Lecture “Automated Reasoning”
(Winter Term 2016/2017)

Midterm Examination

Name: ...........................................................................................................

Student Number: .........................................................................................

Some notes:

• Things to do at the beginning:
  Put your student card and identity card (or passport) on the table.
  Switch off mobile phones.
  Whenever you use a new sheet of paper (including scratch paper), first
  write your name and student number on it.

• Things to do at the end:
  Mark every problem that you have solved in the table below.
  Stay at your seat and wait until a supervisor staples and takes your
  examination text.
  Note: Sheets that are accidentally taken out of the lecture room are
  invalid.

Sign here: Good luck!

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**Problem 1 (Superposition Refutation)** (4 points)

Refute the following clause set by superposition where you may apply the reduction rules Condensation, and Subsumption Resolution. Use the ordering $P_4 \succ P_3 \succ P_2 \succ P_1$. You may also make use of a selection function.

1. $P_2 \lor P_4$
2. $P_1 \lor P_4$
3. $\neg P_2 \lor P_1$
4. $P_1 \lor \neg P_3$
5. $P_1 \lor \neg P_3$
6. $\neg P_3 \lor P_2$
7. $\neg P_1 \lor P_3$
8. $\neg P_4 \lor P_3$
9. $\neg P_1 \lor P_2 \lor P_3$
10. $\neg P_2 \lor \neg P_3$
11. $\neg P_4 \lor \neg P_2 \lor P_3$
Problem 2 (Superposition Model Building) \hspace{1cm} (4 + 2 + 2 = 8 points)

Consider the below clause set with atom ordering \( P_5 \succ P_4 \succ P_3 \succ P_2 \succ P_1 \).

\[
\begin{array}{cccc}
1 & P_1 \lor P_2 \lor P_2 & 2 & \neg P_1 \lor \neg P_2 & 3 & \neg P_2 \lor \neg P_3 \\
4 & P_1 \lor P_3 & 5 & P_4 \lor P_5 \lor P_1 & 6 & \neg P_4 \lor P_1 \\
7 & \neg P_4 \lor P_2 & 8 & \neg P_5 \lor P_2 & 9 & \neg P_5 \lor \neg P_3 \\
\end{array}
\]

(a) Compute \( N_I \).

(b) Determine the minimal false clause in \( N_I \). Perform the respective superposition inference on the clause. Add the derived clause to \( N \) resulting in \( N' \) and compute \( N'_I \).

(c) Determine the minimal false clause in \( N'_I \). Perform the respective superposition inference on the clause. Add the derived clause to \( N' \) resulting in \( N'' \) and compute \( N''_I \).
Problem 3 \((CDCL)\)  
(7 points)

Check via CDCL whether the below clause set is satisfiable.

\[
\begin{align*}
1 & \quad P_{11} \lor P_{12} & 2 & \quad P_{21} \lor P_{22} & 3 & \quad P_{31} \lor P_{32} \\
4 & \quad P_{41} \lor P_{42} & 5 & \quad \neg P_{11} \lor P_{42} & 6 & \quad \neg P_{42} \lor P_{11} \\
7 & \quad \neg P_{11} \lor \neg P_{21} & 8 & \quad \neg P_{11} \lor \neg P_{31} & 9 & \quad \neg P_{31} \lor \neg P_{41} \\
10 & \quad \neg P_{12} \lor \neg P_{22} & 11 & \quad \neg P_{32} \lor \neg P_{42} & 12 & \quad \neg P_{12} \lor \neg P_{32}
\end{align*}
\]
Problem 4 \((CNF)\) 

Transform the formula 

\[
(P \lor ((Q \leftrightarrow \top) \land \neg R)) \lor (P \leftrightarrow (Q \leftrightarrow \bot))
\]

into CNF using \(\Rightarrow_{ACNF}\).
**Problem 5** *(Tableau)* 

Prove that the formula

\[ (((\neg P \lor \neg R) \rightarrow Q) \rightarrow (\neg Q \rightarrow (P \land R))) \]

is valid using tableau. You may use a tree representation of the tableau.
Problem 6 (Conjectures) \hspace{1cm} (2 + 2 + 2 = 6 points)

Which of the following statements are true or false? Provide a proof or a counter example.

1. If $N_I | N$ then $N$ is saturated up to redundancy.

2. If all clauses in $N$ have at most one positive literal and the CDCL rule Propagate is not applicable to the state $(\epsilon; N; \emptyset; 0; \top)$ then $N$ is satisfiable.

3. If all clauses in $N$ have at most one positive literal and there is no clause in $N$ having only negative literals then $N_I | N$. 
Consider a reasonable CDCL run

$$(\epsilon; N; \emptyset; \top) \Rightarrow^{*}_{\text{CDCL}} (L_1 \ldots L_n; N; \emptyset; k; D)$$

where the last applied rule was Conflict and hence $D \not\in \{\top, \bot\}$. Consider the atom ordering $\text{atom}(L_1) \prec \text{atom}(L_2) \prec \ldots \prec \text{atom}(L_n)$. Prove that any of the subsequent CDCL Resolve steps until backtracking is a Superposition Left inference with respect to $\prec$, where clauses are always condensed.