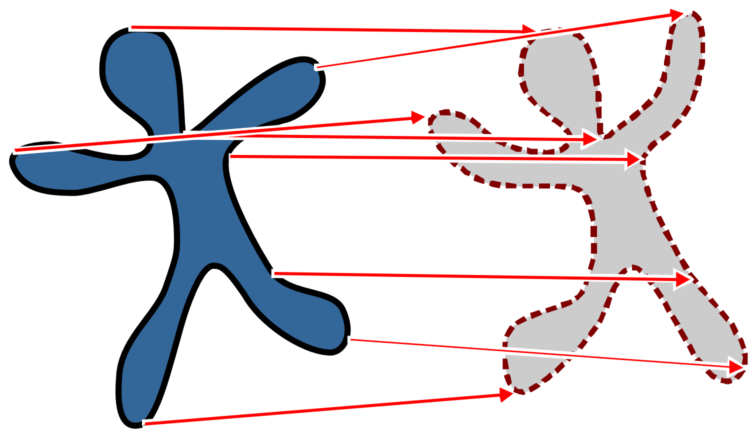
Computing Correspondences in Geometric Datasets



Non-rigid local registration

Animation Reconstruction

Correspondence Problem Classification

How many meshes?

- Two: Pairwise registration
- More than two: multi-view registration

Initial registration available?

- Yes: Local optimization methods
- No: Global methods

Class of transformations?

- Rotation and translation: Rigid-body
- Non-rigid deformations

Overview & Problem Statement

Overview

Animation Reconstruction

- Problem Statement
- Basic algorithm
  - Variational reconstruction
  - Adding dynamics
  - Iterative assembly
  - Results

Real-time Scanners

space-time stereo

courtesy of James Davis, UC Santa Cruz

color-coded structured light

courtesy of Phil Fong, Stanford University

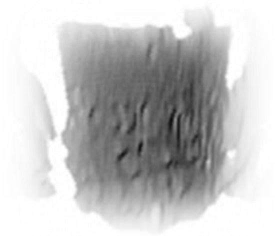
motion compensated structured light

courtesy of Sören König, TU Dresden

Animation Reconstruction

Problems

- Noisy data
- Incomplete data (acquisition holes)
- No correspondences



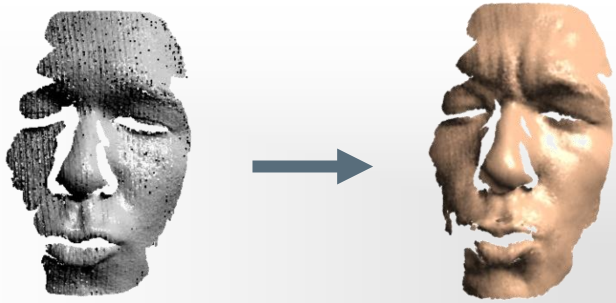
noise

holes

missing correspondences

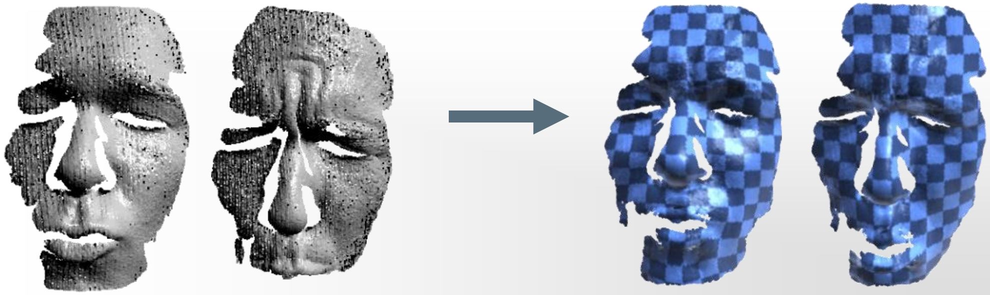
Animation Reconstruction

Remove noise, outliers



Fill-in holes
(from all frames)

Dense correspondences

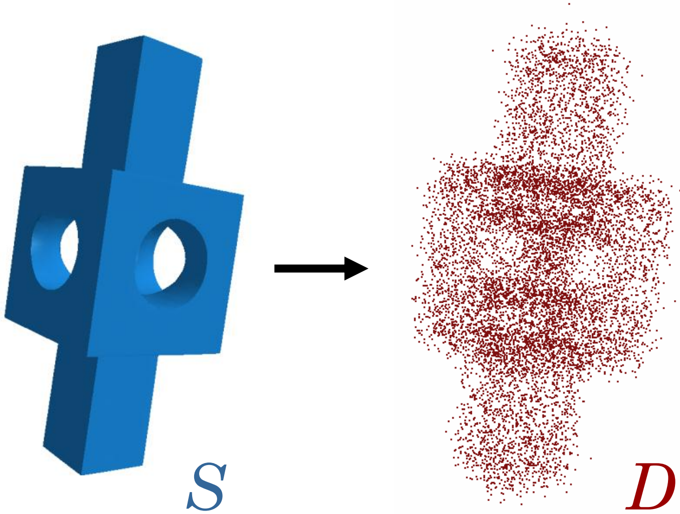


Animation Reconstruction

Overview

Variational Approach:

- S – original model
  D – measurement data



- Variational approach:

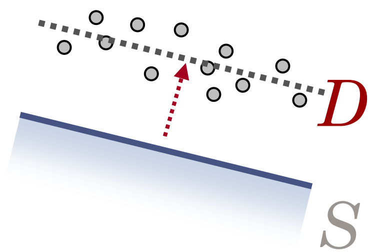
$$E(S \mid D) \sim E(D \mid S) + E(S)$$

measurement   prior

3D Reconstruction

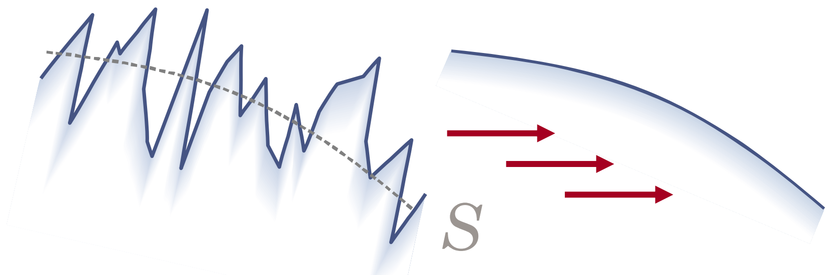
Data fitting:

$E(D \mid S) \sim \Sigma_i \, \mathrm{dist}(S, d_i)^2$



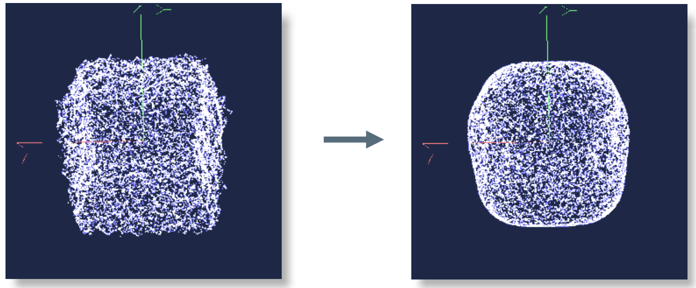
Prior: Smoothness

$E_s(S) \sim \int_S \mathrm{curv}(S)^2$



Optimization:

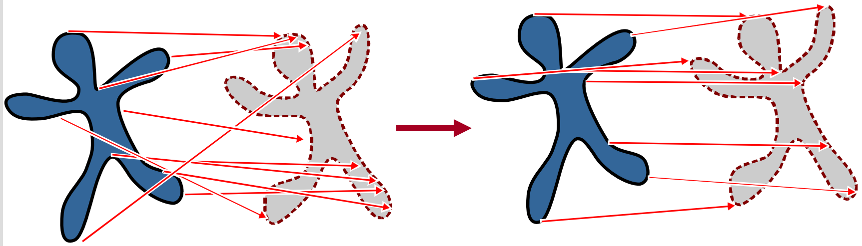
Yields 3D Reconstruction



Two additional priors:

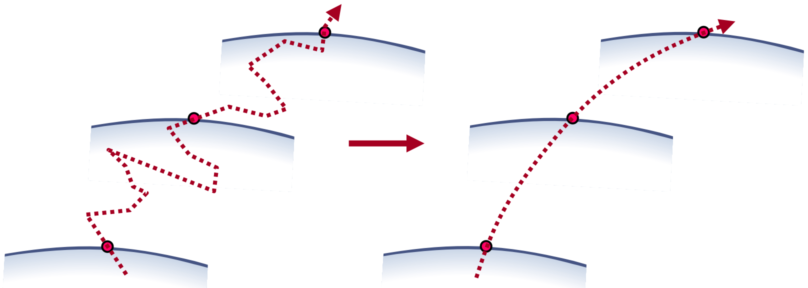
Deformation

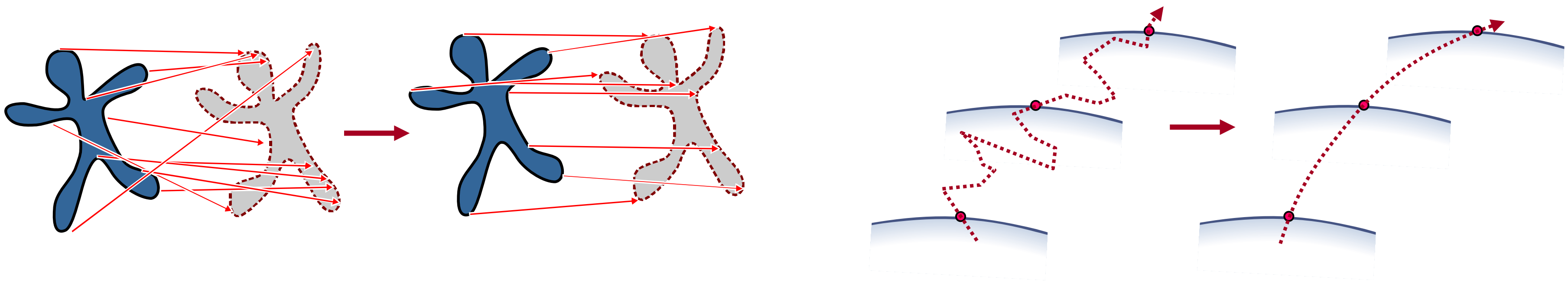
$$E_d(S) \sim \int_S \mathrm{deform}(S_t\,,\ S_{t+1})^2$$



Acceleration

$$E_a(S) \sim \int_{S,t} \ddot{s}(x,\ t)^2$$





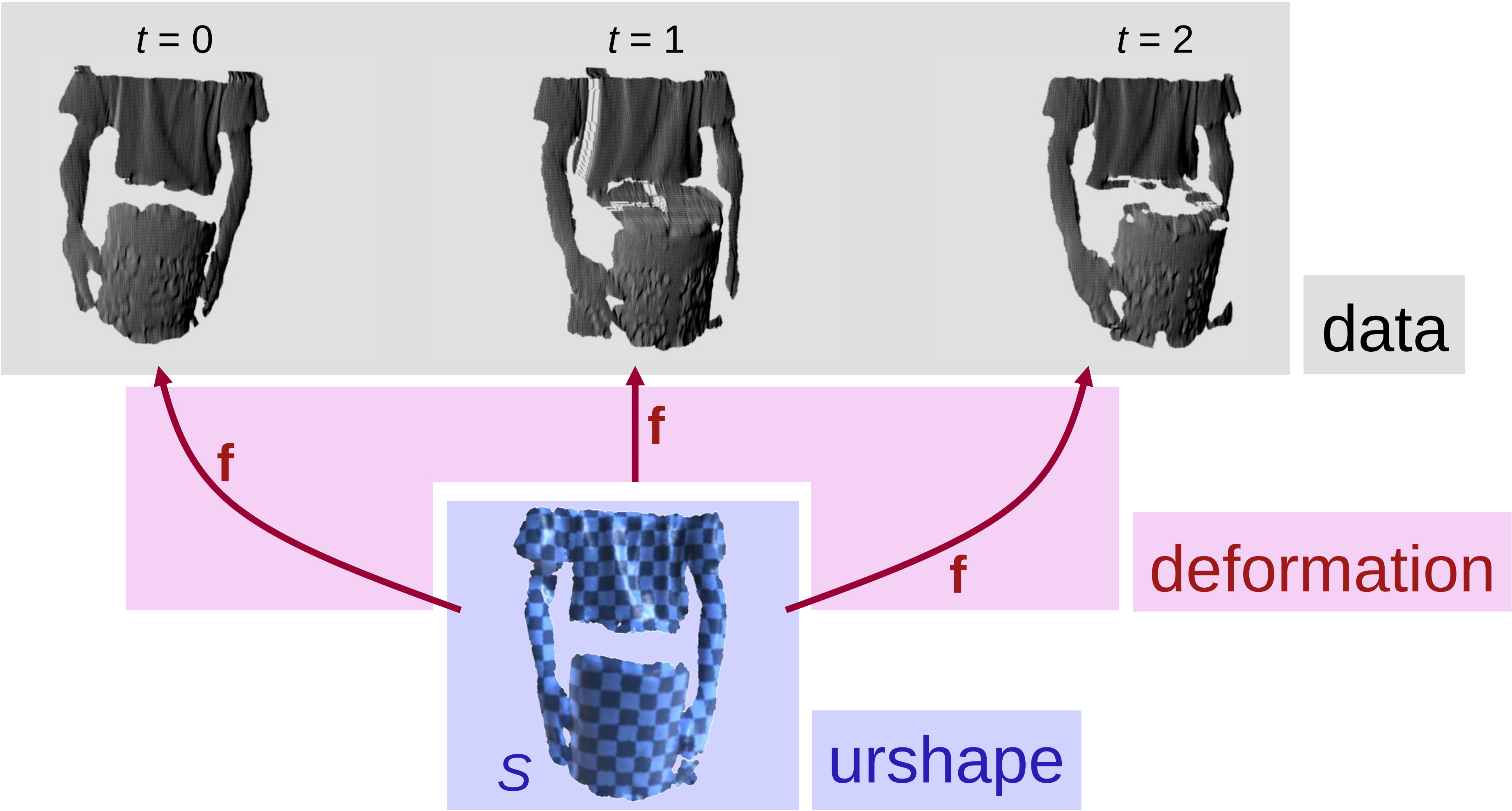
Not just smooth 4D reconstruction!

- Minimize
  - Deformation
  - Acceleration
- This is quite different from smoothness of a 4D hypersurface.

Algorithm Details

Urshape Factorization

Factorization



Components

Variational Model

- Given an initial estimate,
  improve *urshape* and *deformation*

Numerical Discretization

- *Shape*
- *Deformation*

Domain Assembly

- Getting an initial estimate
- *Urshape* assembly

Components

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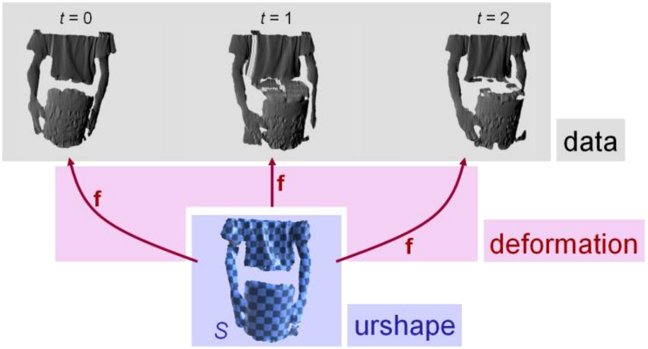
Energy Function

$E(f, S) = E_{data} + E_{deform} + E_{smooth}$



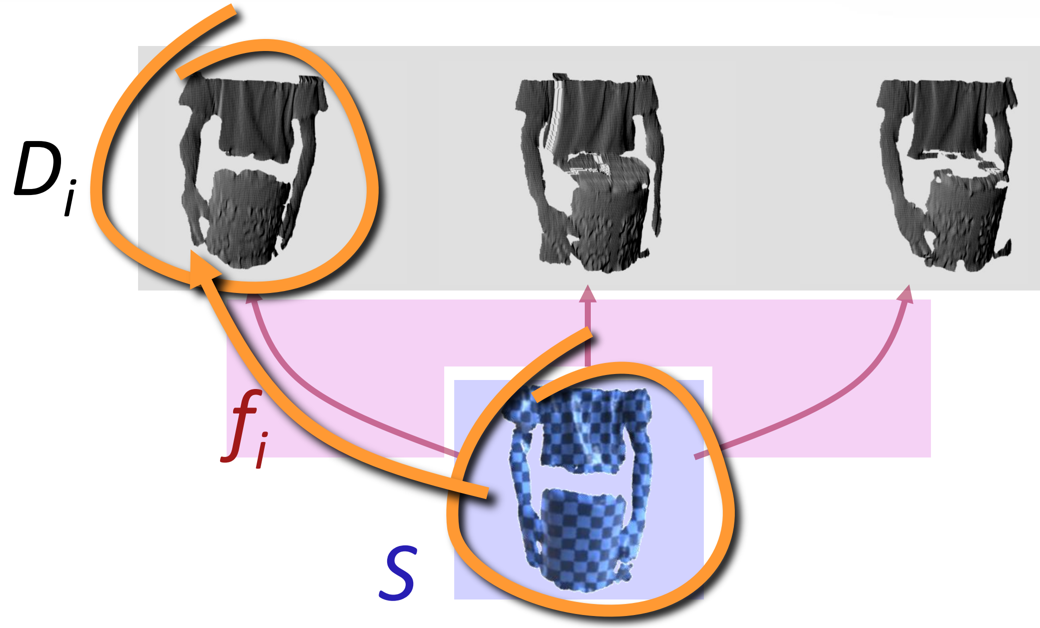
Components

- $E_{data}(f, S)$ – data fitting
- $E_{deform}(f)$ – elastic deformation, smooth trajectory
- $E_{smooth}(S)$ – smooth surface

