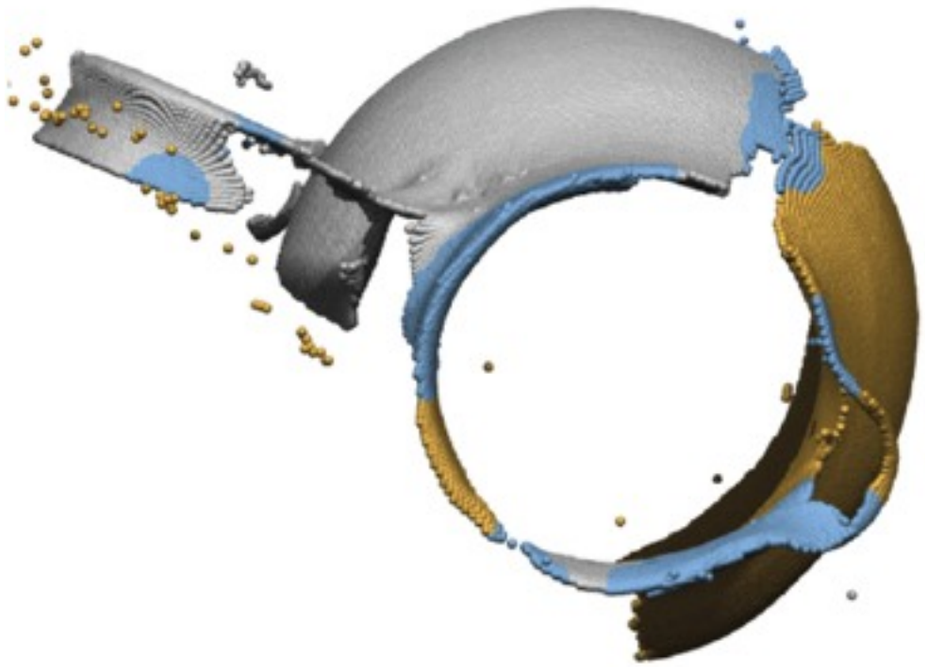
4-Points Congruent Sets for Robust Pairwise Surface Registration



Niloy J. Mitra

Correspondence Problem Classification

How many meshes?

- Two: Pairwise registration
- More than two: multi-view registration

Initial registration available?

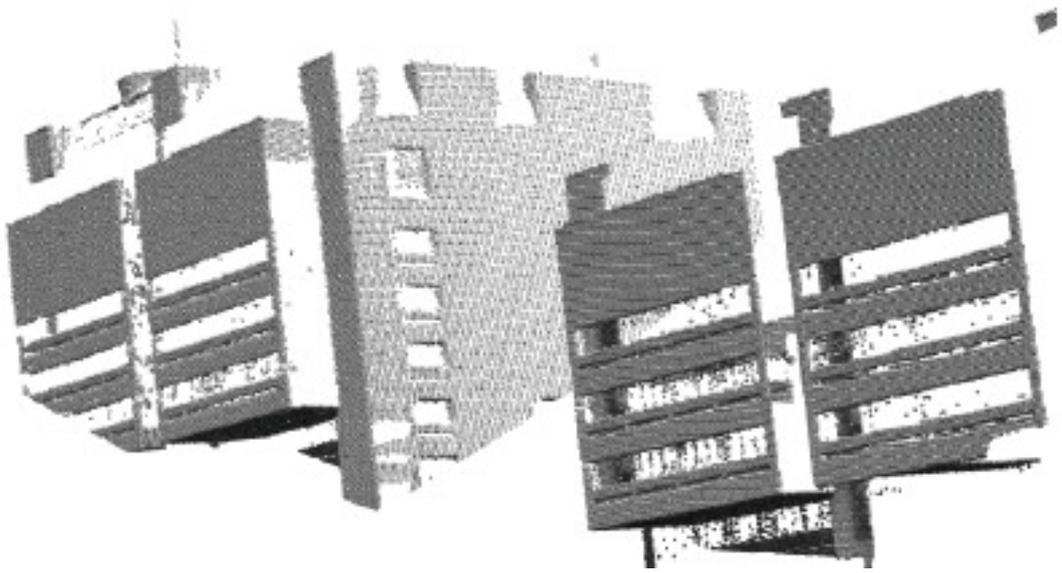
- Yes: Local optimization methods
- No: Global methods

Class of transformations?

