Firstly, we define the classic Herbrand interpretations for formulas without equality.

### 3.5.1 Definition (Herbrand Interpretation)

A *Herbrand Interpretation* (over $\Sigma$) is a $\Sigma$-algebra $\mathcal{H}$ such that

1. $S^\mathcal{H} := T_S(\Sigma)$ for every sort $S \in S$
2. $f^\mathcal{H} : (s_1, \ldots, s_n) \mapsto f(s_1, \ldots, s_n)$ where $f \in \Omega$, $\text{arity}(f) = n$, $s_i \in S^\mathcal{H}$ and $f : S_1 \times \ldots \times S_n \to S$ is the sort declaration for $f$
3. $P^\mathcal{H} \subseteq (S_1^\mathcal{H} \times \ldots \times S_m^\mathcal{H})$ where $P \in \Pi$, $\text{arity}(P) = m$ and $P \subseteq S_1 \times \ldots \times S_m$ is the sort declaration for $P$
3.5.2 Lemma (Herbrand Interpretations are Well-Defined)

Every Herbrand Interpretation is a $\Sigma$-algebra.
3.5.3 Proposition (Representing Herbrand Interpretations)

A Herbrand interpretation $\mathcal{A}$ can be uniquely determined by a set of ground atoms $I$

$$(s_1, \ldots, s_n) \in P^\mathcal{A} \text{ iff } P(s_1, \ldots, s_n) \in I$$
3.5.5 Theorem (Herbrand)

Let $N$ be a finite set of $\Sigma$-clauses. Then $N$ is satisfiable iff $N$ has a Herbrand model over $\Sigma$ iff $\text{ground}(\Sigma, N)$ has a Herbrand model over $\Sigma$. 