


Geometric Modeling

Summer Semester 2010

Summary

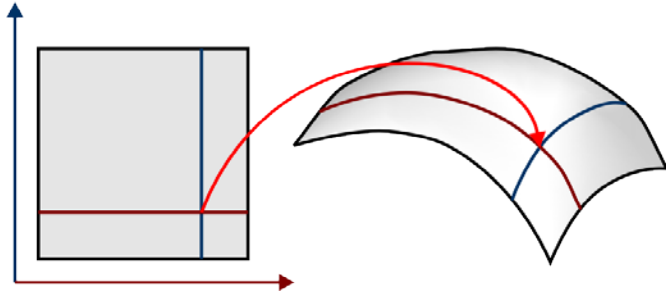
Overview...

Topics:

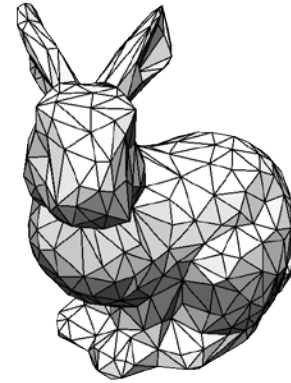
- Introduction
- Mathematical Background
- Interpolation & Approximation
- Polynomial Spline Curves
- Blossoming and Polars
- Rational Spline Curves
- Spline Surfaces
- Triangle Meshes & Multi-Resolution Representations
- Subdivision Surfaces
- Implicit Functions
- Variational Modeling
- Point-Based Modeling
- Summary 

Summary

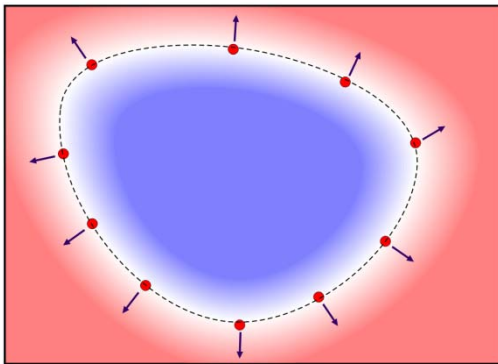
Introduction



Parametric Models



Primitive Meshes



Implicit Models



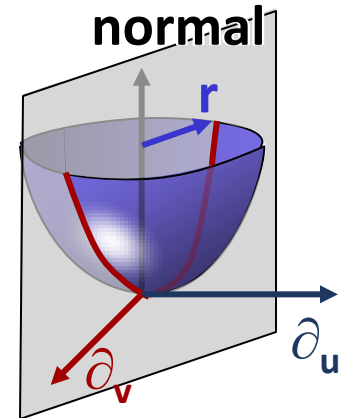
Particle Models

Mathematical Background

Analysis, Numerics & Linear Algebra

Differential Geometry:

- Curve length, curvature, torsion etc.
- Fundamental forms
- Principal curvatures, normal curvature
- Gauss and mean curvature
- Global theorems

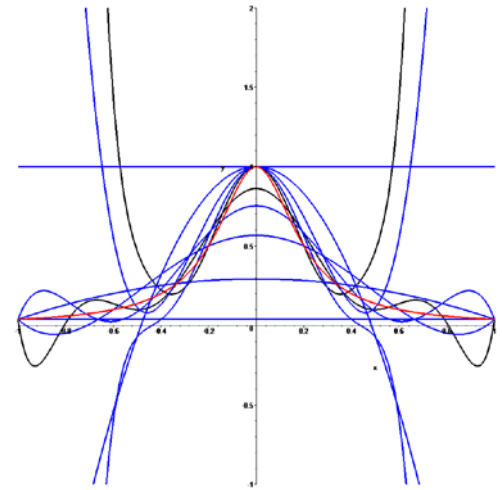
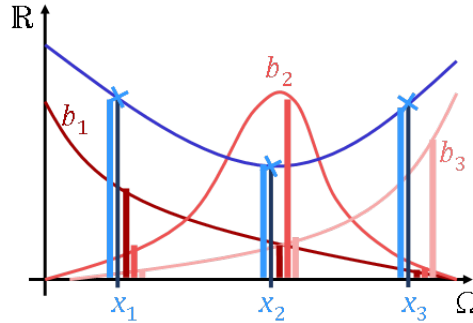


