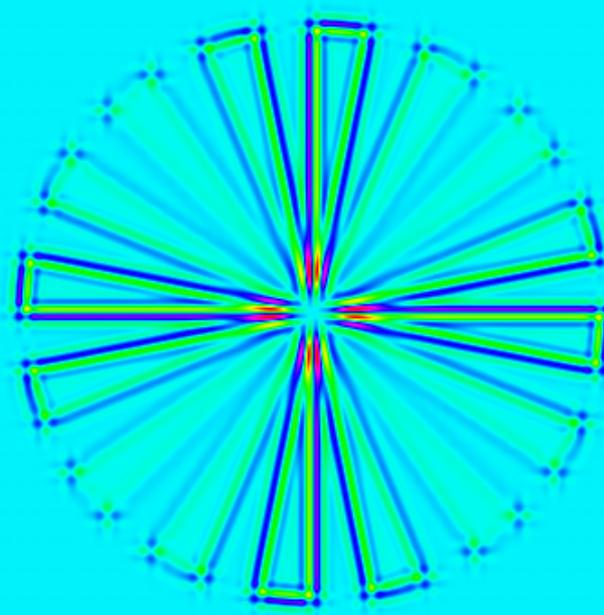
SIFER

# Multi-scale response

(siemens star)



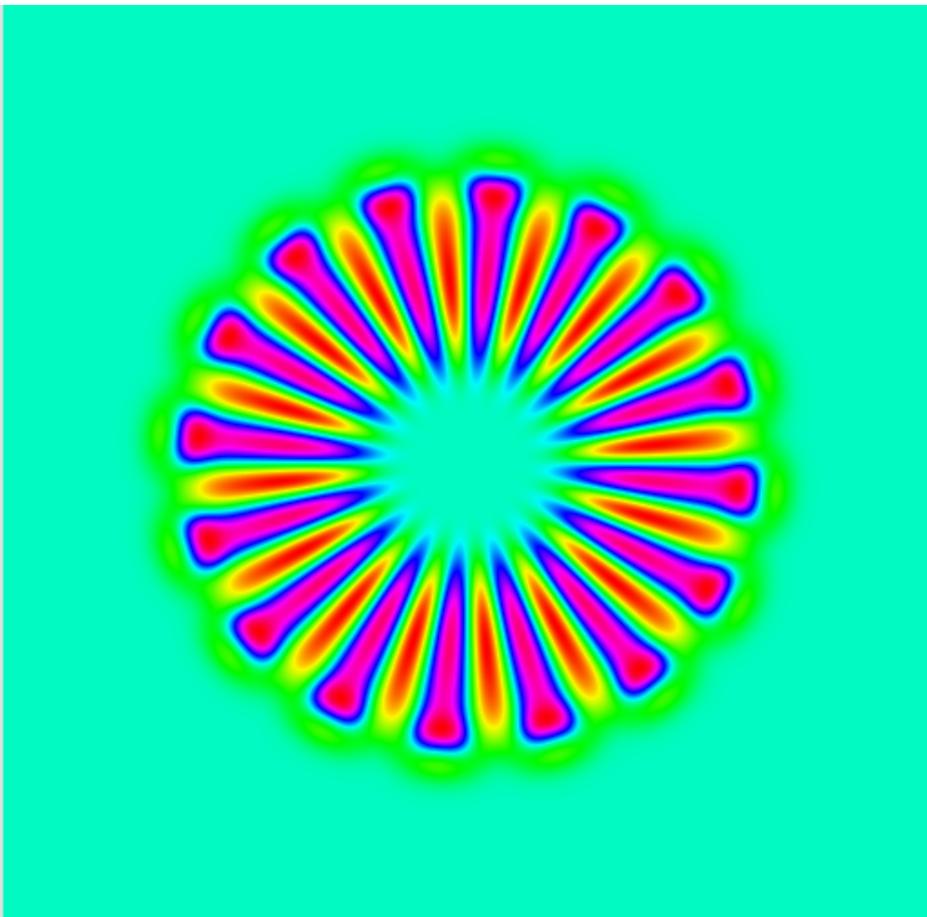
SIFT



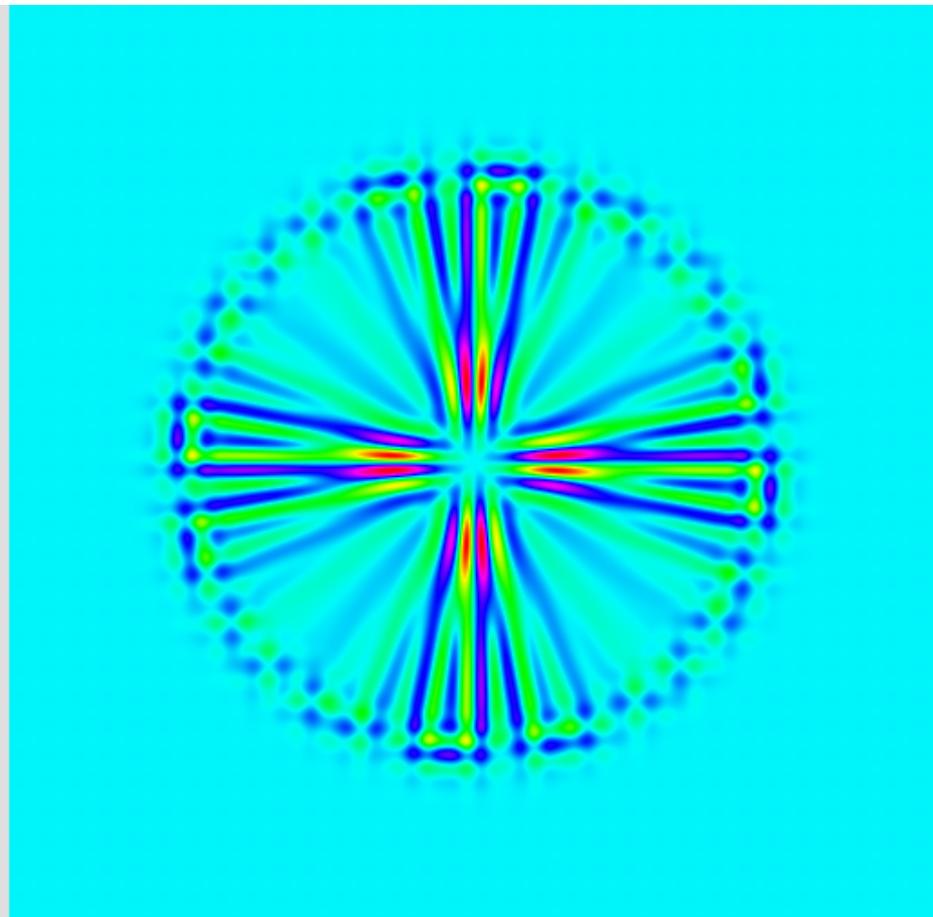
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# Multi-scale response

(siemens star)