- Rotation and translation: Rigid-body
- Non-rigid deformations

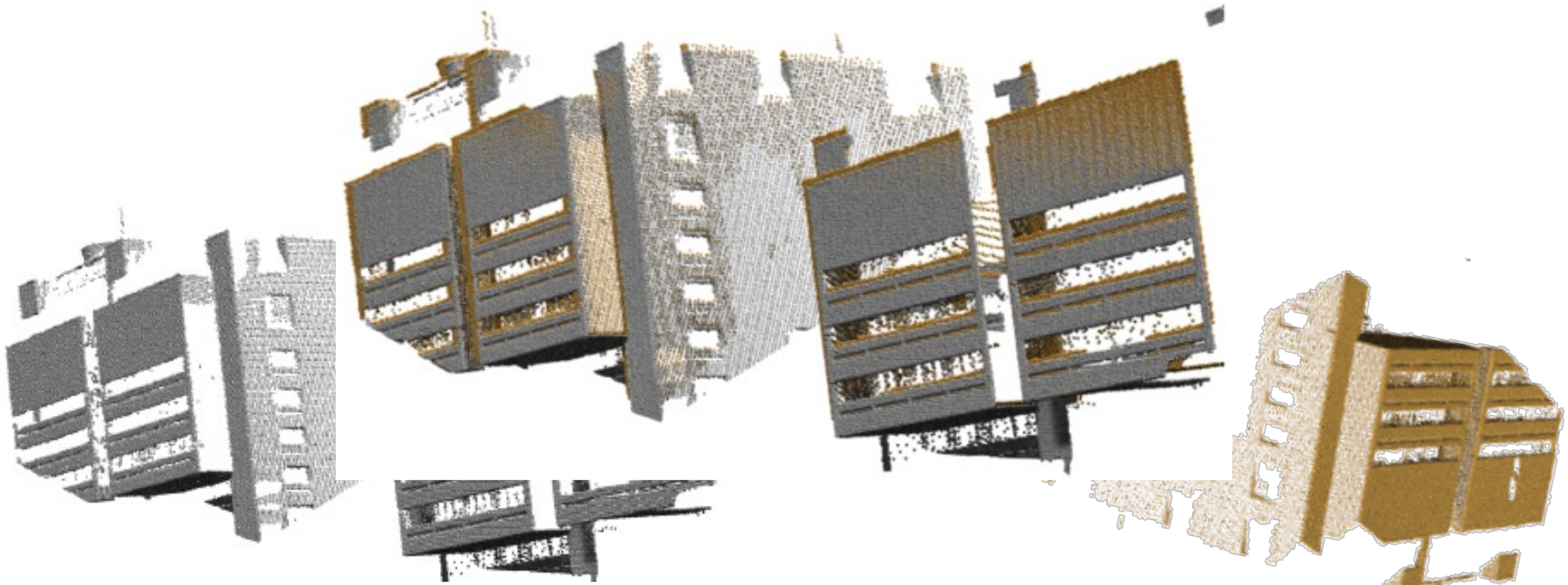
Problem Statement

Given: Two models