Optimize S, f alternatingly

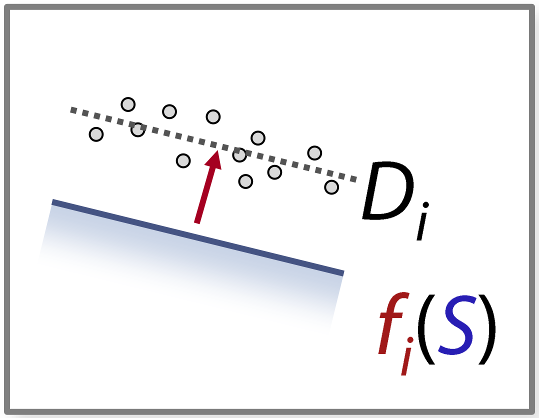
Data Fitting

$$E_{data}(f, S)$$

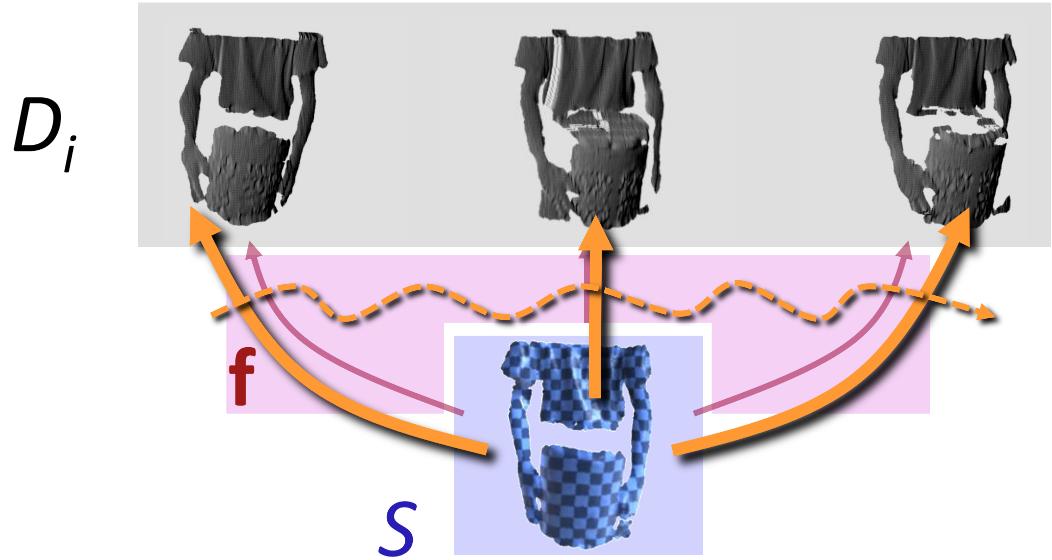


Data fitting

- Necessary: $f_i(S) \approx D_i$
- Truncated squared distance function (point-to-plane)