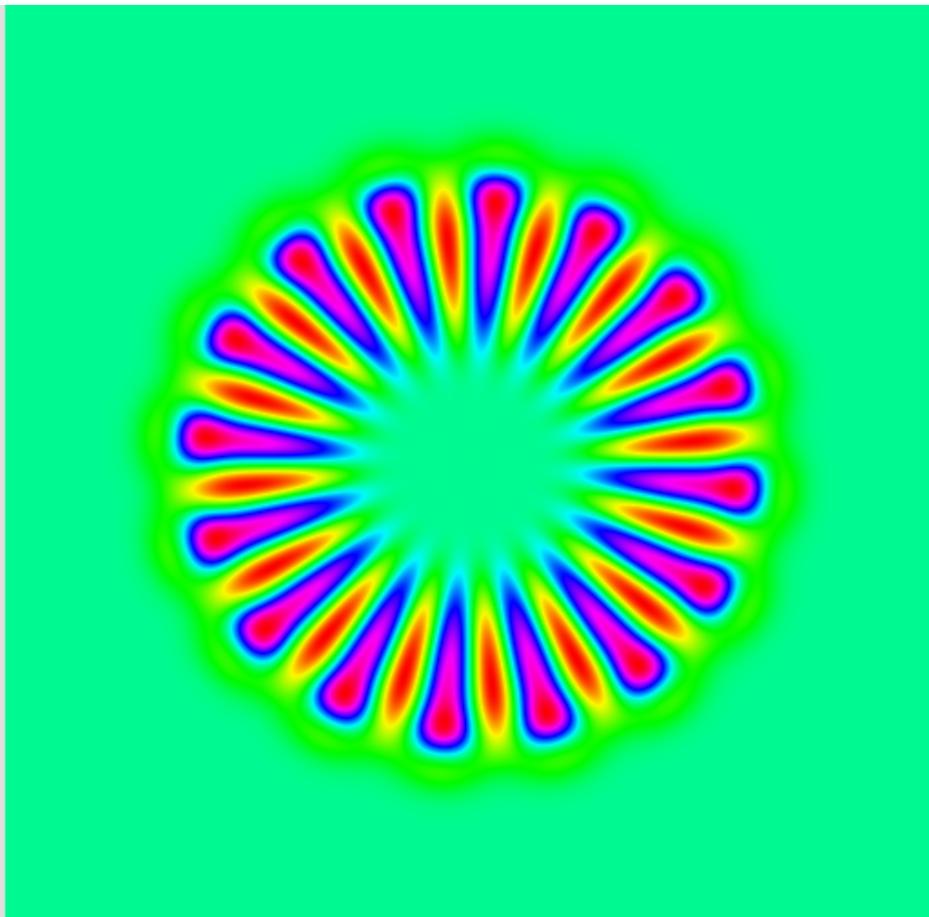
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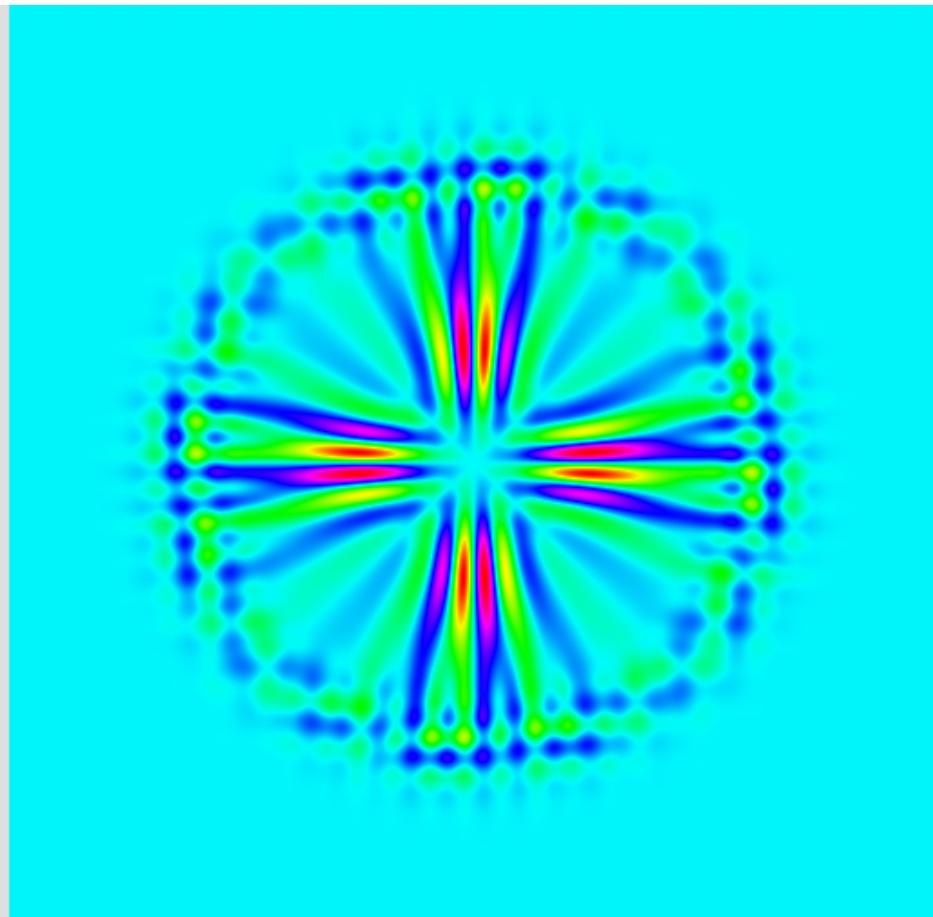
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# Multi-scale response

(siemens star)



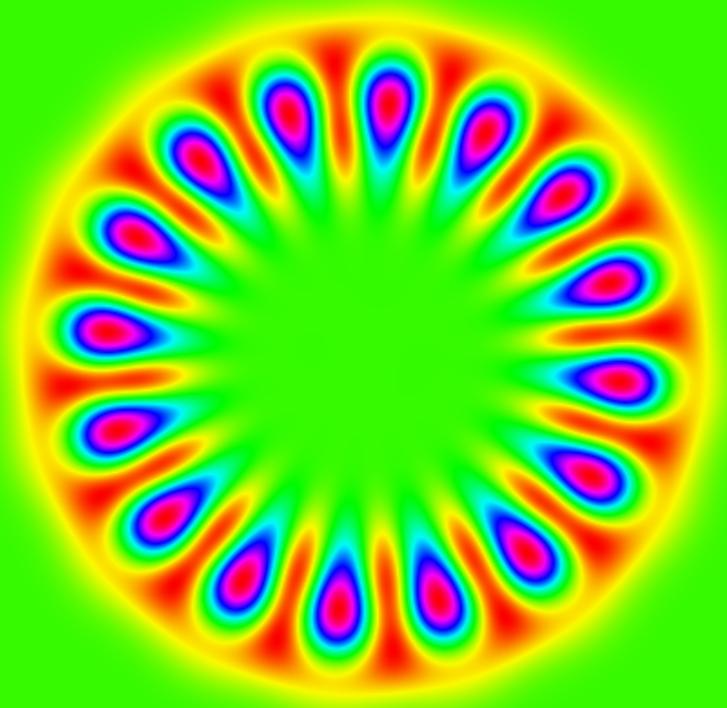
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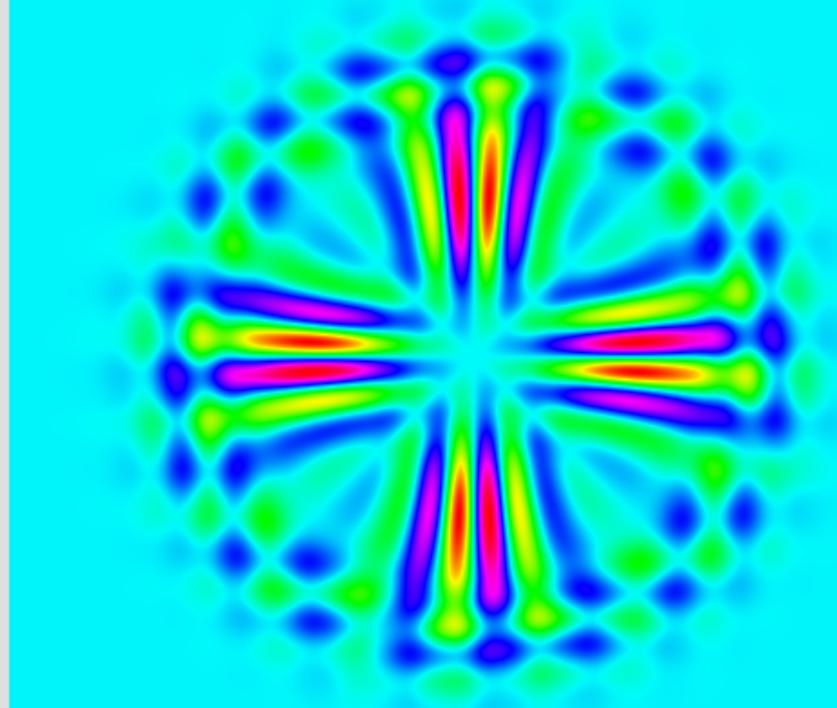
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# Multi-scale response