Problem Statement

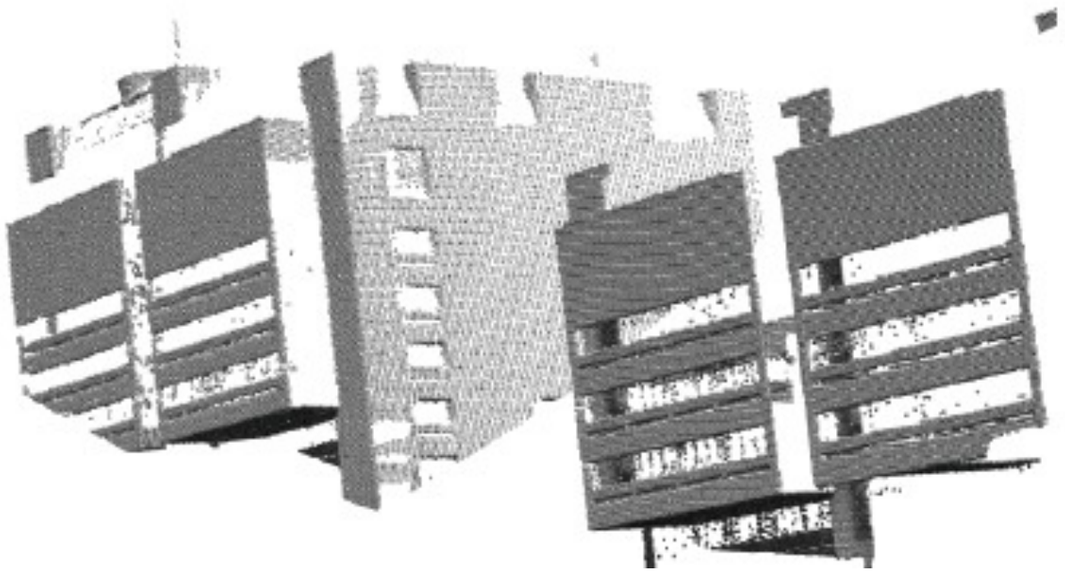
Goal: *Automatically* align the models



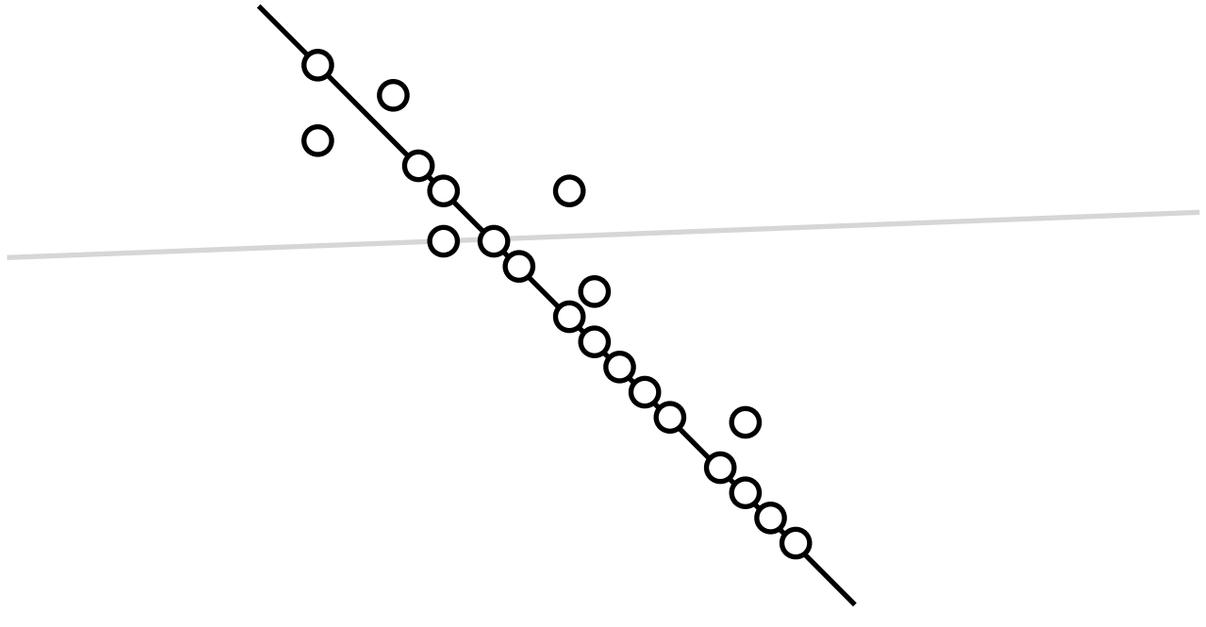
Why is this Hard?