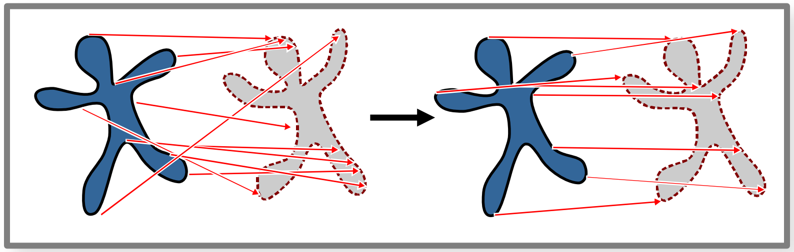
Elastic Deformation Energy



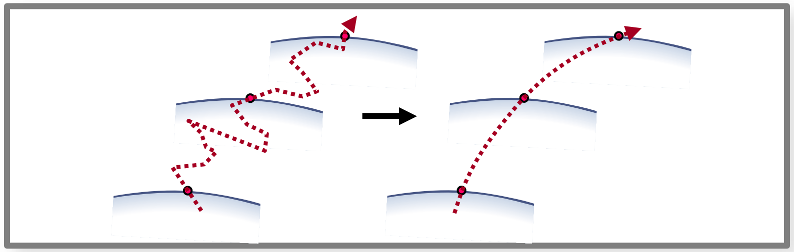
$E_{deform}(f)$

Regularization

- Elastic energy

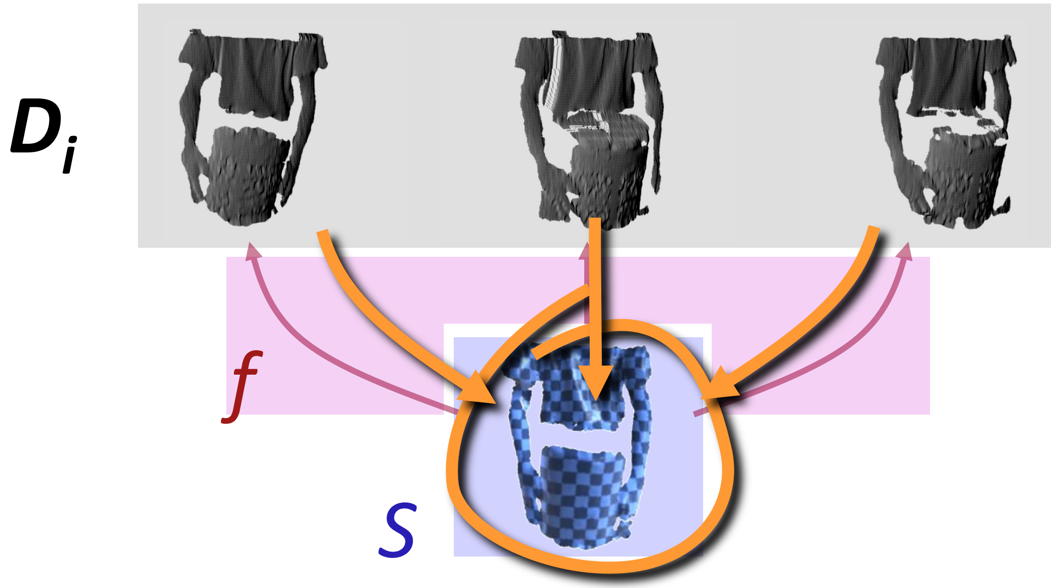


- Smooth trajectories



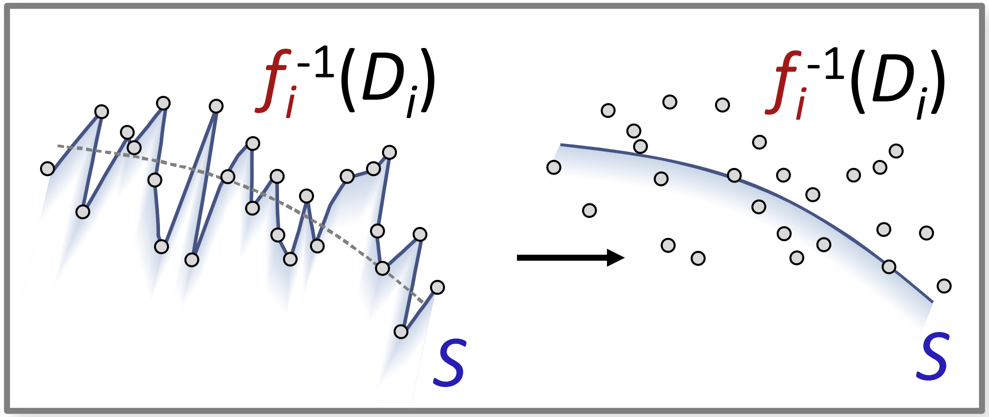
Surface Reconstruction

$E_{smooth}(S)$

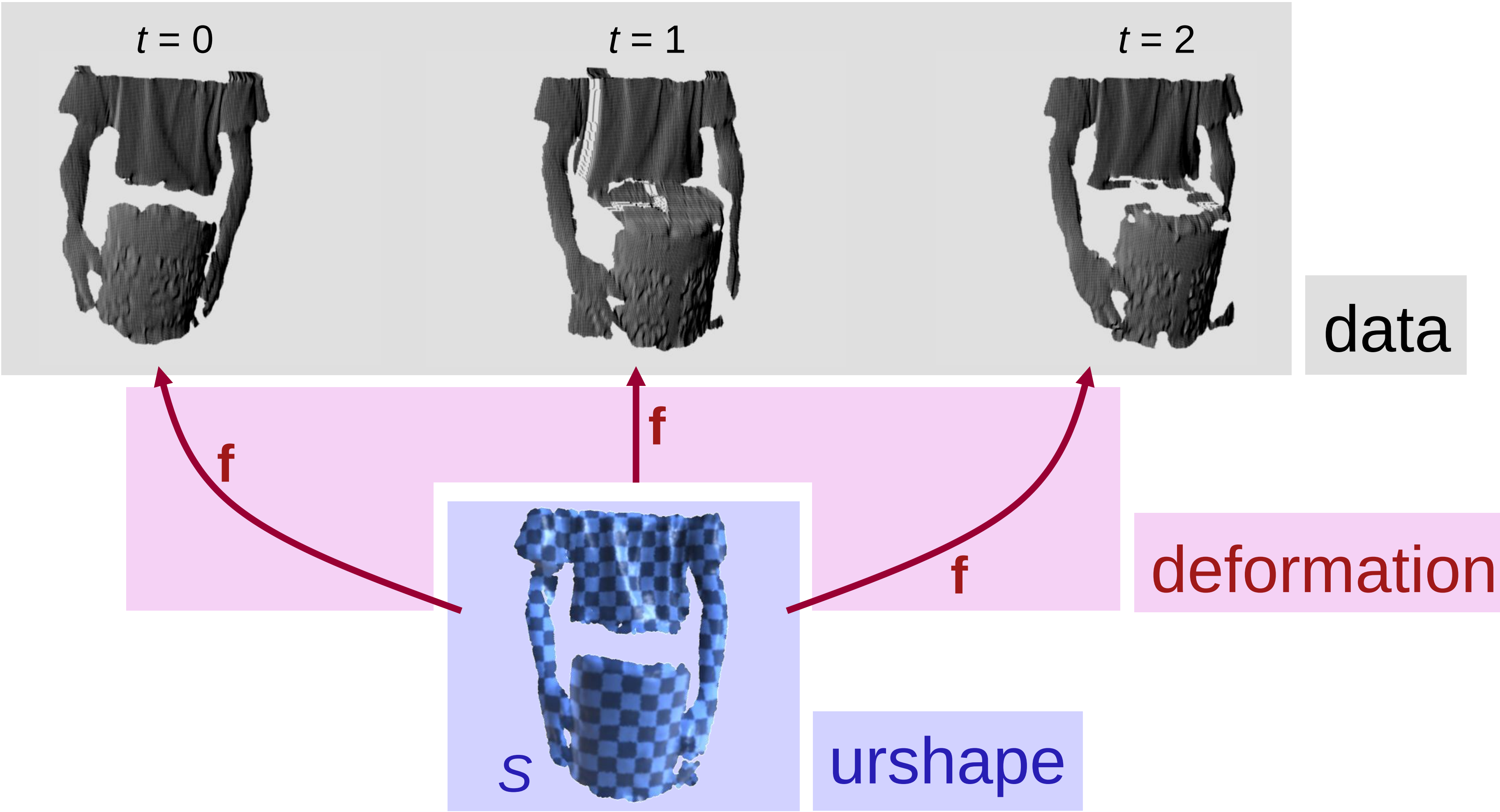


Data fitting

- Smooth surface
- Fitting to noisy data



Factorization



Variational Model

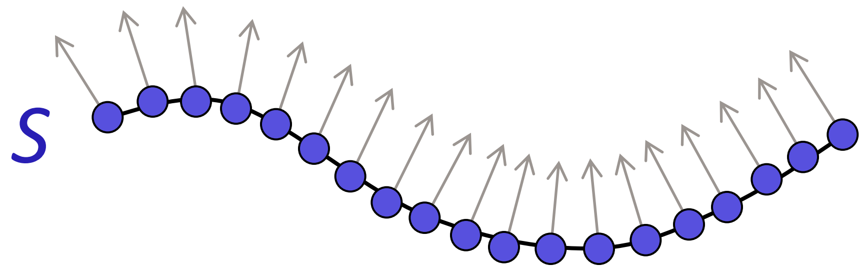
- Given an initial estimate,
  improve *urshape* and *deformation*

Numerical Discretization

- *Shape*
- *Deformation*

Domain Assembly

- Getting an initial estimate
- *Urshape* assembly



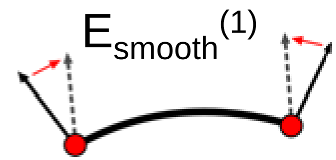
Shape Representation:

- Graph of *surfels* (point + normal + local connectivity)
- E_{smooth} – neighboring planes should be similar

Simple Smoothness Priors:

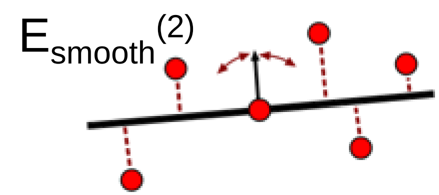
- Similar surfel normals:

$$E^{(1)}_{smooth}(S) = \sum_{surfels} \sum_{neighbors} \left(n_i - n_{i_j}\right)^2, \quad \|n_i\| = 1$$



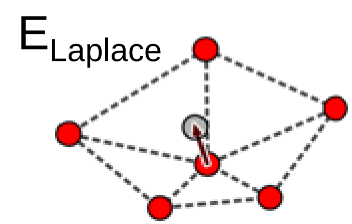
- Surfel positions – flat surface:

$$E^{(2)}_{smooth}(S) = \sum_{surfels} \sum_{neighbors} \left\langle \mathbf{s}_i - \mathbf{s}_{i_j}, \mathbf{n}(\mathbf{s}_i) \right\rangle^2$$



- Uniform density:

$$E_{Laplace}(S) = \sum_{surfels} \sum_{neighbors} \left(\mathbf{s}_i - average\right)^2$$

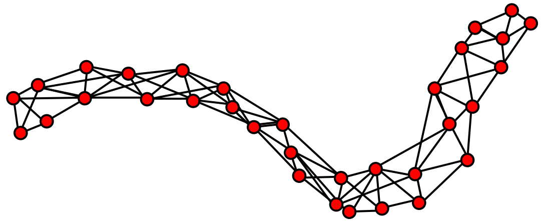


[c.f. Szeliski et al. 93]

Neighborhoods?

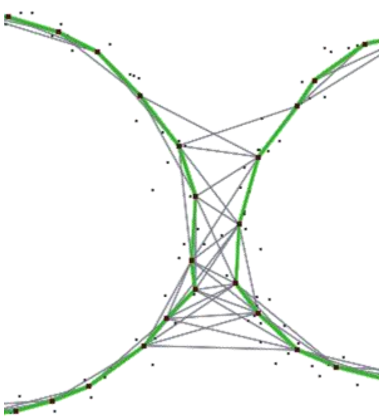
Topology estimation

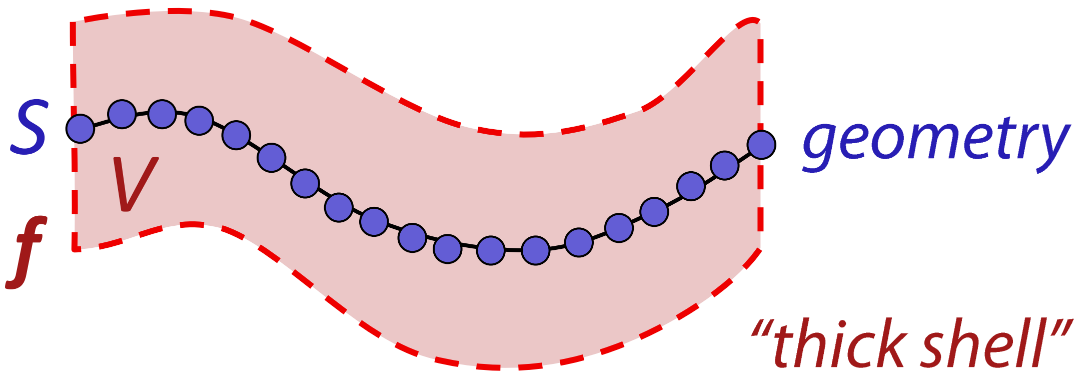
- Domain of *S*, base shape (topology)
- Here, we assume this is easy to get
- In the following
  - *k*-nearest neighborhood graph
  - Typically: $k = 6..20$



Limitations

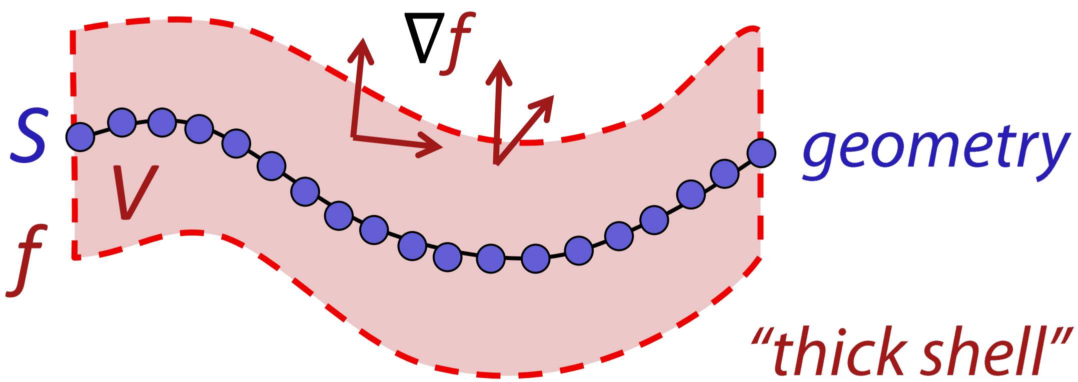
- This requires dense enough sampling
- Does not work for undersampled data