(siemens star)



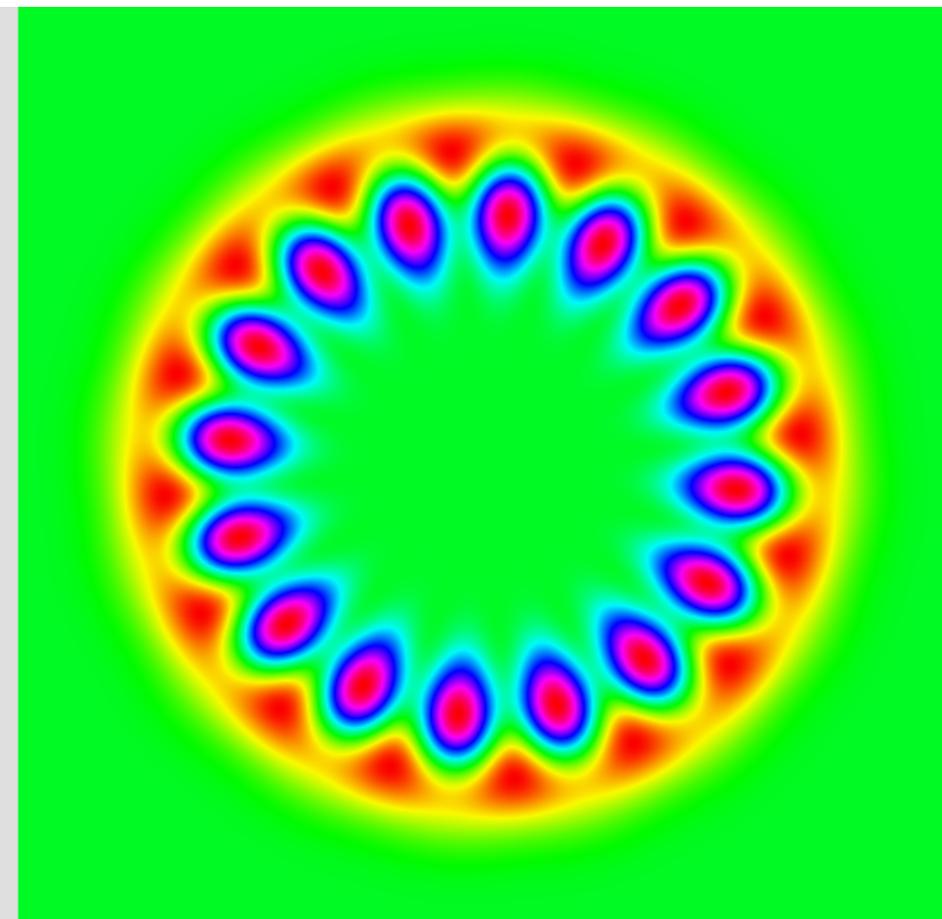
SIFT



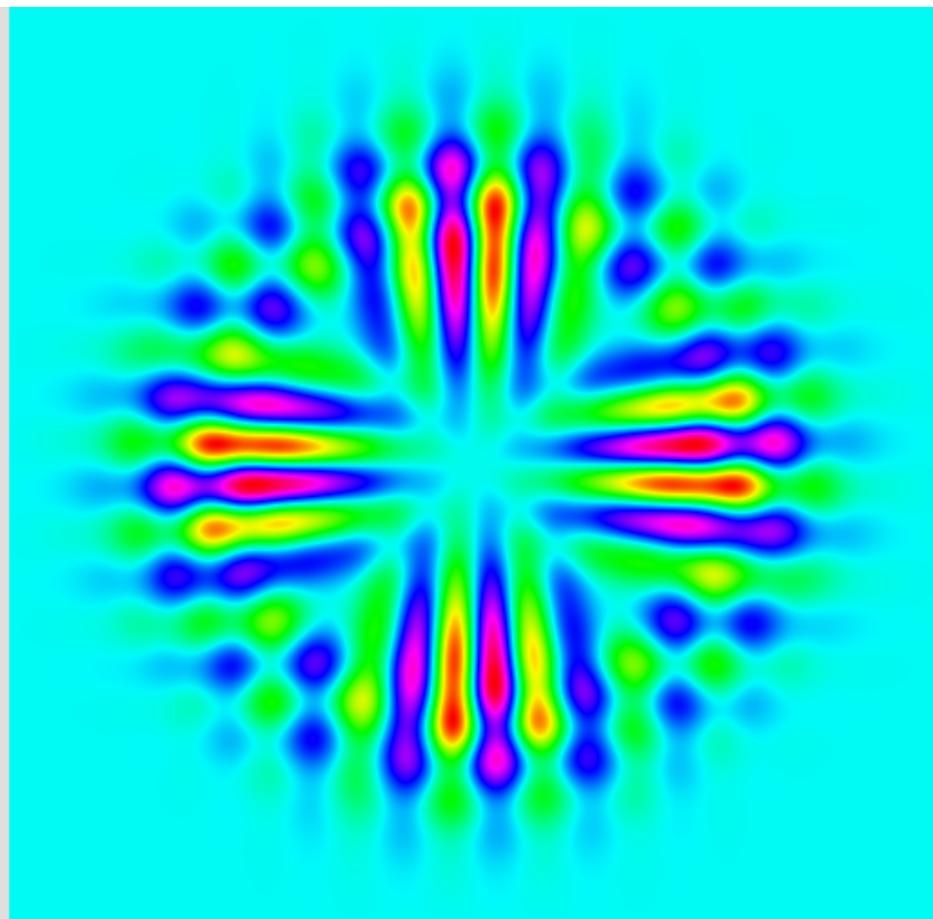
SIFER

# Multi-scale response

(siemens star)



SIFT



SIFER

# Multi-scale response

(graf)