Interpolation & Approximation

Interpolation

Approximation

- Least squares
- Normal equations
- Total least squares / PCA



Normal equations:

$$\begin{pmatrix} \langle \mathbf{b}_1, \mathbf{b}_1 \rangle & \cdots & \langle \mathbf{b}_1, \mathbf{b}_k \rangle \\ \vdots & \ddots & \vdots \\ \langle \mathbf{b}_k, \mathbf{b}_1 \rangle & \cdots & \langle \mathbf{b}_k, \mathbf{b}_k \rangle \end{pmatrix} \begin{pmatrix} \lambda_1 \\ \vdots \\ \lambda_k \end{pmatrix} = \begin{pmatrix} \langle \mathbf{y}, \mathbf{b}_1 \rangle \\ \vdots \\ \langle \mathbf{y}, \mathbf{b}_k \rangle \end{pmatrix}$$

with:

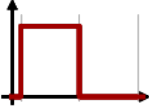
$$\langle \mathbf{b}_i, \mathbf{b}_j \rangle := \sum_{t=1}^n b_i(x_t) \cdot b_j(x_t)$$

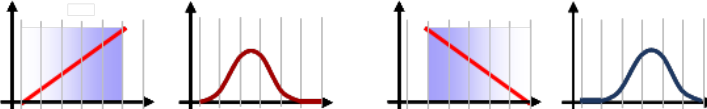
$$\langle \mathbf{y}, \mathbf{b}_i \rangle := \sum_{t=1}^n b_i(x_t) \cdot y_t$$

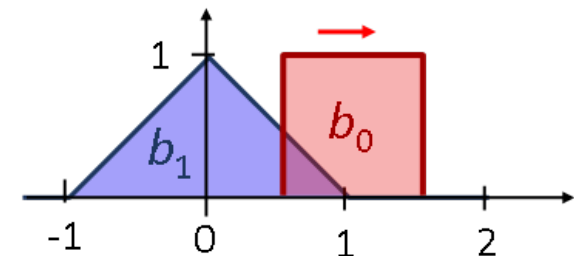
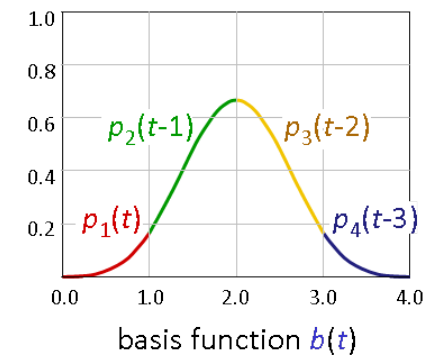
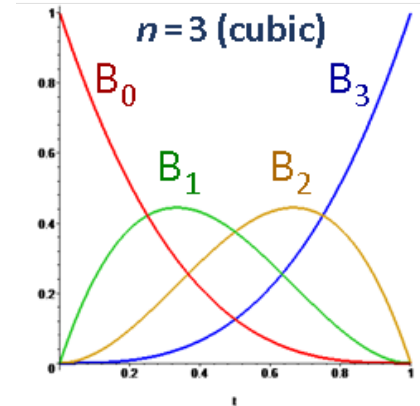
Polynomial Spline Curves

Spline Curves

- General properties (affine invariance, convex hull...)
- Bezier Curves
- B-Splines (Uniform/Non-uniform)
- Other splines (Hermite)

$$N_i^0(t) = \begin{cases} 1, & \text{if } t_{i-1} \leq t < t_i \\ 0, & \text{otherwise} \end{cases}$$


$$N_i^d(t) = \frac{t - t_{i-1}}{t_{i+d-1} - t_{i-1}} N_i^{d-1}(t) + \frac{t_{i+d} - t}{t_{i+d} - t_i} N_{i+1}^{d-1}(t)$$




Polar Forms

Polar Forms / Blossoms:

A *polar form* or *blossom* f of a polynomial F of degree d is a function in d variables:

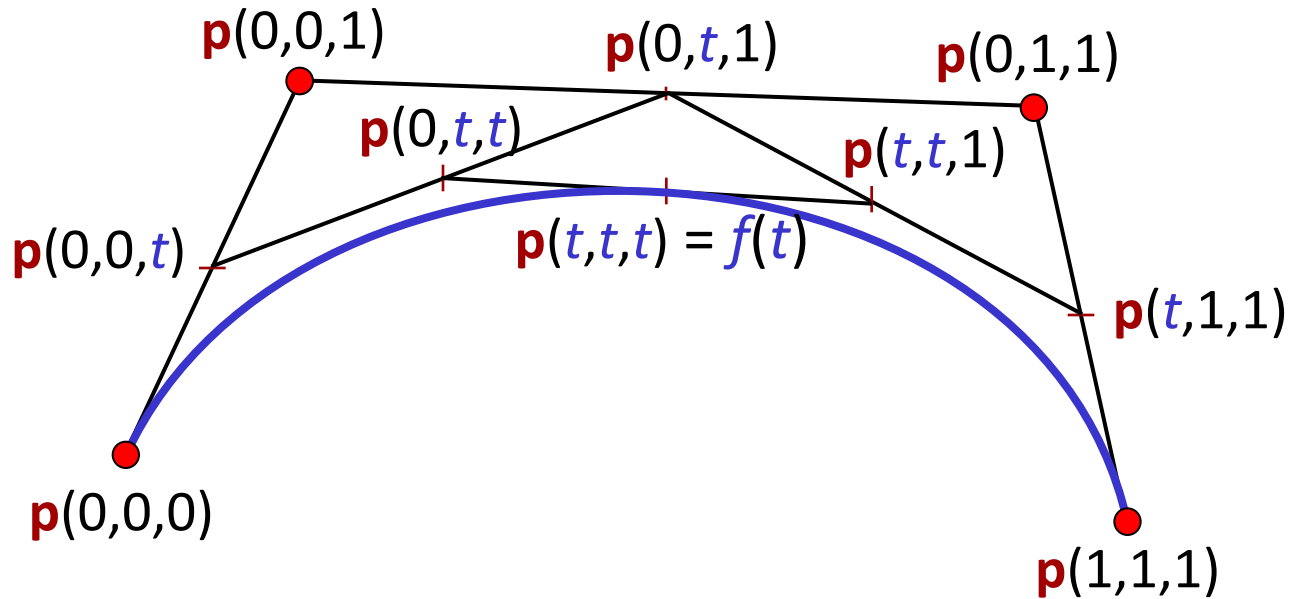
$$F: \mathbb{q} \rightarrow \mathbb{q}$$

$$f: \mathbb{q}^d \rightarrow \mathbb{q}$$

with the following properties:

- *Diagonality*: $f(t, t, \dots, t) = F(t)$
- *Symmetry*: $f(t_1, t_2, \dots, t_d) = f(t_{\pi(1)}, t_{\pi(2)}, \dots, t_{\pi(d)})$
for all permutations of indices π .
- *Multi-affine*: $\sum \alpha_k = 1$
 $\Rightarrow f(t_1, t_2, \dots, \sum \alpha_k t_i^{(k)}, \dots, t_d)$
 $= \alpha_1 f(t_1, t_2, \dots, t_i^{(1)}, \dots, t_d) + \dots + \alpha_n f(t_1, t_2, \dots, t_i^{(n)}, \dots, t_d)$