Volumetric Deformation Model

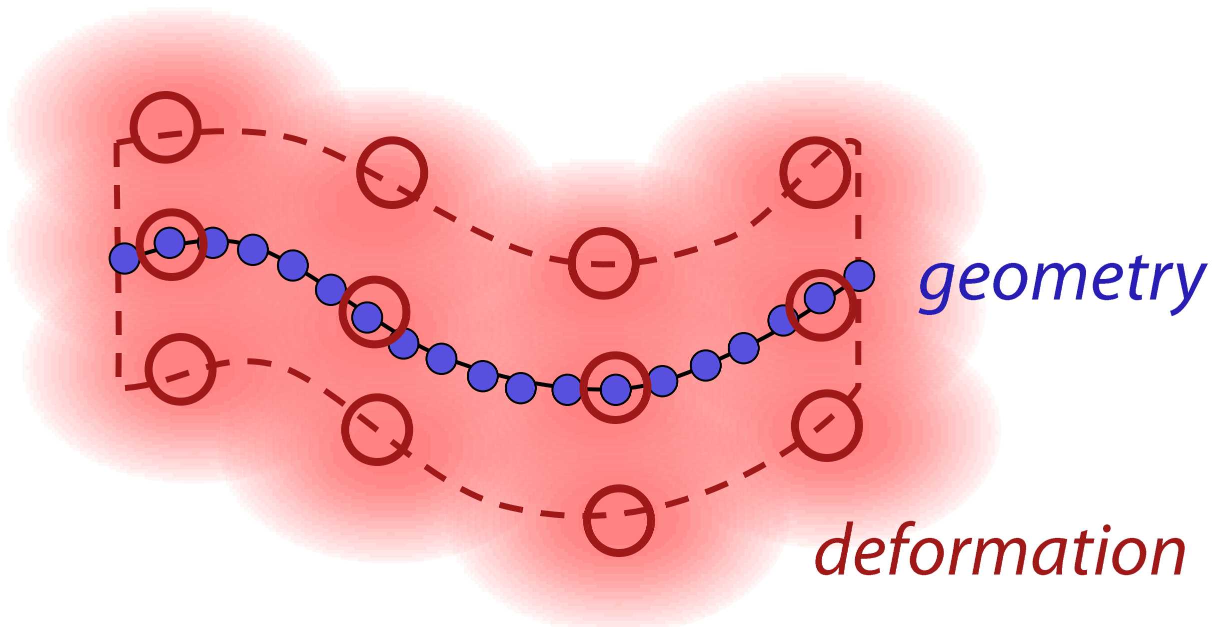
- Surfaces embedded in "stiff" volumes
- Easier to handle than "thin-shell models"
- General – works for non-manifold data



Deformation Energy

- Keep deformation gradients ∇f as-rigid-as-possible
- This means: $\nabla f^{\mathrm{T}} \nabla f = \mathbf{I}$
- Minimize: $E_{deform} = \int_T \int_V \| \nabla f(\mathbf{x},t)^{\mathrm{T}}\, \nabla f(\mathbf{x},t) - \mathbf{I} \|^2 \,\mathrm{d}\mathbf{x}\,\mathrm{d}t$

Discretization

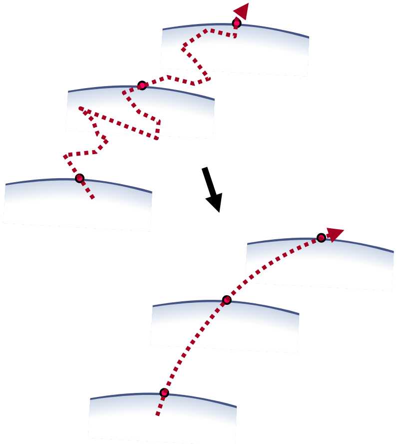


Numerical representation

- Spatial basis functions
- For example: Gaussians, MLS functions, ...

More Regularization

- Volume preservation: $E_{vol} = \int_T \int_V \|\det(\nabla f) - 1\|^2$
  - Stability
- Acceleration: $E_{acc} = \int_T \int_V \|\partial_t^2 f\|^2$
  - Smooth trajectories
- Velocity (weak): $E_{vel} = \int_T \int_V \|\partial_t f\|^2$
  - Damping



Variational Model

- Given an initial estimate, improve *urshape* and *deformation*

Numerical Discretization

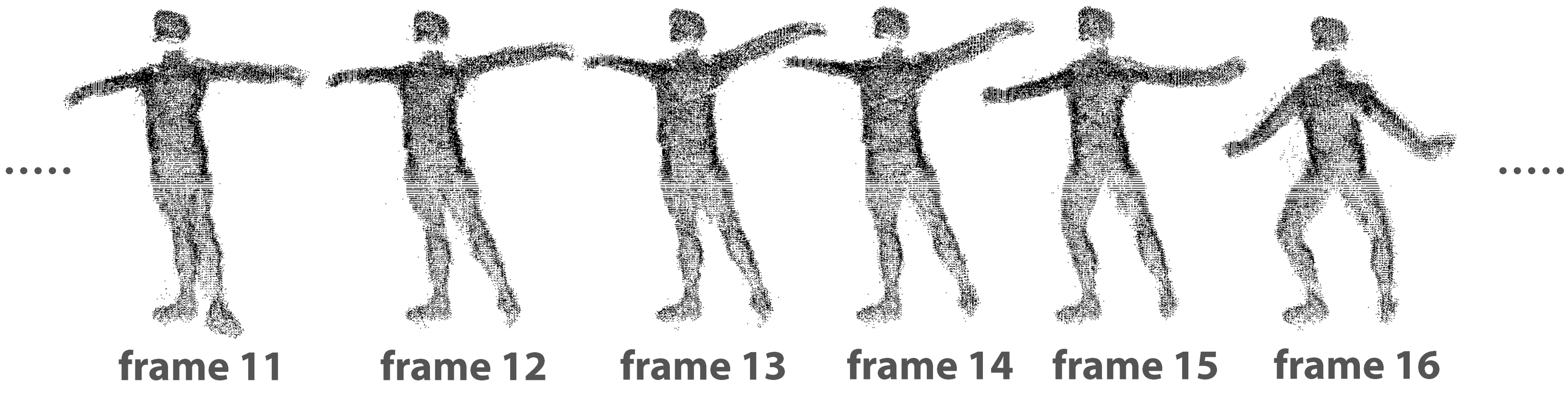
- *Deformation*
- *Shape*

Domain Assembly

- Getting an initial estimate
- *Urshape* assembly

Adjacent frames are similar

- Solve for frame pairs first
- Assemble urshape step-by-step



[data set courtesy of C. Theobald, MPC-VCC]