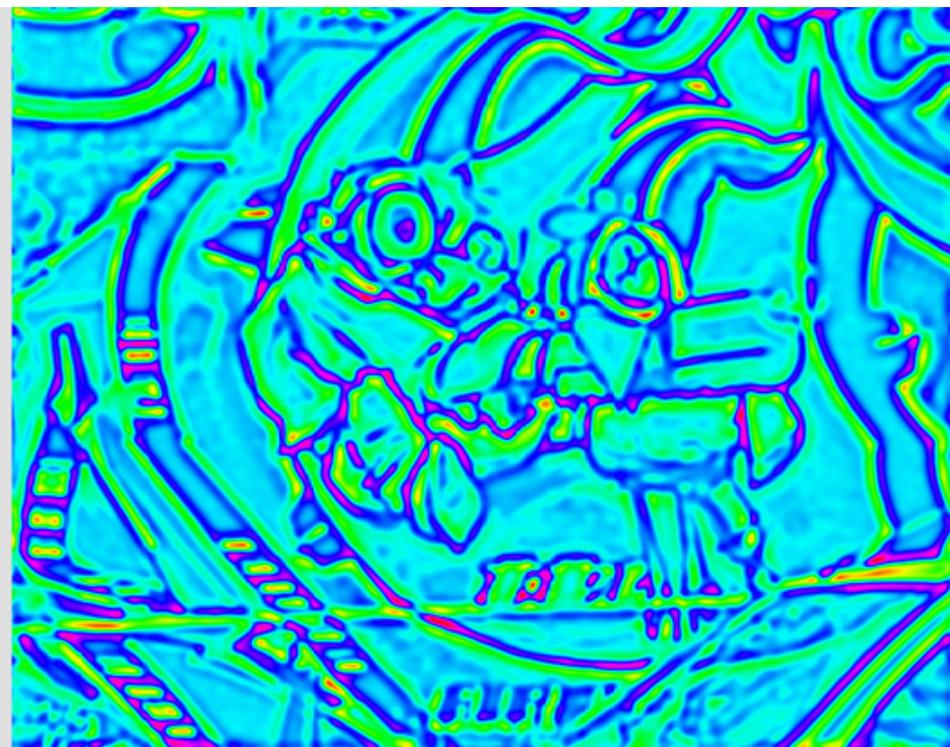


SIFT

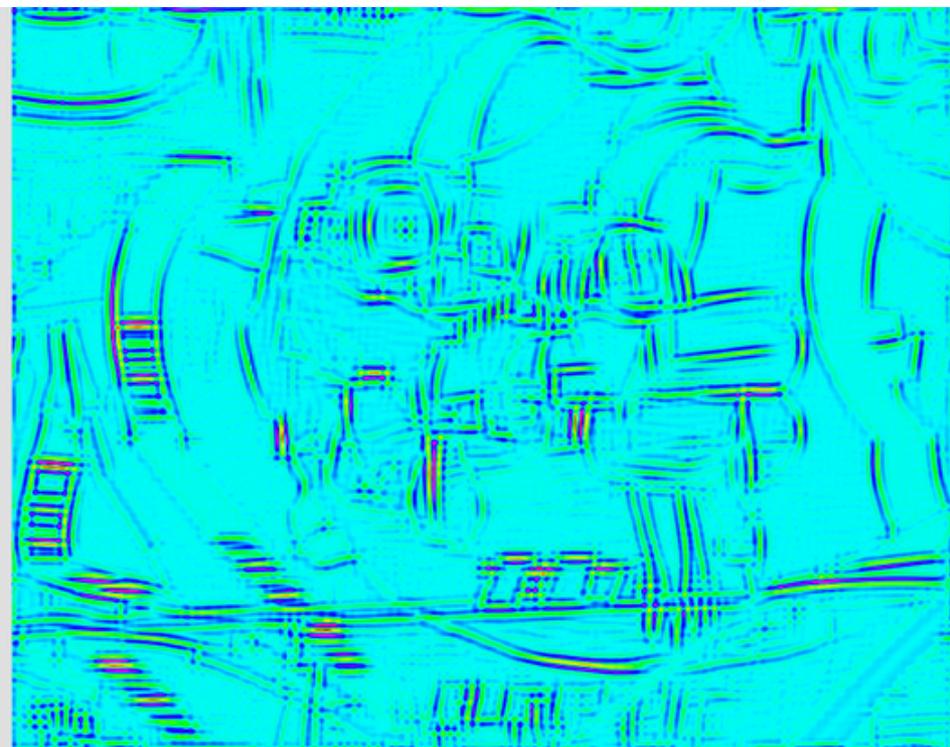


SIFER

# Multi-scale response (graf)

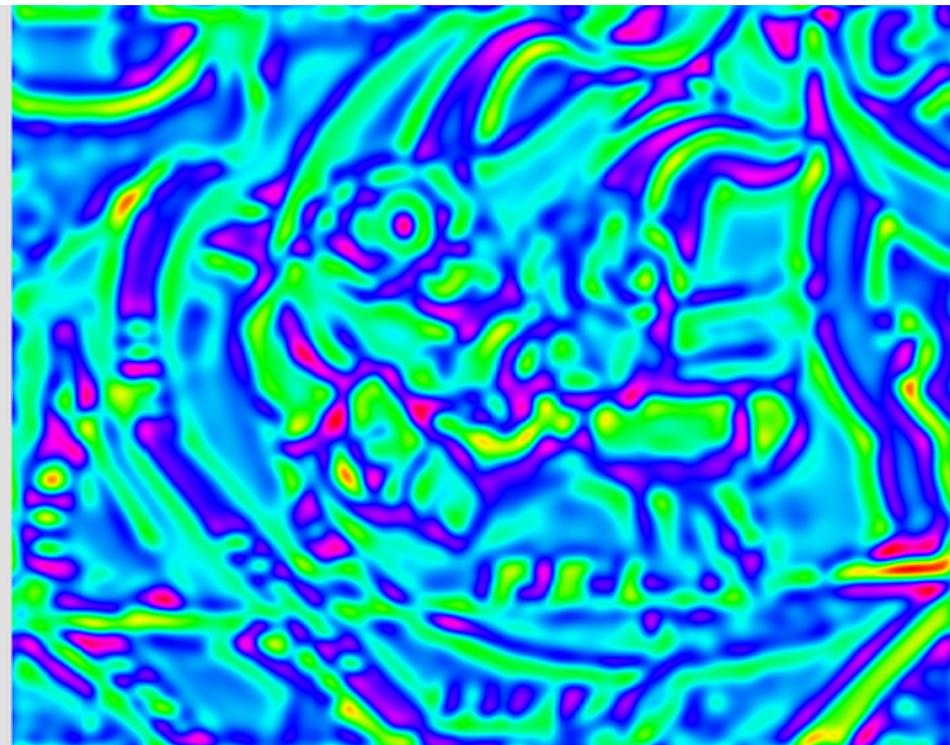


SIFT

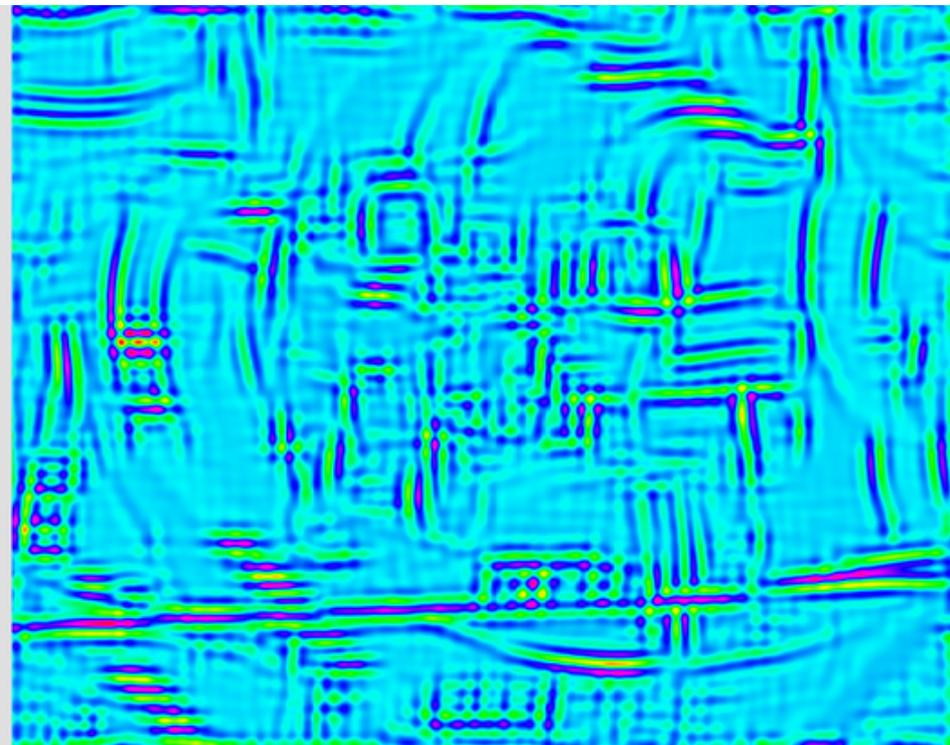


SIFER

# Multi-scale response (graf)

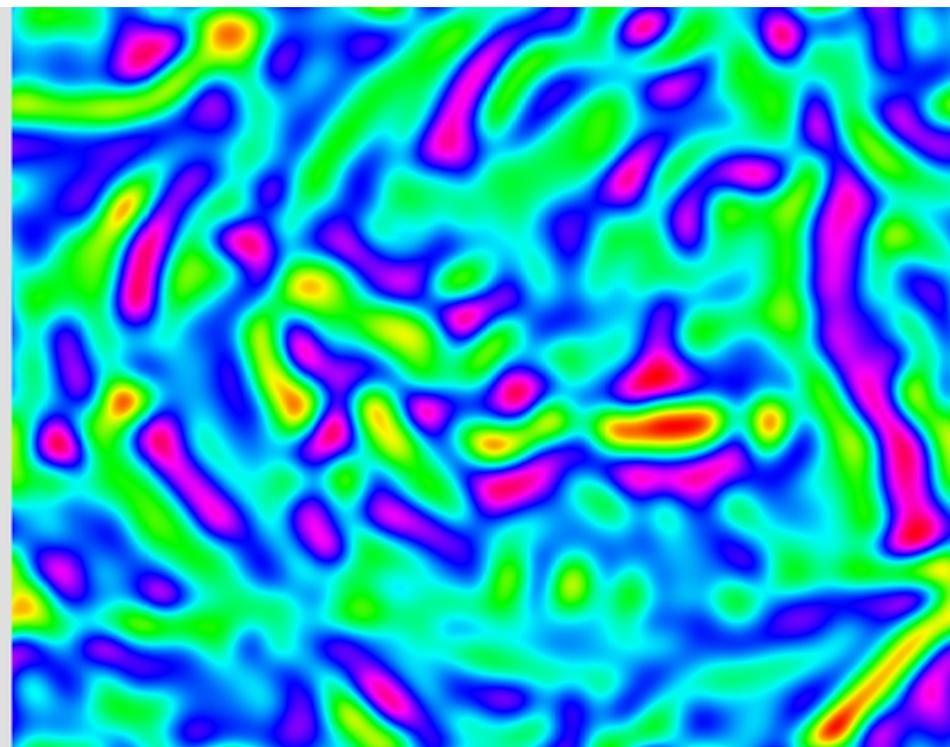


SIFT



SIFER