De Casteljau

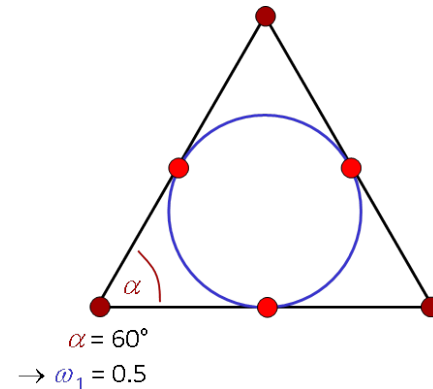
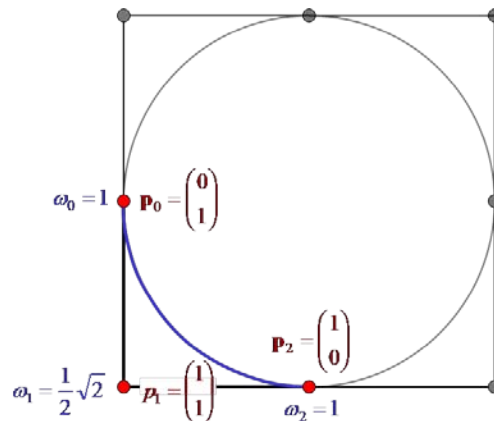
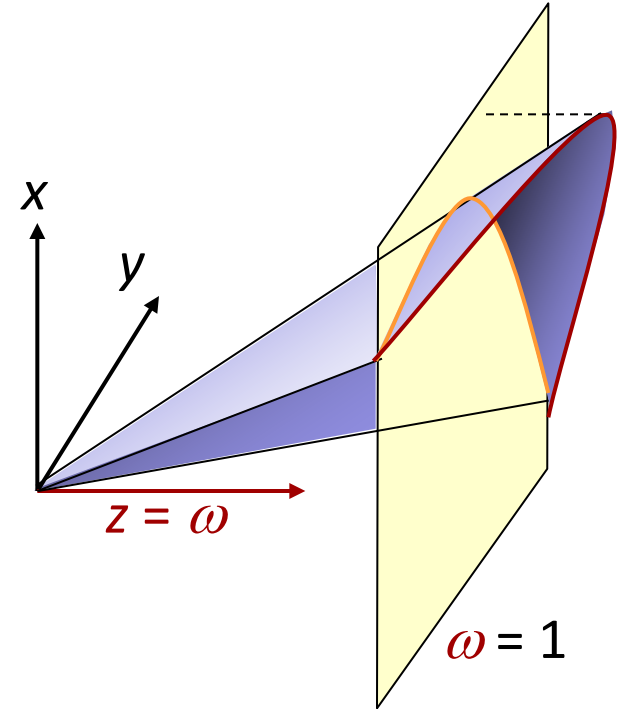


Bezier control points: $p(0,0,0)$, $p(0,0,1)$, $p(0,1,1)$, $p(1,1,1)$

Rational Curves

Rational Curves:

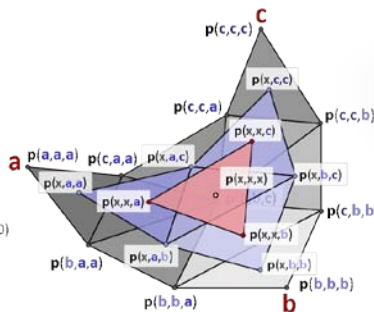
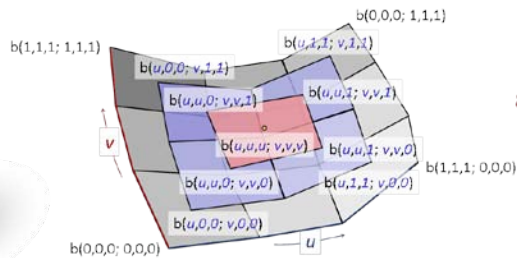
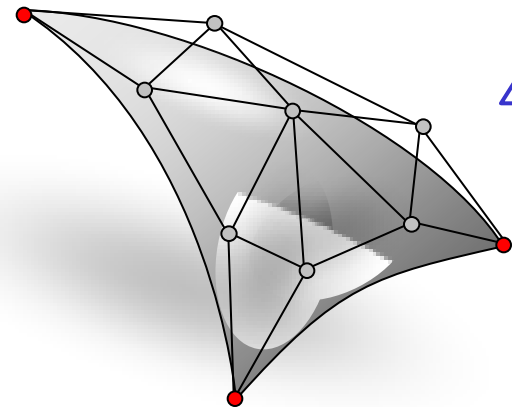
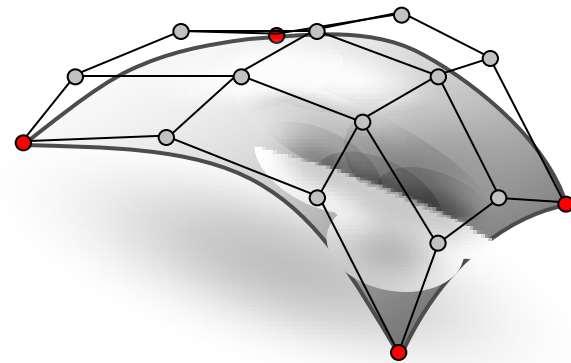
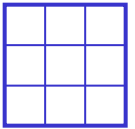
- Conics & Quadrics
- Rational Bezier Splines
- NURBS
- How to create circles