Given: Two scans

- corrupted with ***noise*** and ***outliers***
- in **arbitrary initial poses** with ***unknown overlap***



RANSAC for Line Fitting



Observations:

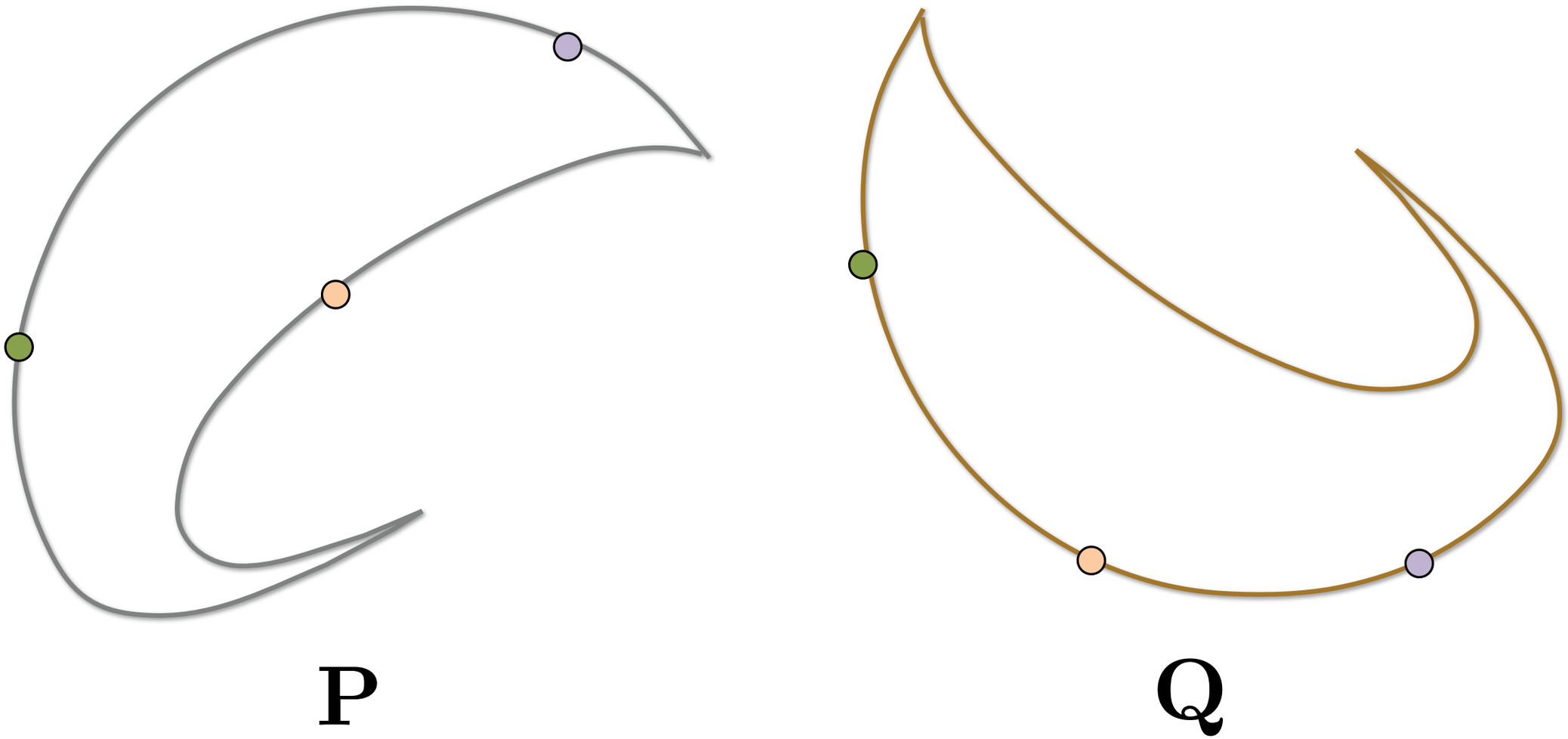
- Problem can be solved by a small number of (correct) points, e.g., 2 in line fitting
- Easy to verify solution (error measurement)