# Multi-scale response (graf)

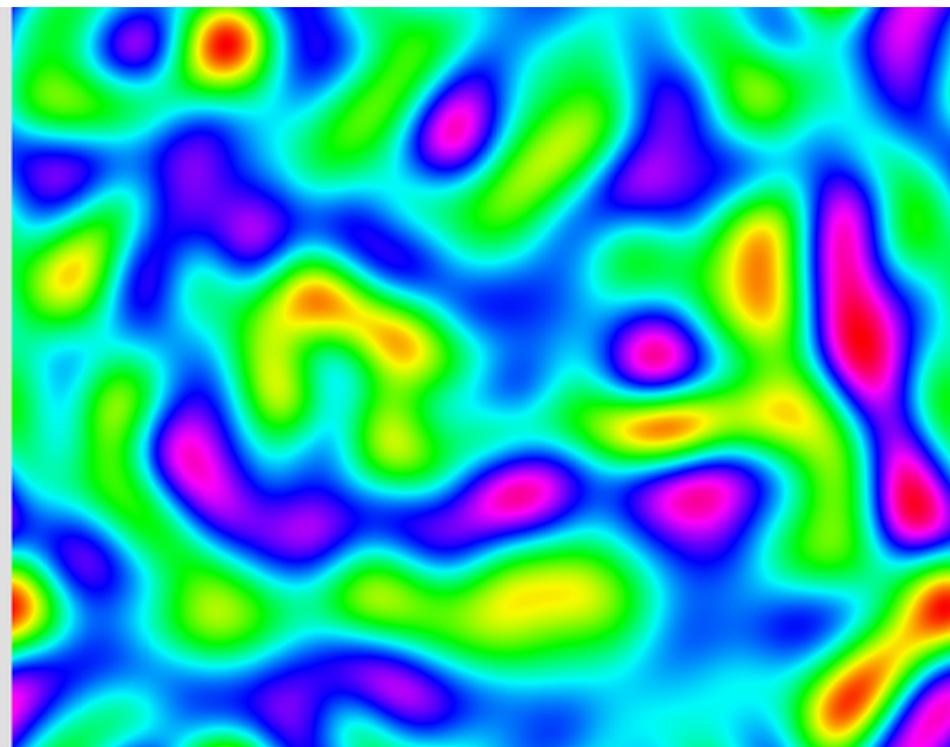


SIFT

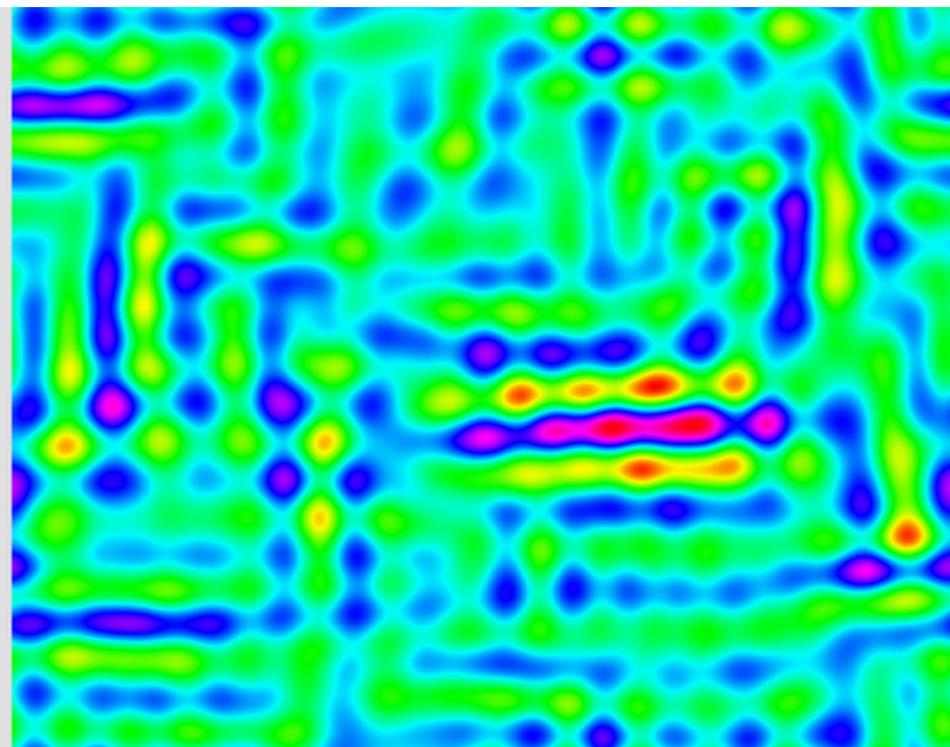


SIFER

# Multi-scale response (graf)



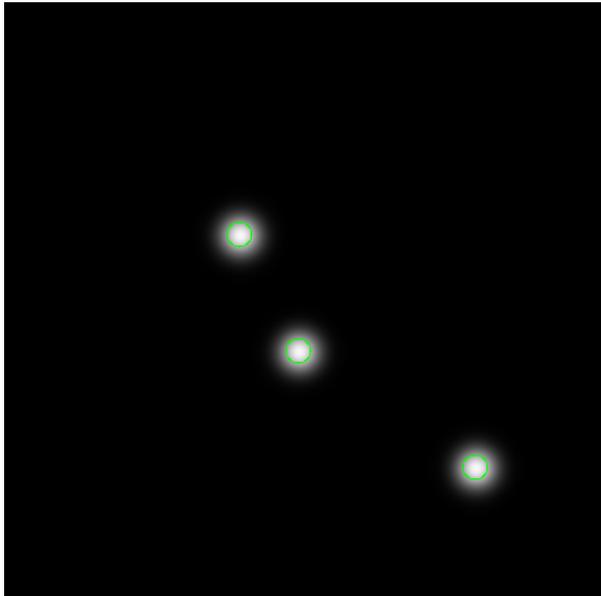
SIFT



SIFER

# Suggested simple correction

$f_k(x, y)$ : the Gaussian window extracted for the description of the keypoint  $(\sigma_k, x_k, y_k, \theta_k)$ .



