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Tutorials for "Automated Reasoning"
Exercise sheet 2

Exercise 2.1: *(3 P)*
Consider a strict ordering $\prec$ on a set M and prove or refute by a counter-example:

1. if for any elements $m_1, m_2 \in M$ there is an element $m \in M$ so that $m_1 \prec m$ and $m_2 \prec m$ then $\prec$ is total
2. if there exists an infinite ascending chain $m_0 \prec m_1 \prec m_2 \prec \dots$ then $\prec$ is not well-founded
3. if $\prec$ is well-founded then $\prec$ is total

Exercise 2.2: *(2+2 P)*
Let $\succ$ be the total ordering on the propositional variables $S \succ R \succ Q \succ P$.

1. For each of the following pairs of triples, determine the lexicographic ordering relationship between the first and the second triple: $((P, P, R)\ ?\ (P, Q, R))$, $((S, P, R)\ ?\ (Q, R, S))$, $((Q, P, P)\ ?\ (P, R, R))$, $((P, Q, P)\ ?\ (P, P, Q))$ generated by the lexicographic extension of $\succ$.
2. Consider the triples from above as multisets and determine the relationship generated by the multiset extension of $\succ$.

Exercise 2.3: *(4 P)*
For an alphabet Σ with a well-founded ordering $>_\Sigma$ let the relation $\succ$ be defined as

$$w \succ w' :\Leftrightarrow |w| > |w'| \text{ or } (|w| = |w'| \text{ and } w >_{\Sigma,\text{lex}} w').$$

Prove that $\succ$ is a well-founded ordering on Σ^*.

Exercise 2.4: *(3 P)*
Let M be the set $\{a, b, c\}$. Provide an ordering $\succ$ of M so that the following statements hold for the multiset extension $(\succ_{\text{mul}})_{\text{mul}}$ of the multiset extension of $\succ$.

1. $\{\{a, b\}, \{c\}\}(\succ_{\text{mul}})_{\text{mul}}\{\{a\}, \{b, c\}\}$ and
2. $\{\{b\}, \{c, c\}\}(\succ_{\text{mul}})_{\text{mul}}\{\{b, b, b\}, \{c\}\}$.

Are there other possible orderings?

Submit your solution in lecture hall E1.3, Room 002 during the lecture on November 04. Please write your name and the date of your tutorial group (Mon, Thu) on your solution.

Joint solutions are not permitted, please submit individually. However, I encourage you working and solving the exercises in a group.