Let ***p*** be the probability of (random) picking a correct point

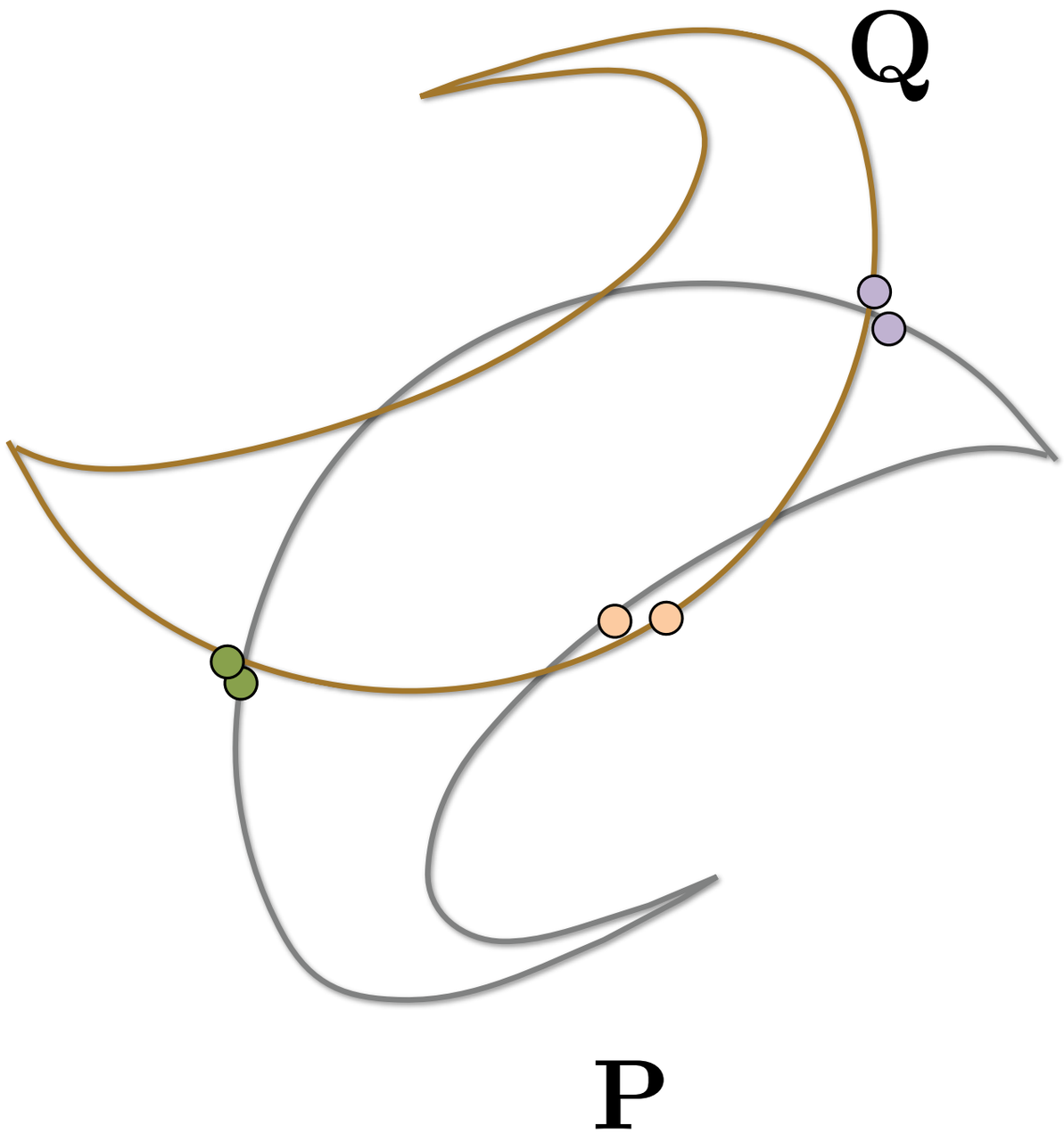
Probability of getting correct result in ***k*** tries