Hierarchical Merging

data

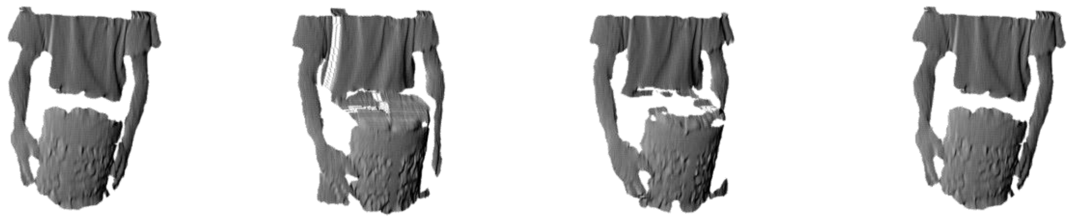
$f(S)$

f

S

Hierarchical Merging

data



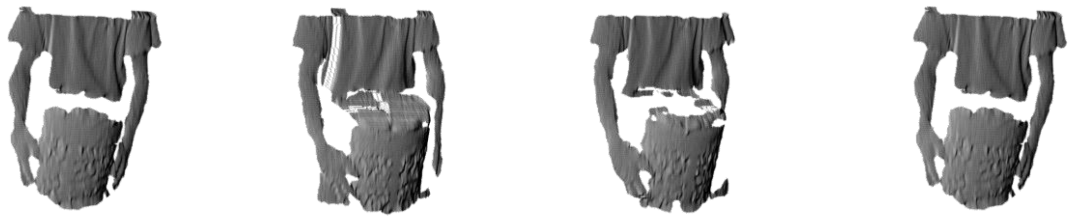
$f(S)$

f

S

Initial Urshapes

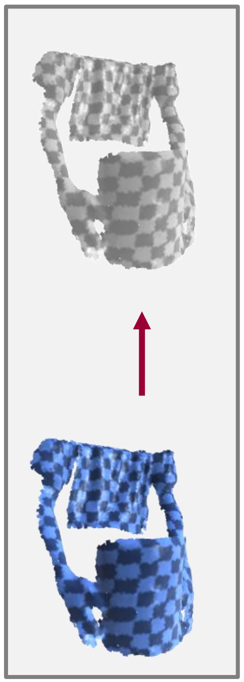
data



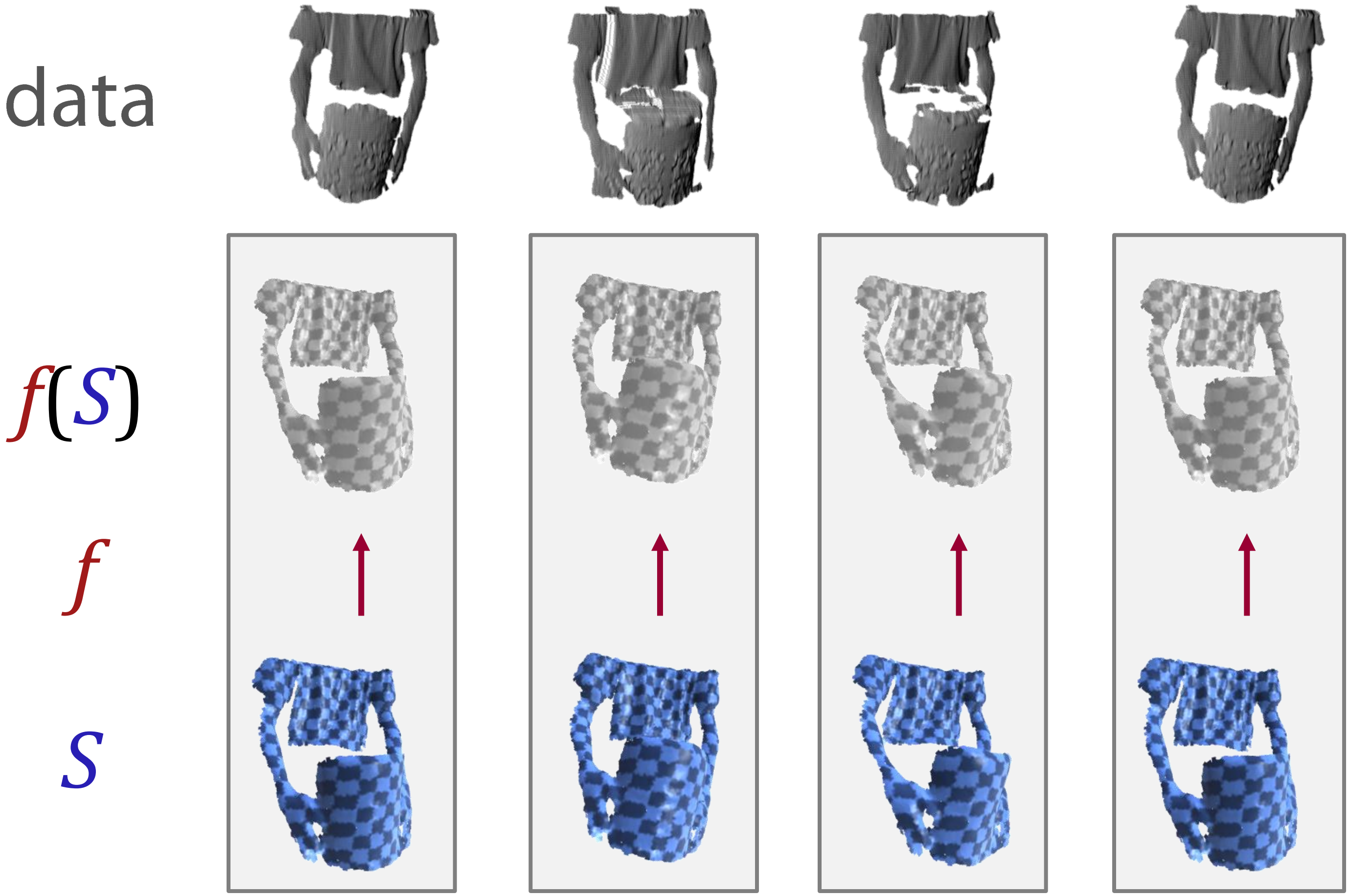
$f(S)$

f

