2.9.5 Definition (Reasonable CDCL Strategy)

A CDCL strategy is *reasonable* if the rules Conflict and Propagate are always preferred over all other rules.

2.9.6 Proposition (CDCL Basic Properties)

Consider CDCL run deriving $(M; N; U; k; C)$ by any strategy but without Restart and Forget. Then the following properties hold:

1. M is consistent.
2. All learned clauses are entailed by N.
3. If $C \notin \{\top, \bot\}$ then $M \models \neg C$.
4. If $C = \top$ and M contains only propagated literals then for each valuation $\mathcal{A}$ with $\mathcal{A} \models N$ it holds that $\mathcal{A} \models M$.
5. If $C = \top$, M contains only propagated literals and $M \models \neg D$ for some $D \in (N \cup U)$ then N is unsatisfiable.
6. If $C = \bot$ then CDCL terminates and N is unsatisfiable.
7. k is the maximal level of a literal in M.
8. Each infinite derivation contains an infinite number of Backtrack applications.

2.9.7 Lemma (CDCL Redundancy)

Consider a CDCL derivation by a reasonable strategy. Then CDCL never learns a clause contained in $N \cup U$.

2.9.9 Lemma (CDCL Soundness)

In a reasonable CDCL derivation, CDCL can only terminate in two different types of final states: $(M; N; U; k; \top)$ where $M \models N$ and $(M; N; U; k; \bot)$ where N is unsatisfiable.

2.9.10 Proposition (CDCL Soundness)

The rules of the CDCL algorithm are sound: (i) if CDCL terminates from $(\epsilon; N; \emptyset; 0; \top)$ in the state $(M; N; U; k; \top)$, then N is satisfiable, (ii) if CDCL terminates from $(\epsilon; N; \emptyset; 0; \top)$ in the state $(M; N; U; k; \bot)$, then N is unsatisfiable.

2.9.11 Proposition (CDCL Strong Completeness)

The CDCL rule set is complete: for any valuation M with $M \models N$ there is a reasonable sequence of rule applications generating $(M'; N; U; k; \top)$ as a final state, where M and M' only differ in the order of literals.

2.9.12 Proposition (CDCL Termination)

Assume the algorithm CDCL with all rules except Restart and Forget is applied using a reasonable strategy. Then it terminates in a state $(M; N; U; k; D)$ with $D \in \{\top, \bot\}$.