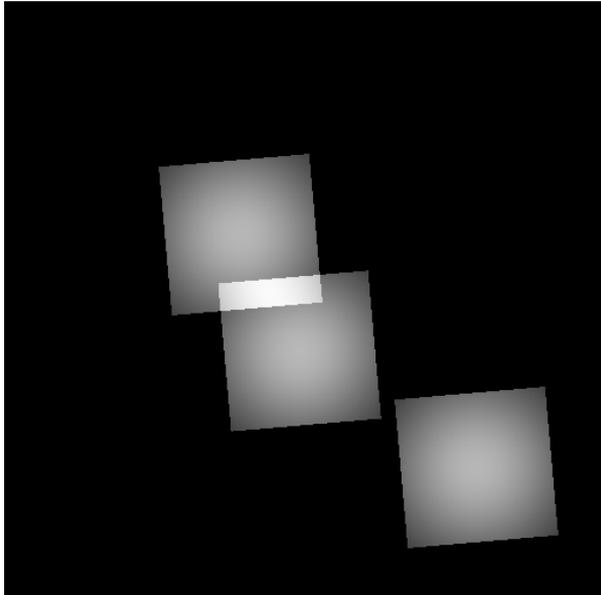
3 detected keypoints



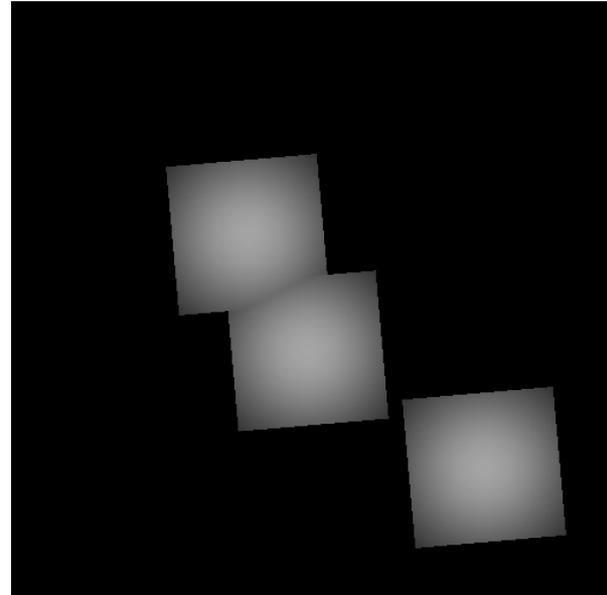
3 detected keypoints

# Suggested simple correction

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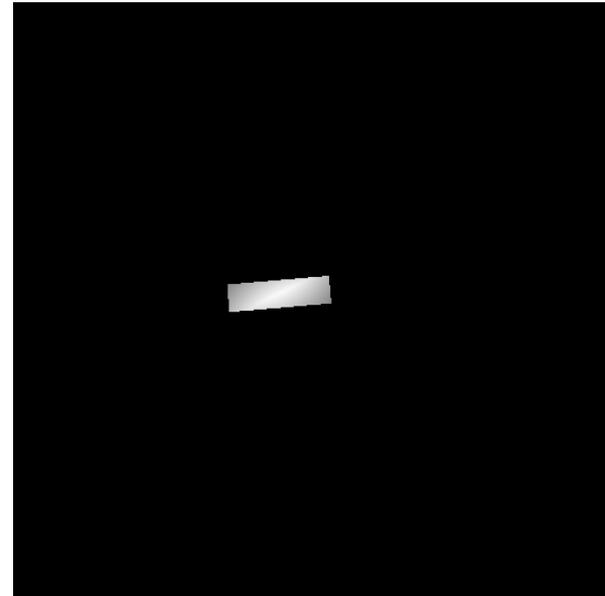
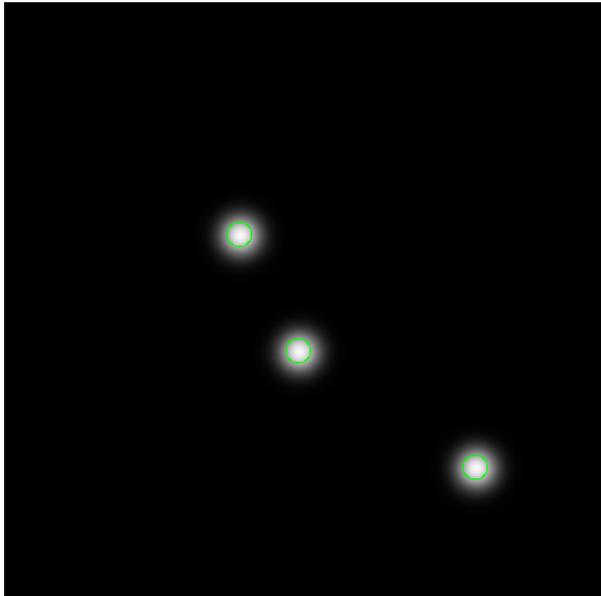
$$\sum_k f_k(x, y)$$



$$\max_k f_k(x, y)$$

# Proposed correction of the repeatability criterion

$\left( \sum_k f_k(x, y) - \max_k f_k(x, y) \right)$  maps the detections redundancy

