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Tutorials for “Automated Reasoning”
Exercise sheet 7

Exercise 7.1: *(1+1+2+1 P)*
Prove the following properties of the superposition partial model construction.

1. For every D with $(C \lor \neg P) \prec D$ we have $\delta_D \neq \{P\}$.
2. If $\delta_C = \{P\}$ then $N_C \cup \delta_C \models C$.
3. If $N_C \models D$ and $D \prec C$ then for all C' with $C \prec C'$ we have $N_{C'} \models D$ and in particular $N_{\mathcal{I}} \models D$.
4. There is no clause C with $P \lor P \prec C$ such that $\delta_C = \{P\}$.

Solution:

1. If $(C \lor \neg P) \prec D$ then P cannot be maximal in D because $P \prec \neg P$. Only strictly maximal literals are produced, so $\delta_D \neq \{P\}$.

2. If $\delta_C = \{P\}$ then $C = C' \lor P$ and so in particular $\{P\} \models C$.

3. If $N_C \models D$ then there are two cases. Firstly, $D = P \lor D'$ and $P \in N_C$. Since the model construction adds only positive literals, also $P \in N_{\mathcal{I}}$ and we are done. Secondly, if $D = \neg P \lor D'$ and $P \notin N_C$ then P cannot be produced by any clause C', $C \prec C'$ because for otherwise we would have $C' = C'' \lor P$ with P strictly maximal and at the same time $\neg P \lor D' \prec C'' \lor P$, a cotradiction.

Exercise 7.2: *(4 P)*
Consider the clause set

$$N = \{\neg P \lor \neg R, R \lor S \lor Q, \neg S \lor R, \neg Q \lor R, R \lor P \lor S\}$$

and the CDCL state $(P^1, N, \emptyset, 1, \top)$. Continue the application of CDCL rules to this state (don’t use Forget, Restart) until a contradiction is derived or a model is found. Hint: prefer Conflict and Propagate over the other rules.

Solution:

$$
\begin{array}{ll}
 & (P^1, N, \emptyset, 1, \top) \\
\Rightarrow^{\text{Propagate}}_{\text{CDCL}} & (P^1 \neg R^{\neg P \vee \neg R}, N, \emptyset, 1, \top) \\
\Rightarrow^{\text{Propagate}}_{\text{CDCL}} & (P^1 \neg R^{\neg P \vee \neg R} \neg S^{\neg S \vee R}, N, \emptyset, 1, \top) \\
\Rightarrow^{\text{Propagate}}_{\text{CDCL}} & (P^1 \neg R^{\neg P \vee \neg R} \neg S^{\neg S \vee R} \neg Q^{\neg Q \vee R}, N, \emptyset, 1, \top) \\
\Rightarrow^{\text{Conflict}}_{\text{CDCL}} & (P^1 \neg R^{\neg P \vee \neg R} \neg S^{\neg S \vee R} \neg Q^{\neg Q \vee R}, N, \emptyset, 1, R \vee S \vee Q) \\
\Rightarrow^{\text{Resolve}}_{\text{CDCL}} & (P^1 \neg R^{\neg P \vee \neg R} \neg S^{\neg S \vee R}, N, \emptyset, 1, R \vee S) \\
\Rightarrow^{\text{Resolve}}_{\text{CDCL}} & (P^1 \neg R^{\neg P \vee \neg R}, N, \emptyset, 1, R) \\
\Rightarrow^{\text{Backtrack}}_{\text{CDCL}} & (R^R, N, \{R\}, 0, \top) \\
\Rightarrow^{\text{Propagate}}_{\text{CDCL}} & (R^R \neg P^{\neg P \vee \neg R}, N, \{R\}, 0, \top)
\end{array}
$$

The partial valuation $\{R, \neg P\}$ already satisfies N, so any further application of Decide, Propagate will do.

Exercise 7.3: *(3+3+3 P)*
Consider the clause set

$N = \{P \vee Q \vee S, P \vee Q \vee \neg S, P \vee \neg Q \vee S, P \vee \neg Q \vee \neg S, \neg P \vee Q \vee S, \neg P \vee Q \vee \neg S, \neg P \vee \neg Q \vee S, \neg P \vee \neg Q \vee \neg S\}$

and refute it by

1. Semantic Tableaux
2. Propositional Superposition with Redundancy
3. CDCL

Solution:

1. Straightforward

2. Applying Subsumption Resolution pairwise to the clauses of N from left to right on S generates

$$P \vee Q, P \vee \neg Q, \neg P \vee Q, \neg P \vee \neg Q$$

doing this once more on Q results in

$$P, \neg P$$

and finally in

$$\perp$$

3. I show the start ... the rest is then similar to 7.2.

$$
\begin{array}{ll}
 & (\epsilon, N, \emptyset, 0, \top) \\
\Rightarrow^{\text{Decide}}_{\text{CDCL}} & (P^1, N, \emptyset, 1, \top) \\
\Rightarrow^{\text{Decide}}_{\text{CDCL}} & (P^1 Q^2, N, \emptyset, 2, \top) \\
\Rightarrow^{\text{Propagate}}_{\text{CDCL}} & (P^1 Q^2 S^{\neg P \vee \neg Q \vee S}, N, \emptyset, 2, \top) \\
\Rightarrow^{\text{Conflict}}_{\text{CDCL}} & (P^1 Q^2 S^{\neg P \vee \neg Q \vee S}, N, \emptyset, 2, \neg P \vee \neg Q \vee \neg S) \\
\Rightarrow^{\text{Resolve}}_{\text{CDCL}} & (P^1 Q^2, N, \emptyset, 2, \neg P \vee \neg Q) \\
\Rightarrow^{\text{Backtrack}}_{\text{CDCL}} & (P^1 \neg Q^{\neg P \vee \neg Q}, N, \{\neg P \vee \neg Q\}, 1, \top) \\
\Rightarrow^{\text{Propagate}}_{\text{CDCL}} & (P^1 \neg Q^{\neg P \vee \neg Q} S^{\neg P \vee Q \vee S}, N, \{\neg P \vee \neg Q\}, 1, \top) \\
\Rightarrow^{\text{Conflict}}_{\text{CDCL}} & (P^1 \neg Q^{\neg P \vee \neg Q} S^{\neg P \vee Q \vee S}, N, \{\neg P \vee \neg Q\}, 1, \neg P \vee Q \vee \neg S) \\
\Rightarrow^{\text{Resolve}}_{\text{CDCL}} & (P^1 \neg Q^{\neg P \vee \neg Q}, N, \{\neg P \vee \neg Q\}, 1, \neg P \vee Q) \\
\Rightarrow^{\text{Resolve}}_{\text{CDCL}} & (P^1, N, \{\neg P \vee \neg Q\}, 1, \neg P) \\
\Rightarrow^{\text{Backtrack}}_{\text{CDCL}} & (\neg P^{\neg P}, N, \{\neg P \vee \neg Q, \neg P\}, 0, \top) \\
 & \quad \vdots
\end{array}
$$

Actually this shows a typical phenomenon of CDCL runs: the calculus produces directly one after the other learned clauses that subsume each other, i.e., they get stronger, here $\neg P \vee \neg Q$ and $\neg P$.