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Randomized Algorithms and Probabilistic Analysis of Algorithms

Summer 2016

Exercise Set 5

Exercise 1: (5 Points)
Consider the gambler's ruin problem where the game is unfair. The player loses with probability 2/3 and wins with probability 1/3. The player starts at j and finishes at 0 or n. Compute the probability q_j that the player ends at n when starting at j.

Exercise 2: (5 Points)
Consider the gambler's ruin problem as in class, i.e., the game is fair. The player starts at zero and ends either at $-\ell_1$ or at $+\ell_2$. Show that the expected number of games played is $\ell_1\ell_2$.

Exercise 3: (5 Points)
A cat and a mouse each independently take a random walk on a connected, undirected, non-bipartite graph. They start at different nodes, and make one transition at each time step. Prove that the expected number of steps taken until they meet is $O(nm^2)$. Hint: consider a Markov chain whose states are pairs (a, b), where a is the position of the cat and b is the position of the mouse.

Exercise 4: (5 Points)
The n dimension cube has vertices 2^n vertices corresponding to the bitstrings of length n. Two vertices are connected if the corresponding bitstrings differ in exactly one bit. Let $x \in \{0, 1\}^n$ be a state. At each step choose a coordinate i uniformly at random and set the i-th bit of x to 0 or 1 with probability 1/2 each. In other words, we probability 1/2, one stays in x, and with probability 1/2 one flips the i-th bit.
Show that this chain is rapidly mixing. Define the appropriate coupling.