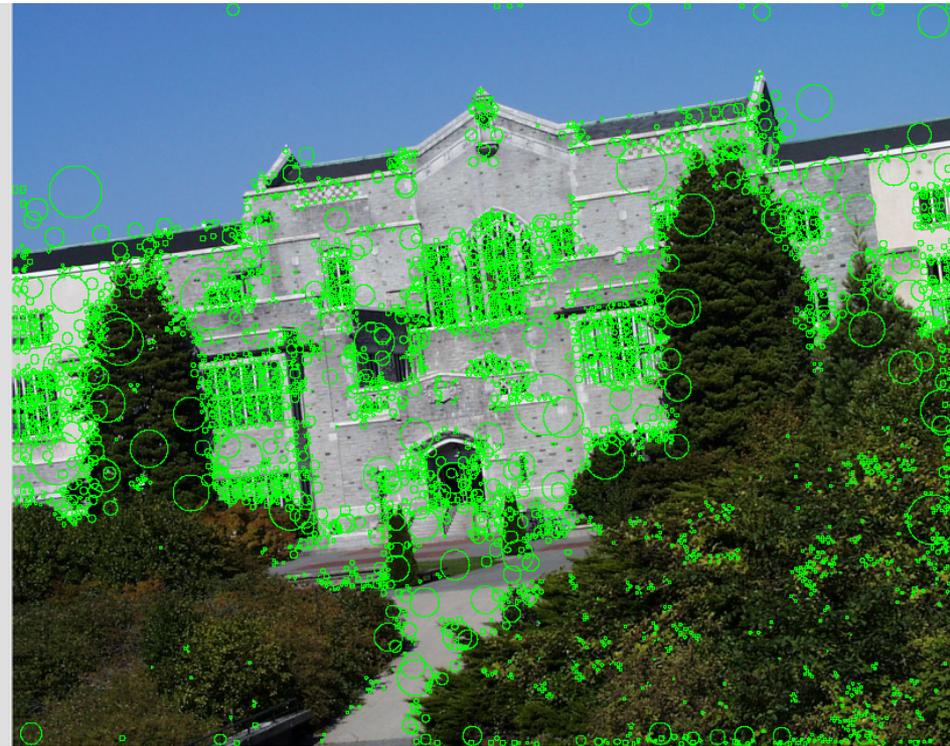


$\left( \sum_k f_k(x, y) - \max_k f_k(x, y) \right)$

# Proposed correction of the repeatability criterion: Detections overlap



SIFT

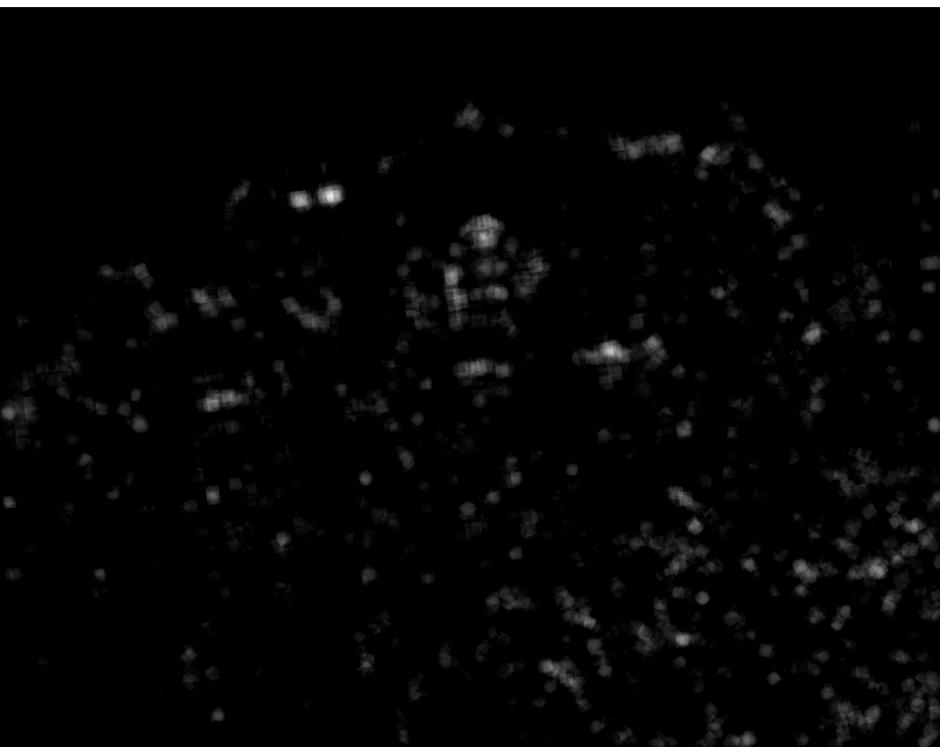


SIFER

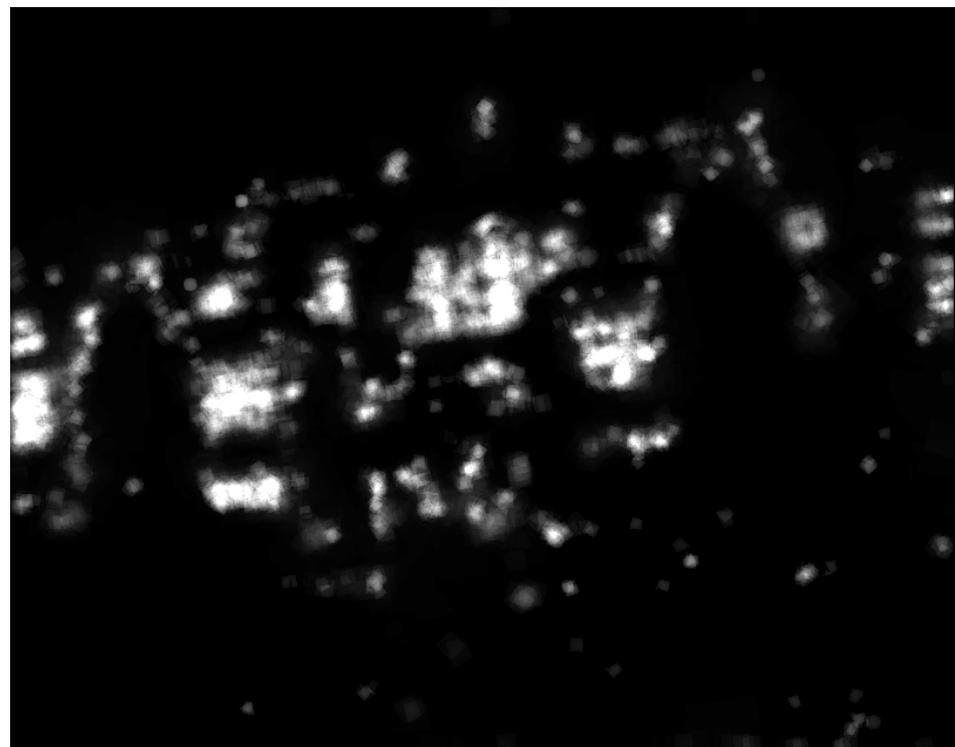
Detection maps

# Suggested simple correction

Detections overlap



SIFT



SIFER

$$\left( \sum_k f_k(x, y) - \max_k f_k(x, y) \right)$$

# Suggested simple correction

$$\int_{\Omega} \sum_k f_k(x, y) dx dy = \text{number of detections}$$

$$\int_{\Omega} \max_k f_k(x, y) dx dy \approx \text{number of detections without overlap}$$

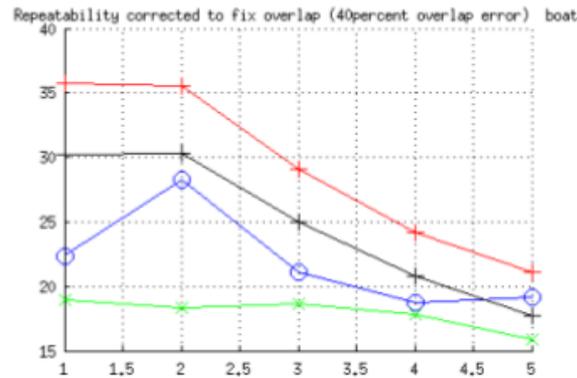
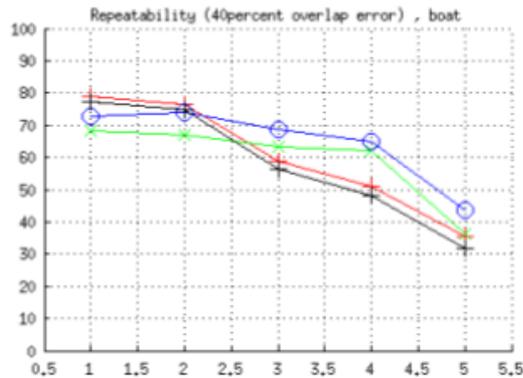
$$\text{repeatability rate} = \frac{\int_{\Omega} \max_{k \in K_{\text{rep}}} f_k(x, y) dx dy}{\text{total number of detection in use}}$$

$K_{\text{rep}}$ : set of repeated keypoints.

