max planck institut informatik

Universität des Saarlandes
**FR Informatik**

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**Tutorials for "Automated Reasoning"**
**Exercise sheet 2**

**Exercise 2.1:** *(4 P)*
Determine which of the following formulas are valid/satisfiable/unsatisfiable:

1. $(P \land Q) \rightarrow (P \lor Q)$
2. $(P \lor Q) \rightarrow (P \land Q)$
3. $(\neg P \rightarrow Q) \rightarrow ((\neg P \rightarrow \neg Q) \rightarrow P)$
4. $\neg(P \rightarrow \neg P)$
5. $\neg\big(P \lor \neg(P \land Q)\big)$
6. $(P \lor \neg Q) \land \neg(\neg P \rightarrow \neg Q)$
7. $\big((P \rightarrow Q) \land (\neg P \rightarrow R)\big) \rightarrow (Q \lor R)$
8. $\neg(P \lor Q) \leftrightarrow (\neg P \land \neg Q)$

In case of satifiability present a model and a counter-model. In case of validity/unsatisfiability present a semantic argument.

**Exercise 2.2:** *(4 P)*
Consider the new connective $\oplus$ where $\mathcal{A}(\phi \oplus \psi) = 1$ iff exactly one of $\mathcal{A}(\phi) = 1$ or $\mathcal{A}(\psi) = 1$.

1. Define $\oplus$ in terms of the known connectives, i.e., define a valid formula $(\phi \oplus \psi) \leftrightarrow \chi$ where $\chi$ does not contain $\oplus$.
2. The operator $\oplus$ is extended from binary to $n$-ary by $\mathcal{A}(\phi_1 \oplus \ldots \oplus \phi_n) = 1$ iff $\sum_{i=1}^{n} \mathcal{A}(\phi_i)$ is odd. Define the $n$-ary operator in terms of the known connectives.

Justify both results.

**Exercise 2.3:** (*4 P*)
Prove the validity of the following formulas using tableau.

1. $(P \to (Q \to R)) \to ((P \to Q) \to (P \to R))$
2. $(P \to Q) \to ((R \vee P) \to (R \vee Q))$

**Exercise 2.4:** (*6 P*)
Let $\phi$ contain propositional variables $P_1, \dots, P_n$. Let $\phi'$ be obtained from $\phi$ by replacing every occurrence of a $P_i$ by $\neg P_i$. Then prove: $\phi$ is valid iff $\phi'$ is.

Submit your solution in lecture hall E1.3, Room 001 during the lecture on November 15. Please write your name and the date of your tutorial group (Mon, Wed) on your solution.

Joint solutions, prepared by up to three persons together, are allowed (but not encouraged). If you prepare your solution jointly, submit it only once and indicate all authors on the sheet.