Exercise 13.1: \((2+2\ P)\)
Consider the following ground clause set

\[
\{ P(g(a)) \lor R(a,b), \neg R(a,b) \lor P(c), \neg P(g(a)) \lor \neg P(c) \}
\]

and the ground superposition calculus.

1. Prove that with an empty selection function there is always an inference possible, independently which instance of KBO, LPO is chosen.

2. For both KBO, LPO: provide a selection function and an instance of the respective ordering such that no superposition inference is possible.

Exercise 13.2: \((2+2\ P)\)
Consider the below clause set with a KBO where every symbol has weight 1 and \(P \succ Q \succ g \succ b \succ a\), atoms are considered with respect to the KBO like terms and an empty selection function.

\[
N = \{ \neg P(a) \lor Q(g(a)), \neg Q(b) \lor P(a), P(a) \lor \neg P(g(a)), P(b) \lor P(g(a)) \}
\]

1. Compute \(N_\mathcal{I}\).

2. Perform the inference between the minimal false clause and its productive counterpart and recompute the partial model with respect to the changed clause set.
Exercise 13.3: (4 P)
Refute the below clauses by first-order superposition, with ordering KBO where every symbol has weight 1 and $R \succ Q \succ f \succ g \succ b \succ a$, atoms are considered with respect to the KBO like terms and an empty selection function.

$$R(b, g(a)), \neg R(x, y) \lor \neg R(y, x), \neg R(x, y) \lor Q(f(x, y)), \neg Q(x) \lor \neg Q(f(y, g(x))), Q(a)$$

Exercise 13.4: (3 P)
Apply the reduction rules Subsumption, Tautology Deletion, Subsumption Resolution, and Condensing exhaustively to the below clauses:

$$P(x) \lor R(x, y) \lor P(a), \neg P(a) \lor R(x, z) \lor R(y, z), P(x) \lor R(x, y), P(x) \lor \neg P(y) \lor \neg R(x, y)$$

Submit your solution in lecture hall E1.3, Room 002 during the lecture on February 10. Please write your name and the date of your tutorial group (Mon, Thu) on your solution.

Joint solutions are not permitted, please submit individually. However, I encourage you working and solving the exercises in a group.