Exercise 12.1: (4+4 P)
Consider the following valid first-order formula, as usual one sort \( S \) for everything,
\[
(\exists x. \forall y. (F(y, x) \leftrightarrow F(y, y))) \rightarrow \neg \forall w. \exists x. \forall y. (F(y, x) \leftrightarrow \neg F(y, w))
\]
and show validity by first-order superposition.

1. Transform the formula, i.e., its negation, into CNF using \( \Rightarrow_{ACNF} \).
2. Refute the resulting clause set by first-order superposition. You may freely choose an appropriate ordering and selection function.

Exercise 12.2: (4+4 P)
Consider a signature with function symbols \( g, a \) and \( E := \{g(g(g(g(a)))) \approx a, g(g(x)) \approx x\} \), as usual one sort \( S \) for everything.

1. Prove that \( g(a) \approx_E a \) using the inference system \( \Rightarrow_E \).
2. Prove that \( g(a) \approx_E a \) using the inference system \( \Rightarrow_{KBC} \). Choose an appropriate ordering, KBO or LPO.

Exercise 12.3: (2 P)
Compute the Craig interpolant, following the proof presented in the lecture, for the two propositional clause sets \( N_\phi = \{\neg P \lor Q \lor R, \neg Q \lor S, \neg R \lor S, P \lor \neg T\} \) and \( N_\psi = \{\neg T \lor S\} \). Note that \( N_\phi \models N_\psi \).

Submit your solution in lecture hall E1.3, Room 001 during the lecture on February 14. Please write your name and the date/time of your tutorial group (Wed-Fabian, Wed-Tobias) on your solution.

Joint solutions, prepared by up to three persons together, are allowed (but not encouraged). If you prepare your solution jointly, submit it only once and indicate all authors on the sheet.