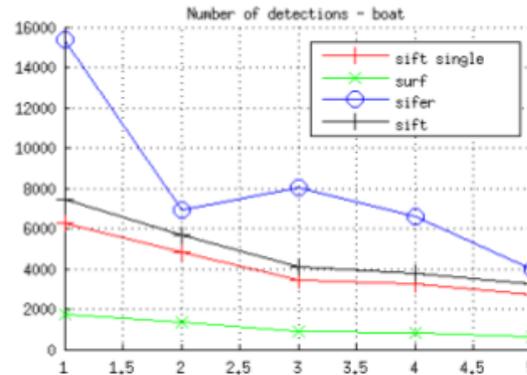
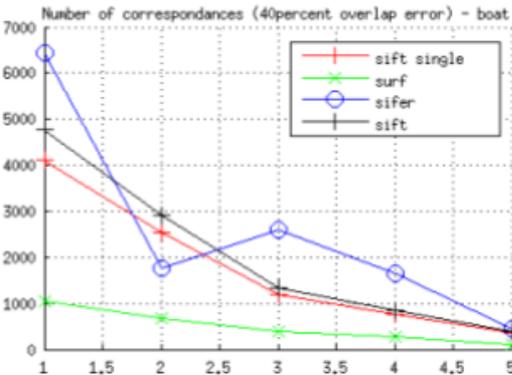
# The new repeatability curves

## SIFT, SURF, SIFER

perturbation: rotation and scale

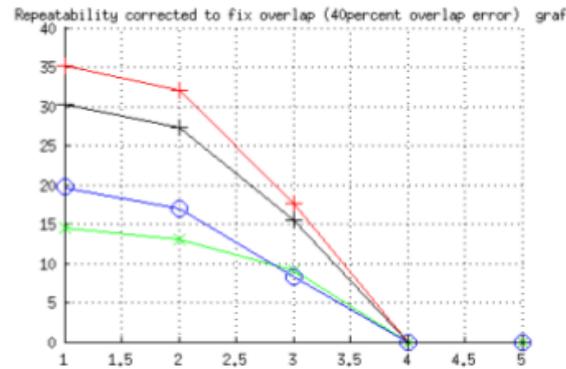
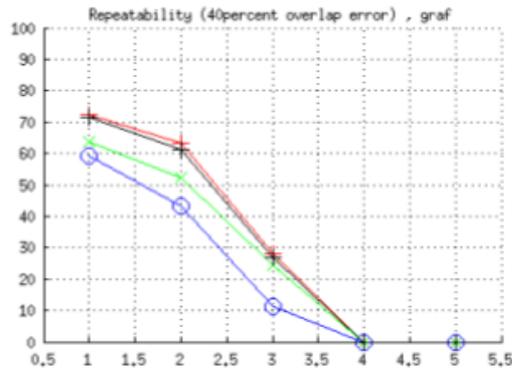


New repeatability curves

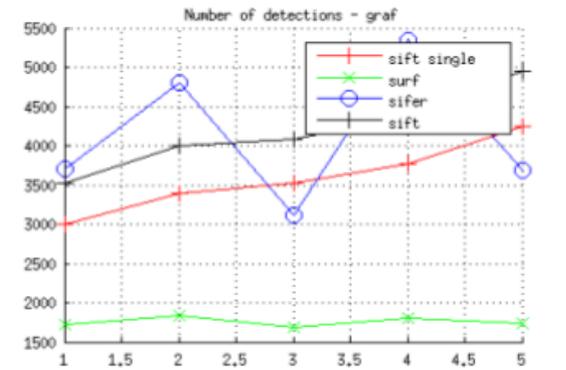
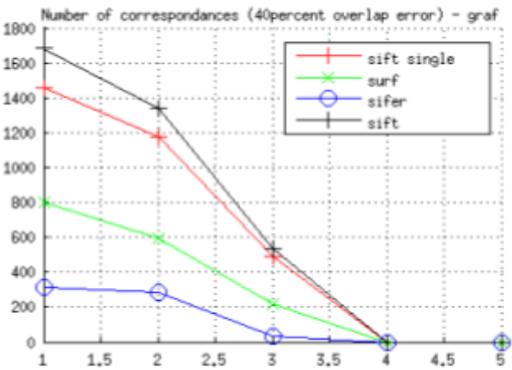


# The new repeatability curves SIFT, SURF, SIFER

perturbation: tilt



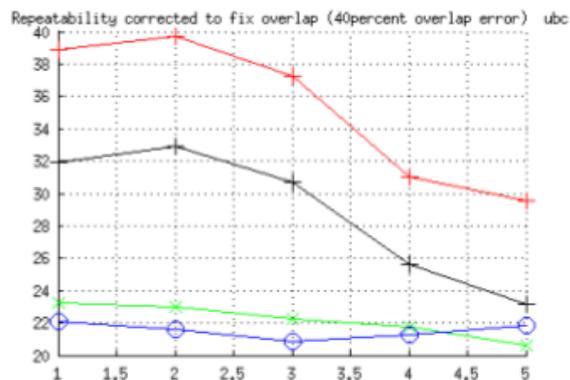
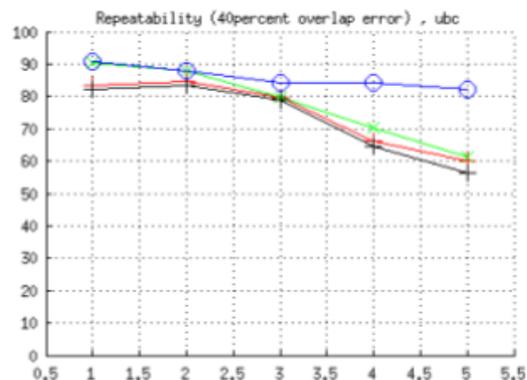
New repeatability curves



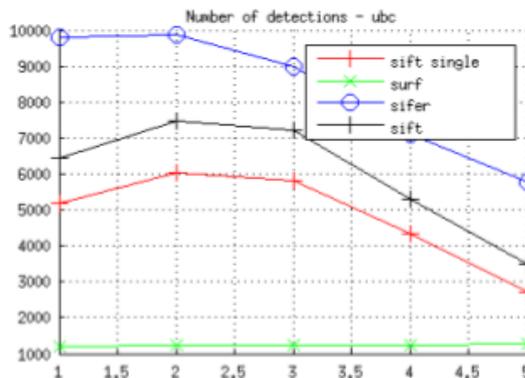
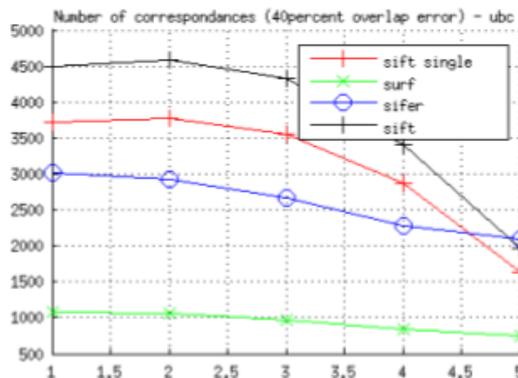
# The new repeatability curves

## SIFT, SURF, SIFER

perturbation: JPG compression



New repeatability curves



# Conclusion and open problems

**Conclusion:** One cannot be satisfied with the proliferation of unprincipled detectors/descriptors. For many of them, the benchmark data demonstrating that they “win” may well be misleading.

## Open problem 1:

By simply modifying the parameters of the most invariant method (so far SIFT), one may reach improvements in the performance curves equivalent to those obtained by modifying the detector/descriptor pair. All things equal, we will prefer the really invariant methods.

## Open problem 2:

Make the mathematical theory and check if the numerous interest-point/descriptor methods are really scale invariant or not in the Lowe sense. Classify them by their proven invariances.

## Open problem 3:

SIFER seems to suggest that scale invariance is not necessary. Thus, all homothety-invariant families of filters are candidates to construct keypoints! If this is true, the chase is open for the best filter family. SIFER is just one of them.

## Open problem/proposition 4:

Should we not check repeatability, invariance, and robustness on a short list of reliable patterns where we can also view and discuss the position of interest points?

## Open question 5: make requested invariances and benchmarks match!!!

Why are we testing and comparing detectors/descriptors for invariances that they do not have in theory?? (E.g. blur invariance or affine invariance)

# Detections maps



SIFT



SURF



SIFER

# Detections maps



SIFT



SURF



SIFER