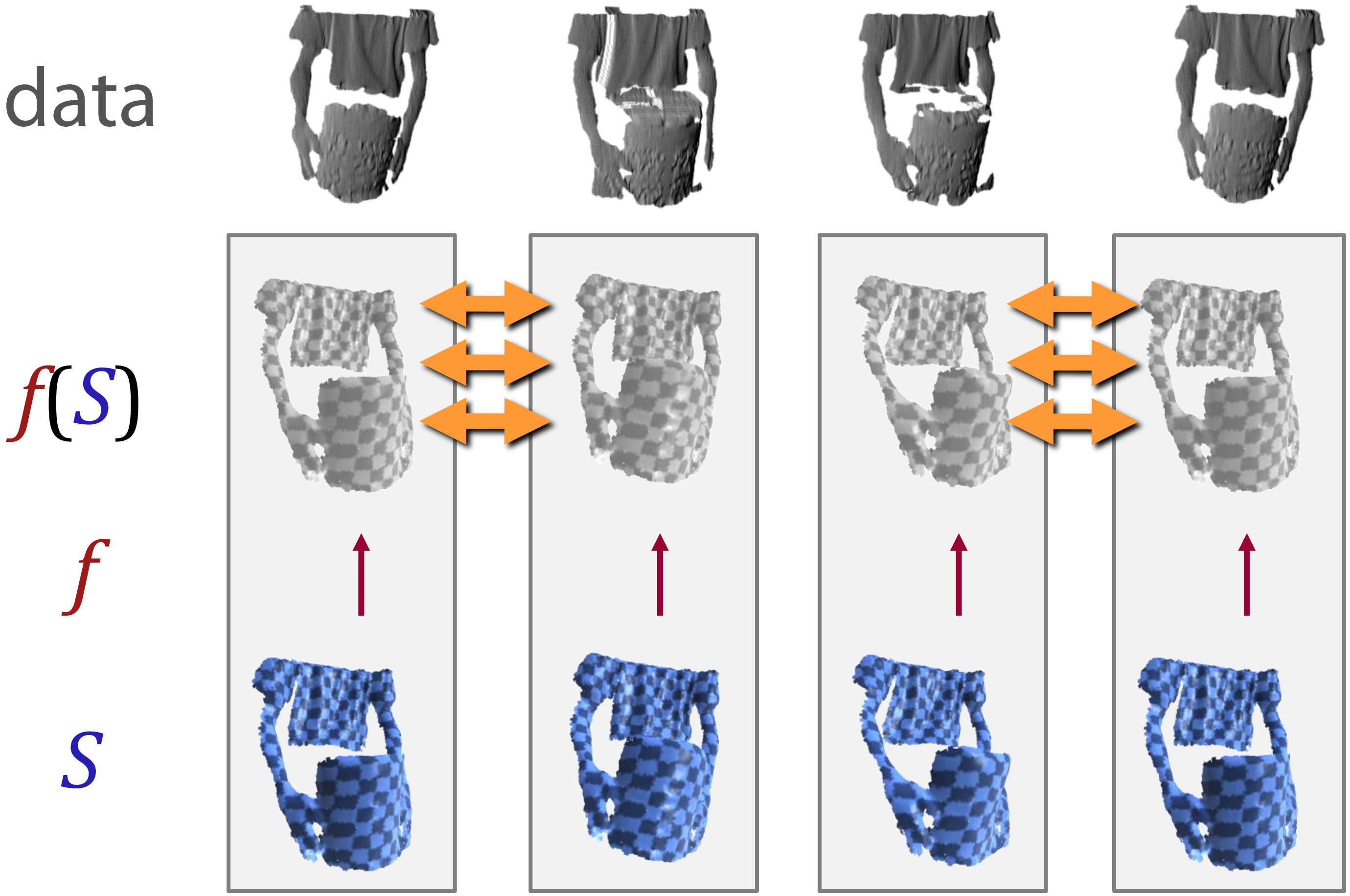
S



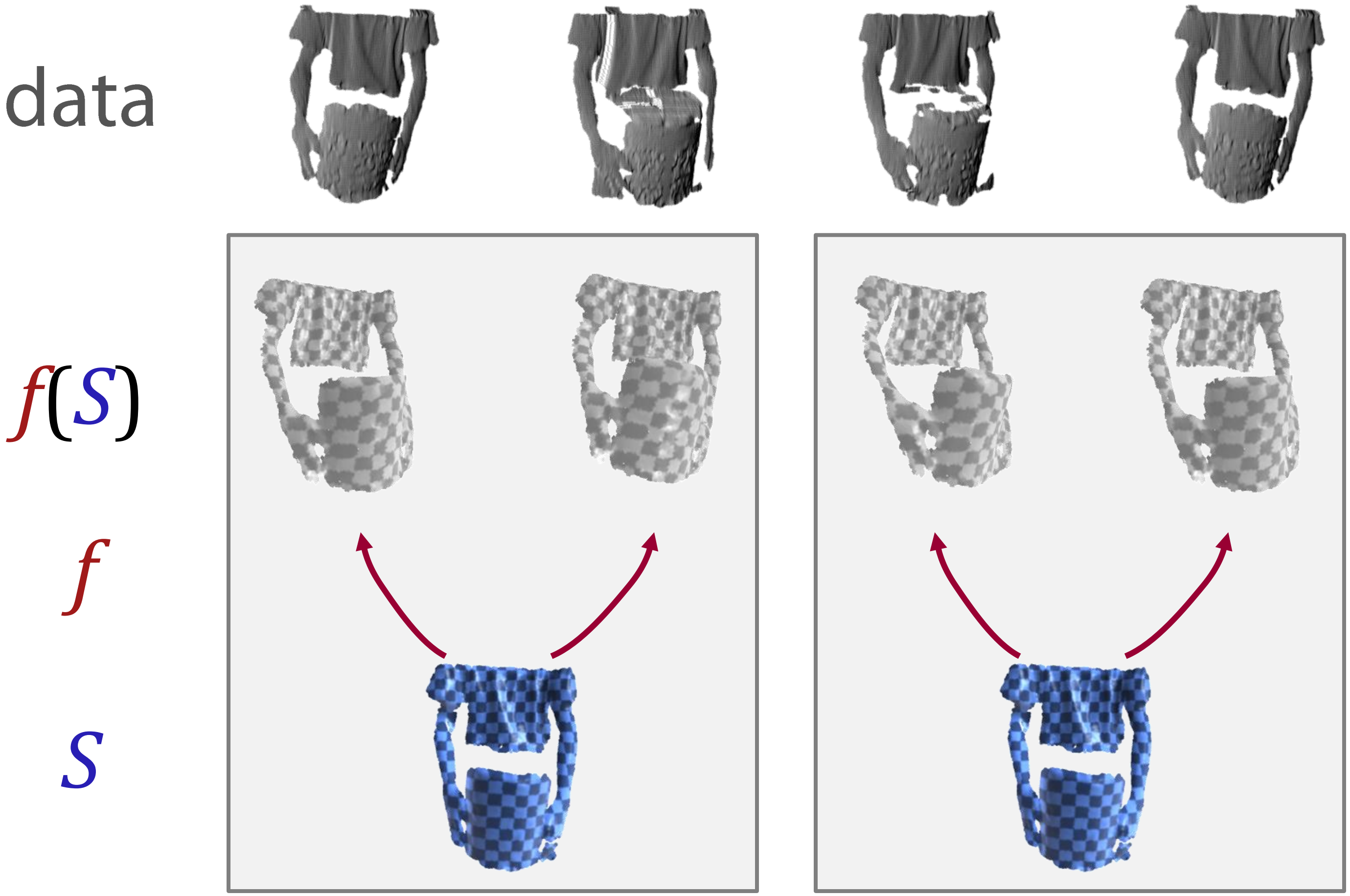
Initial Urshapes



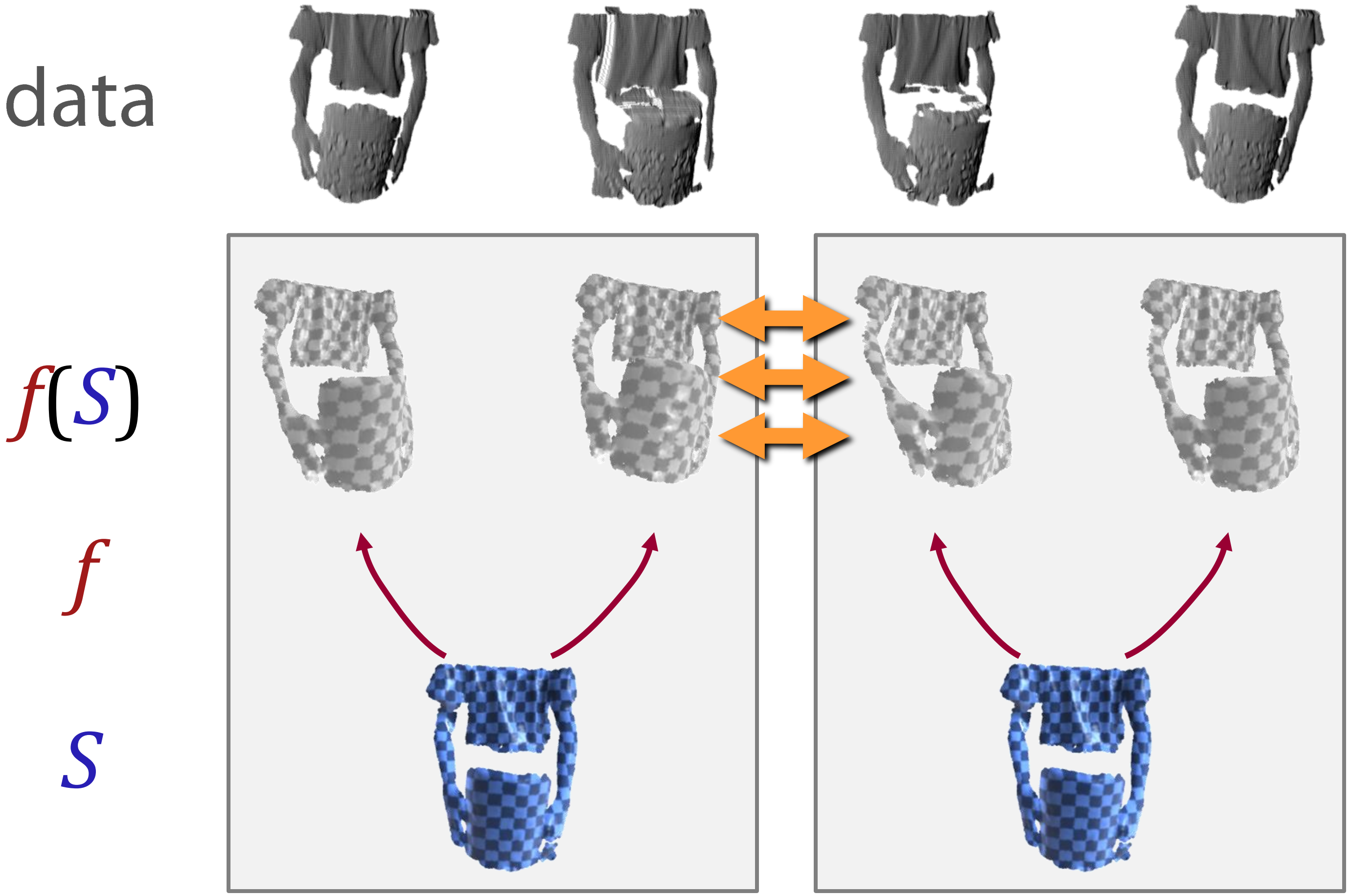
Alignment



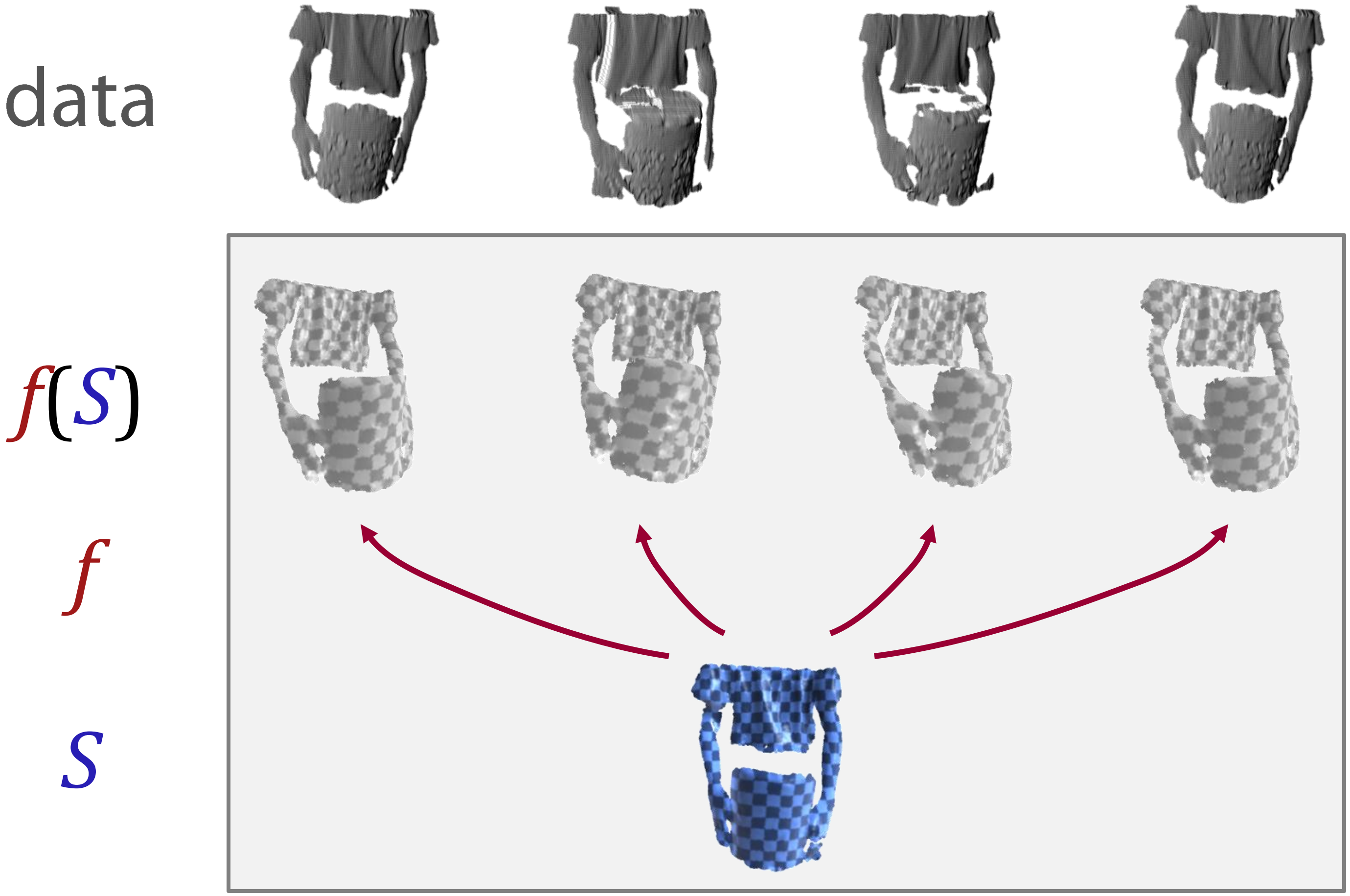
Align & Optimize



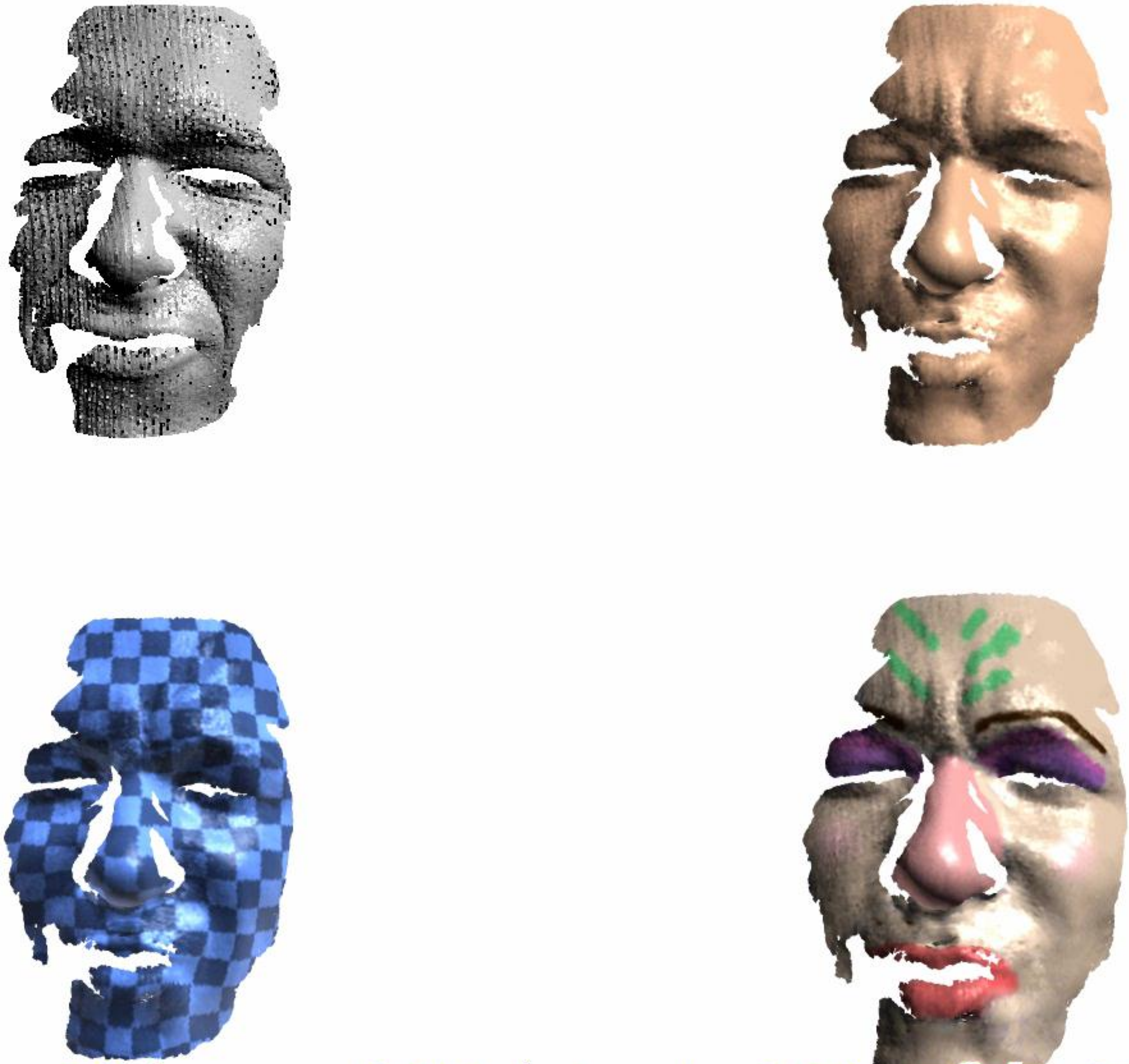
Hierarchical Alignment



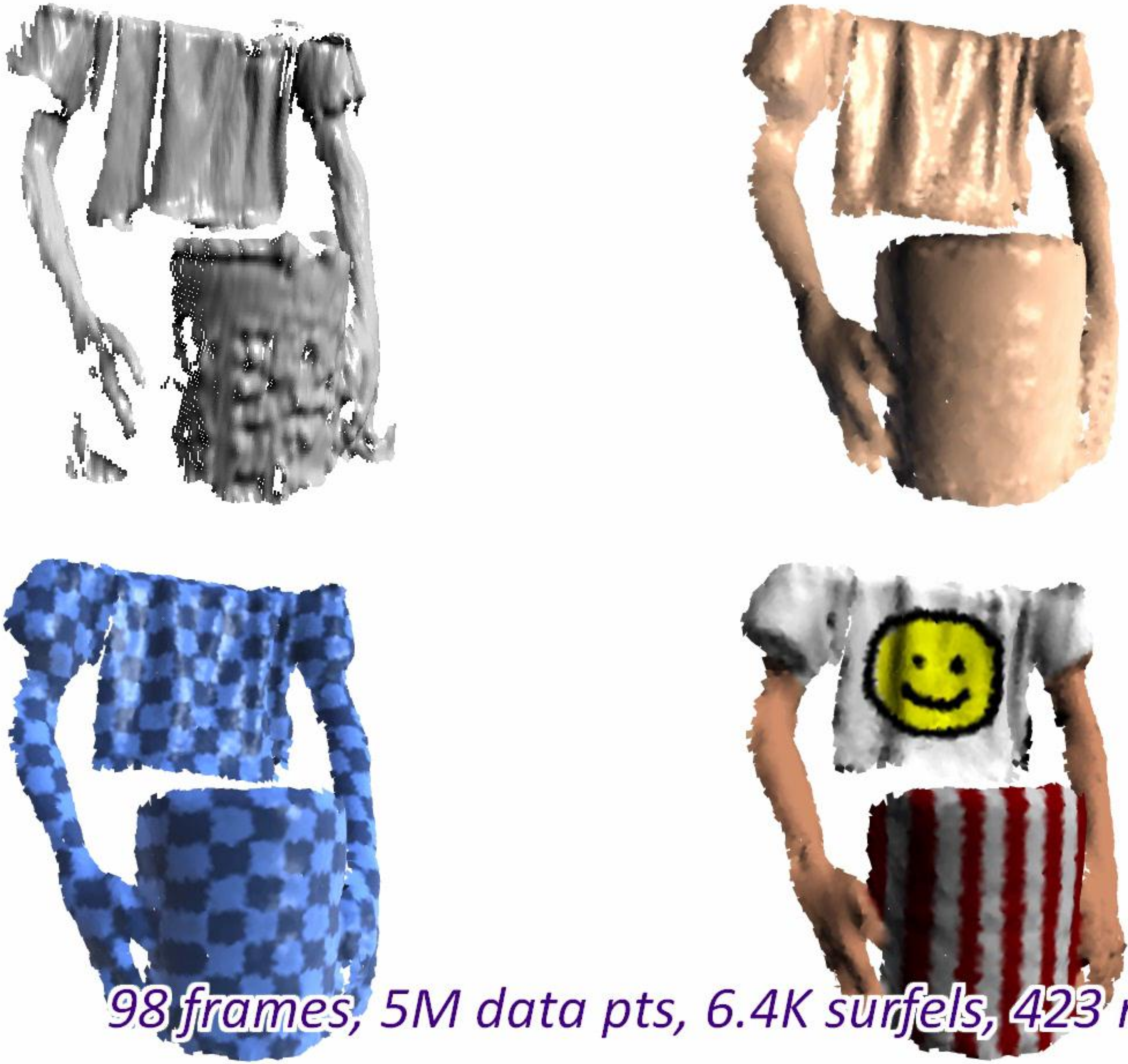
Hierarchical Alignment



Results

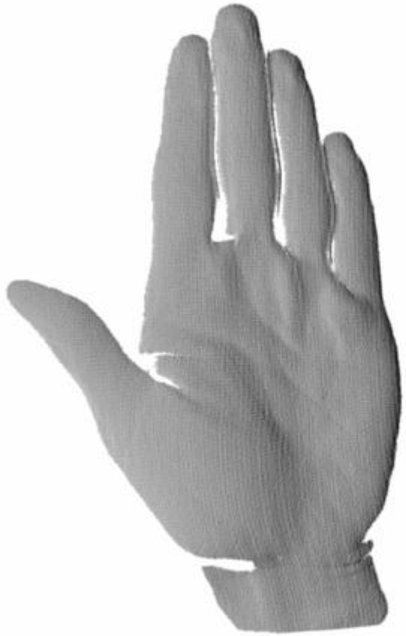


79 frames, 24M data pts, 21K surfels, 315 nodes



98 frames, 5M data pts, 6.4K surfels, 423 nodes

120 frames,
30M data pts,
17K surfels,
1,939 nodes



34 frames,
4M data pts,
23K surfels,
414 nodes