

This part of the course will follow the book Approximation Algorithms by Vijay V. Vazirani [Vaz00]. I will cover.

## 1 The Set Cover Problem

**The Greedy Algorithm, pages 15 to 17:** Proof of Lemma 2.3: Vijay writes:

In any iteration, the leftover sets of the optimal solution can cover the remaining elements at a cost of at most  $\text{OPT}$ . Therefore, among these sets, there must be one of having cost-effectiveness of at most  $\text{OPT}/|\bar{C}|$ , where  $\bar{C}$  is the set of uncovered elements.

I had a hard time understanding this sentence. It is supposed to prove

$$\min_{S \in \mathcal{S}: |S \cap \bar{C}|} \frac{c(S)}{|S \cap \bar{C}|} \leq \frac{\text{OPT}}{|\bar{C}|}.$$

An extended argument is as follows.

$$\begin{aligned} \frac{\text{OPT}}{|\bar{C}|} &= \frac{\sum_{S \in \text{OPT}} c(S)}{|\bar{C}|} = \frac{\sum_{S \in \text{OPT}} \sum_{e \in S \cap \bar{C}} \frac{c(S)}{|S \cap \bar{C}|}}{|\bar{C}|} \\ &= \frac{\sum_{e \in \bar{C}} \sum_{S \in \text{OPT}: e \in S} \frac{c(S)}{|S \cap \bar{C}|}}{|\bar{C}|} \\ &\geq \min_{e \in \bar{C}} \sum_{S \in \text{OPT}: e \in S} \frac{c(S)}{|S \cap \bar{C}|} \\ &\geq \min_{e \in \bar{C}} \min_{S \in \text{OPT}: e \in S} \frac{c(S)}{|S \cap \bar{C}|} \\ &\geq \min_{S \in \mathcal{S}: |S \cap \bar{C}|} \frac{c(S)}{|S \cap \bar{C}|} \end{aligned}$$

Here is a simpler argument. Consider the leftover sets in  $\text{OPT}$ , i.e., their intersection with  $\bar{C}$ . Modify them as follows: as long as an element is covered by more than one element in  $\text{OPT}$ , remove the element from all but one. After this modification, the sets are disjoint and still cover  $\bar{C}$ . The modification worsens the cost-effectiveness of the sets. Since the sets are disjoint, the average cost of covering an element is  $\text{OPT}/|\bar{C}|$ . Consider an element  $e$  which is covered at a cost no larger than average and the set  $S$  covering it. This set has a cost-effectiveness of no more than  $\text{OPT}/|\bar{C}|$ .

**Set Cover Via Dual Fitting, pages 108 to 110:**

**Rounding Applied to Set Cover, pages 118 to 120:**

**Set Cover via the Primal-Dual Scheme: pages 124 to 128:**

**The Steiner Problem and Variants: pages 27 to 30, 197 – 230:**

## **2 EATCS-Award 2010**

Have a look at <http://www.eatcs.org/index.php/eatcs-award/623>

## **References**

[Vaz00] V.V. Vazirani. *Approximation Algorithms*. Springer, 2000.