Lecture “Automated Reasoning”
(Winter Term 2016/2017)
Final Examination

Name: .................................................................................................

Student Number: .................................................................................

Some notes:

- Things to do at the beginning:
  Put your student card and identity card (or passport) on the table.
  Switch off mobile phones.
  Whenever you use a new sheet of paper (including scratch paper), first
  write your name and student number on it.

- Things to do at the end:
  Mark every problem that you have solved in the table below.
  Stay at your seat and wait until a supervisor staples and takes your
  examination text.
  Note: Sheets that are accidentally taken out of the lecture room are
  invalid.

Sign here: Good luck!

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Problem 1 \textit{(Superposition Refutation)} (4 points)

Refute the following set of clauses by superposition, including all redundancy rules. You can freely choose an ordering and selection function. As usual one sort for everything and $x, y, z$ are all variables.

1. $\neg R(x, y) \lor \neg R(y, z) \lor R(x, z)$
2. $\neg R(x, x)$
3. $R(x, g(x))$
4. $\neg R(x, y) \lor R(y, x)$
Problem 2 (Superposition Model Building) (4 + 2 = 6 points)
Consider the below clause set $N$ over predicate $R$, function $g$ and constant $a$ with respect to an LPO with precedence $g \succ R \succ a$. As usual one sort for everything and $x, y$ are variables.

1. Compute $N \prec R(g(a), g(a))$ and determine the minimal false clause.
2. Do the respective superposition inference with the minimal false clause, add it to $N$ giving $N'$ and recompute $(N') \prec R(g(a), g(a))$. 

1. $\neg R(x, y) \lor R(y, x)$
2. $\neg R(x, x)$
3. $R(x, g(x))$
4. $\neg R(g(a), a)$
Problem 3 \((CDCL)\) (5 points)

Check satisfiability of the below propositional clauses using \(\Rightarrow_{CDCL}\).

1 \(\neg P_4 \lor P_3\)  2 \(\neg P_3 \lor P_4\)  3 \(P_1 \lor P_2 \lor P_4\)
4 \(\neg P_3 \lor \neg P_4\)  5 \(\neg P_1 \lor \neg P_4 \lor P_2\)  6 \(\neg P_2 \lor \neg P_4 \lor P_1\)
7 \(\neg P_1 \lor \neg P_2 \lor P_4\)
Problem 4 \((CNF)\)  
(6 points)  
Transform the following formula into CNF using \(\Rightarrow_{ACNF}\). As usual one sort for everything.

\[
\forall x. \exists y. \forall z. \exists u. (R(x, y) \rightarrow (R(g(u), g(z)) \leftrightarrow R(u, z)))
\]
Problem 5 \((Tableau)\) (4 points)
Prove validity of the following formula using standard Tableau (don’t use free-variable Tableau). As usual one sort for everything.

\[
[(\exists x.\forall y R(x, y)) \land (\forall x, y. (R(x, y) \rightarrow R(y, x)))] \rightarrow \exists x.\forall y R(y, x)
\]
Problem 6 (Knuth Bendix Completion)

Apply $\Rightarrow_{\text{KBC}}$ to the following set of equations. Choose an appropriate ordering. As usual one sort for everything.

$$E = \{ f(g(x), x) \approx h(x), \ f(g(x), h(y)) \approx f(x, y), \ h(a) \approx a \}$$
Problem 7 (Conjectures) (2 + 2 + 2 = 6 points)

Which of the following statements are true or false? Provide a proof or a counter example.

1. Let $s, t$ be two terms with unifier $\sigma$. Then every term in $\text{codom}(\sigma)$ is a subterm of $s$ or a subterm of $t$.

2. Let $C \lor A$ and $D \lor \neg A$ be two first-order ground clauses. Let $A$ and $\neg A$ be strictly maximal literals in their respective clauses. Then the clause $C \lor D$, the result of a superposition left inference, is smaller than both parent clauses.

3. Let $N$ be a set of satisfiable ground clauses. Assume $N$ is saturated by superposition up to redundancy where in every clause containing a negative literal, one negative literal is always selected. Then $N_I = \{ A \mid A$ is a positive unit clause in $N \}$. 
Problem 8 (Standard Unification) (4 points)

Let $s, t$ be two linear, unifiable terms such that $\text{vars}(s) \cap \text{vars}(t) = \emptyset$. Recall a term is linear, if every variable occurs at most once in the term. Let
\[
\{s = t\} \Rightarrow^*_{\text{SU}} \{x_1 = l_1, \ldots, x_n = l_n\}
\]
be a derivation resulting in the above solved form. Prove that each $l_i$ is either a subterm of $s$ or $t$. 