Spline Surfaces

Spline Surfaces

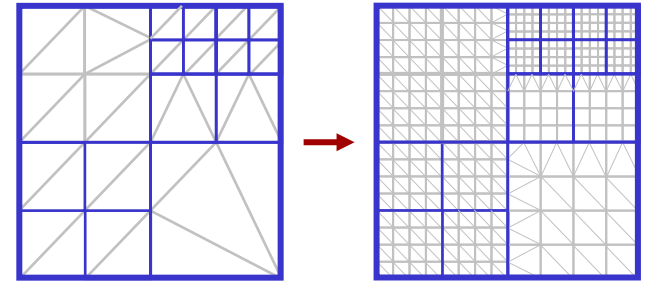
- Tensor product surfaces
- Total degree surfaces
- Surfaces of revolution
- Trimmed patches



Triangle Meshes & Multires

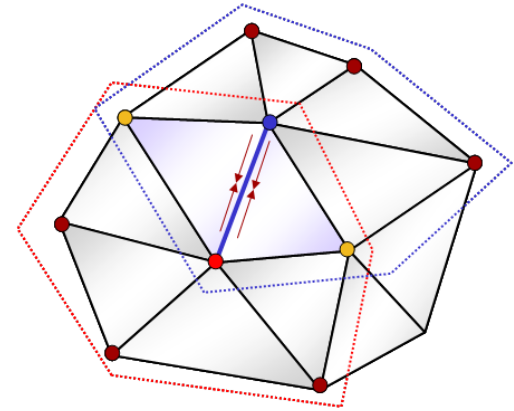
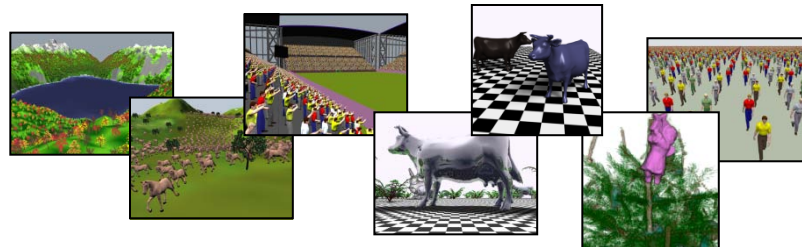
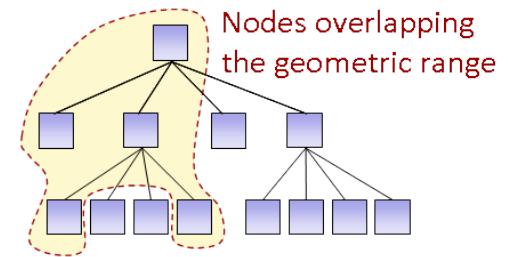
Triangle Meshes:

- Data structures
- Triangulations



Multi-Resolution Techniques:

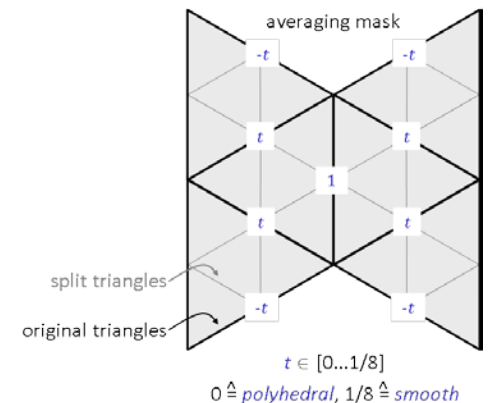
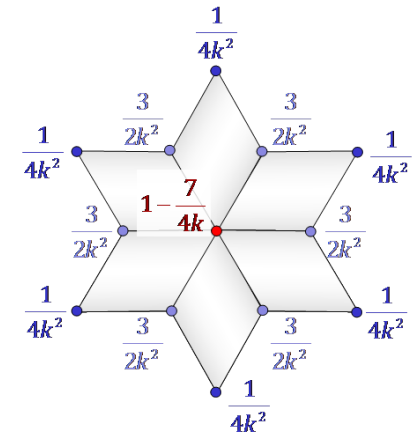
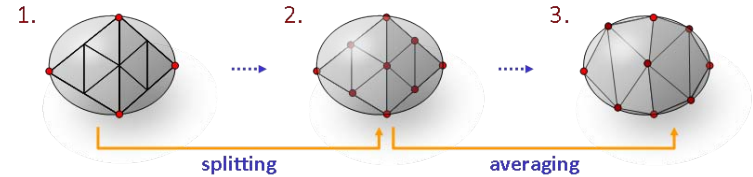
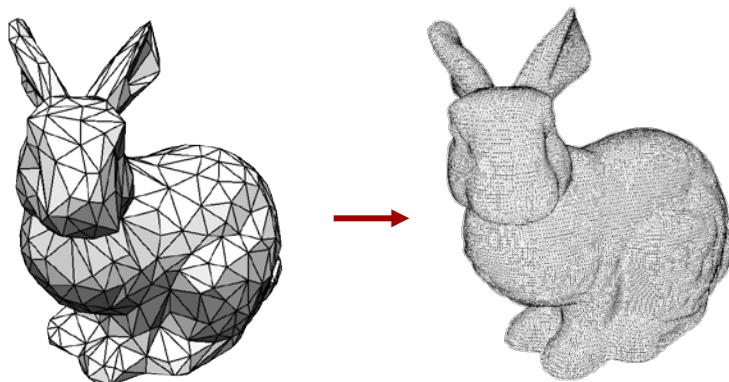
- Hierarchical data structures & algorithms
- Mesh simplification
- Point-based hierarchies



Subdivision Surfaces

Subdivision Surfaces

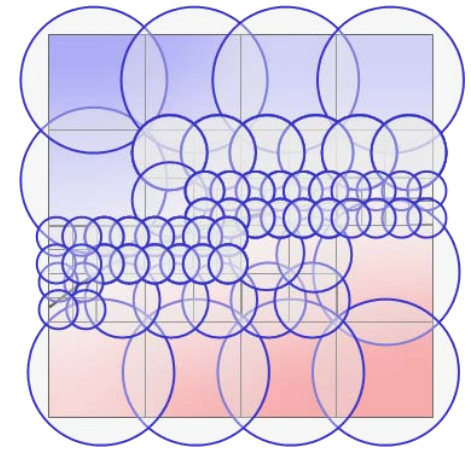
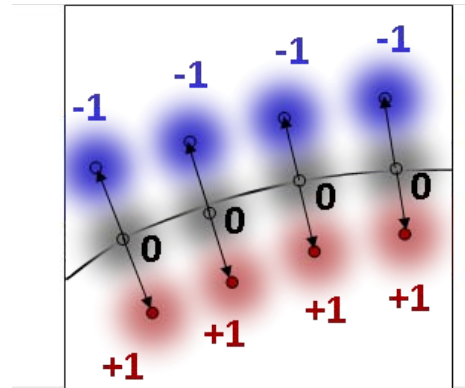
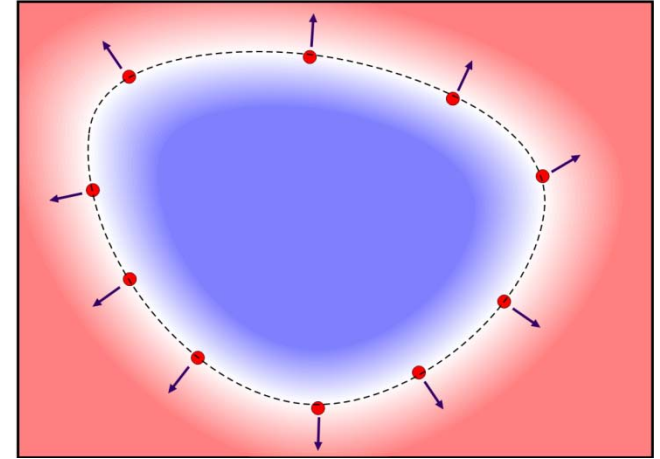
- B-Spline subdivision
- Spectral analysis of subdivision
- Extraordinary vertices
- General subdivision rules
- Wavelets



