

$$E = \{g(a) \approx b, a \approx b\} \quad \Sigma = \{S, \{a, b, g\}, \neq\}$$

$$E \models g(b) \approx b \quad \forall R \subseteq R \text{ s.t. } g(b) \downarrow_R b \downarrow_R$$

$$\text{First try: } R_0 = \{g(a) \mapsto b, a \mapsto b\}$$

$$\downarrow \text{Let } g(b) \downarrow_{R_0} = g(b), \quad b \downarrow_{R_0} = b$$

$$R_1 = R_0 \cup \{g(b) \mapsto b\} \neq$$

$$g(b) \downarrow_{R_1} = b = b \downarrow_{R_1}$$

$$E_c = \{g(a) \approx b, a \approx b\} \quad \Sigma = \{S, \{a, b, g\}, \emptyset\}$$

$$E_c \models g(b) \approx b$$

$$E_c \stackrel{\text{Symmetry}}{\Rightarrow}_{E_c} E_c \cup \{b \approx a\} = E_1$$

$$\stackrel{\text{Transitivity}}{=}_{E_1} E_1 \cup \{g(a) \approx a\} = E_2$$

$$\stackrel{\text{Congruence}}{=}_{E_2} E_2 \cup \{g(a) \approx g(b)\} = E_3$$

$$\stackrel{\text{Symmetry}}{=}_{E_3} E_3 \cup \{g(b) \approx g(a)\} = E_4$$

$$\stackrel{\text{Transitivity}}{=}_{E_4} E_4 \cup \{g(b) \approx b\}$$

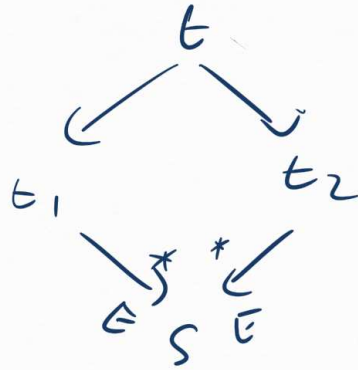
$$E = \{g(a) \approx b, a \approx b\}$$

$$[g(a)] = \{a, b, g(a), g(b), g(g(a)), g(g(b)) \dots\}$$

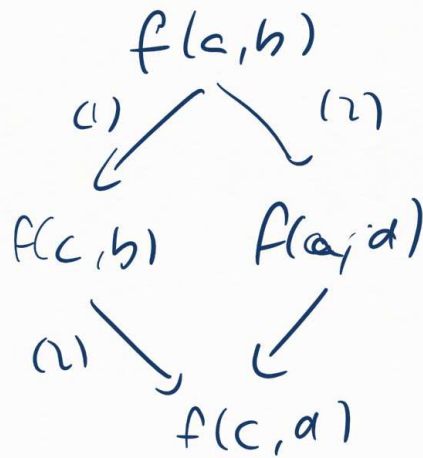
$$= [g(b)] = [a] \dots$$

$$S^{\Gamma} = \{[a]\}$$

local confluence

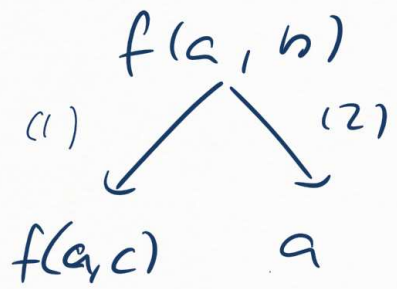


disjunctive rewrites



$a \mapsto c$  (1)  
 $b \mapsto d$  (2)

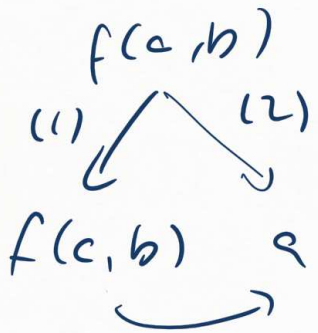
overlap



$$\underline{b} \mapsto c \quad (1)$$

$$f(x, \underline{b}) \mapsto x \quad (2)$$

=> is not confluent



$$a \mapsto c \quad (1)$$

$$f(x, b) \mapsto a \quad (2)$$

=> is confluent

Critical pair  $\begin{matrix} l_1 \rightarrow r_1 \\ l_2 \rightarrow r_2 \end{matrix} \langle r_1 \sigma_1, (l_2 \sigma_2) [\sigma_2 \sigma_1] \rho \rangle$

$$R = \left\{ \underbrace{f(f(x, x), y)}_{l_1} \rightarrow \underbrace{g(x)}_{r_1}, \underbrace{f(z, y')}_{l_2} \rightarrow \underbrace{z}_{r_2} \right\}$$

1)  $f(f(x, x), y)$  and  $f(z, y')$  overlap  $\rho = \varepsilon$

$$\sigma_1 = \{z \mapsto f(x, x), y' \mapsto y\}$$

$$cp = \langle g(x) \sigma_1, z \sigma_1 \rangle = \langle g(x), f(x, x) \rangle$$

2)  $f(x, x)$  and  $f(z, y')$  overlap,  $\sigma_2 = \{z \mapsto x, y' \mapsto x\}$

$$cp = \langle g(x) \sigma_2, f(z \sigma_2, y') \rangle = \langle g(x), f(x, x) \rangle$$