Implicit Surfaces

Implicit Surfaces:

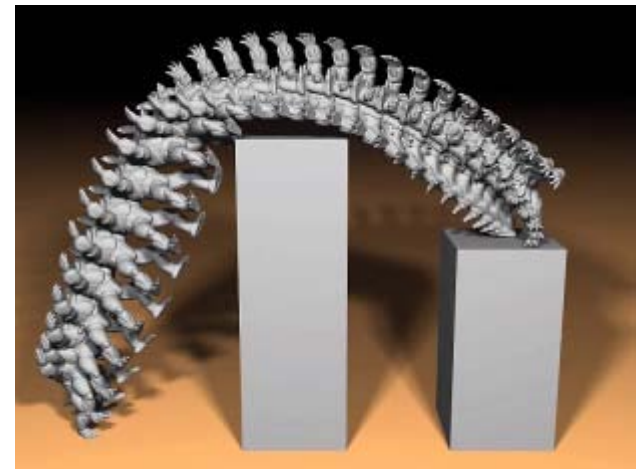
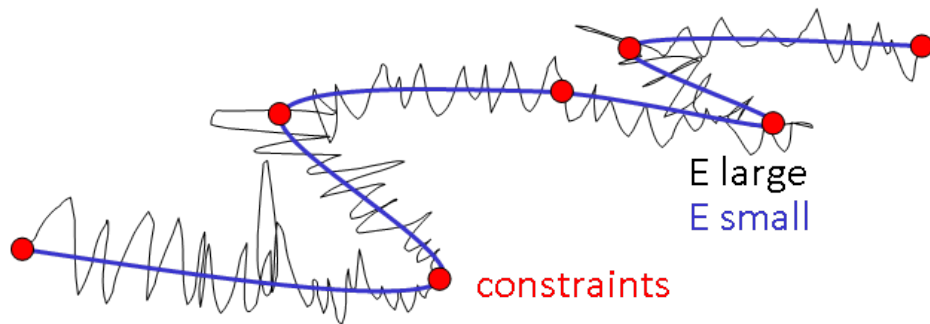
- Mathematical background
- Differential properties
- Representations
- Data Fitting



Variational Modeling

Variational Modeling

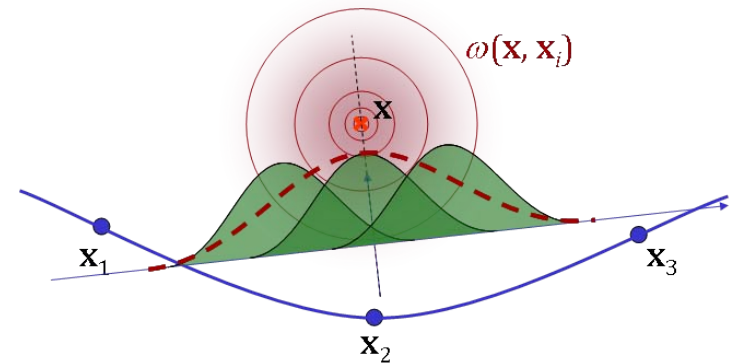
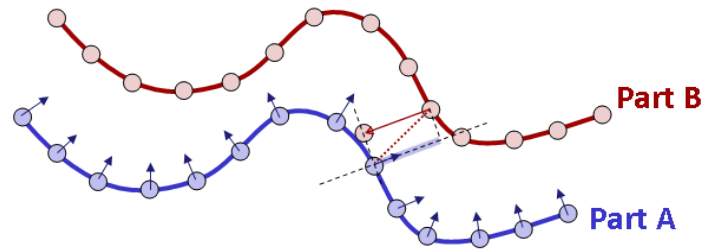
- Energy minimization on function spaces under constraints
- Numerical discretization, finite elements
- Variational Toolkit: Functionals, soft & hard constraints
- Euler-Lagrange equation
- Applications
 - Surface modeling
 - Dynamic modeling



Point-Based Modeling

Point-based modeling:

- 3D scanners
- Surface smoothing & reconstruction
- Surface matching (ICP)
- Moving least squares
- Point-based modeling



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