$$(1-p) \qquad (1-p)^k \qquad \text{Prob. of success: } 1-(1-p)^k$$

Alignment Approach



Alignment Approach



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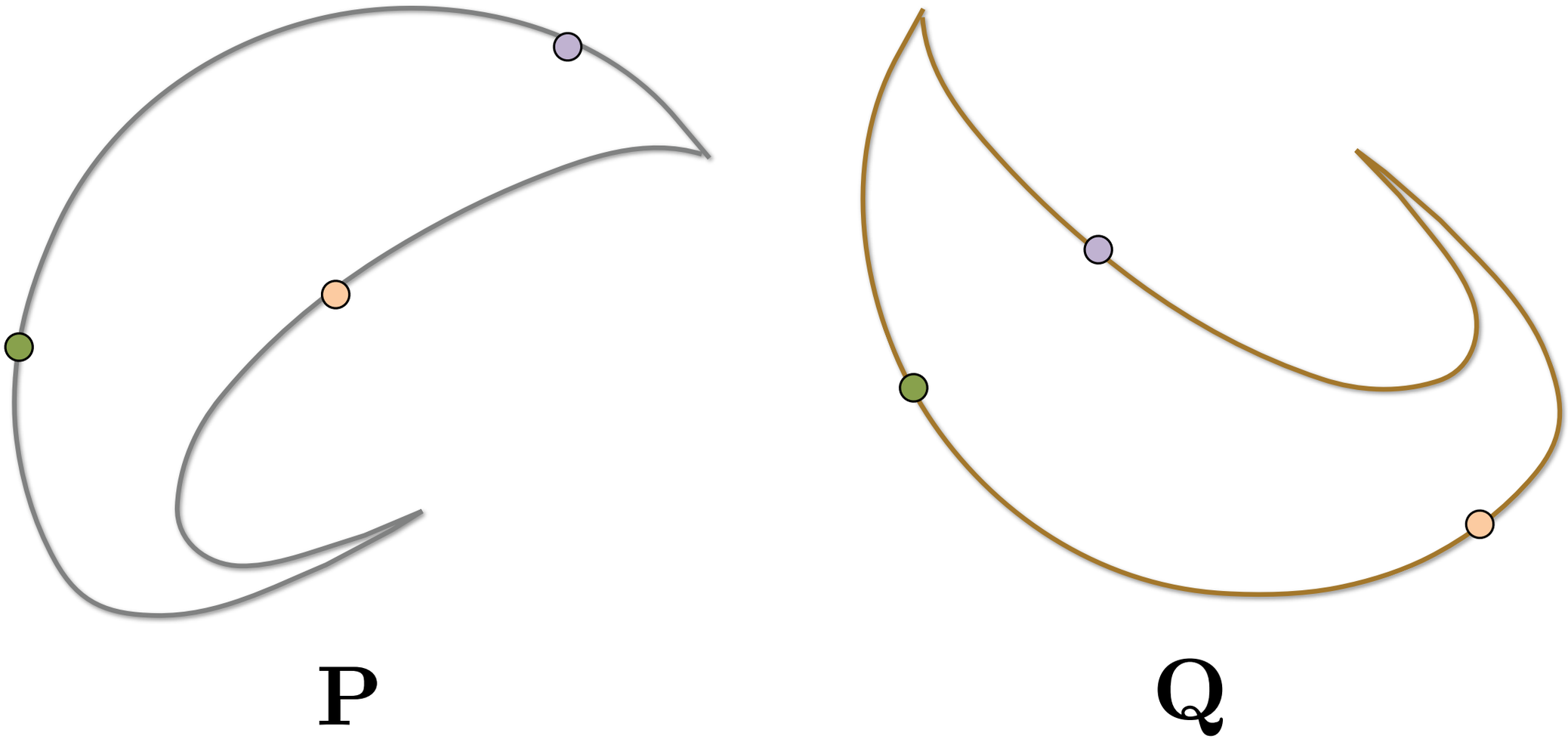

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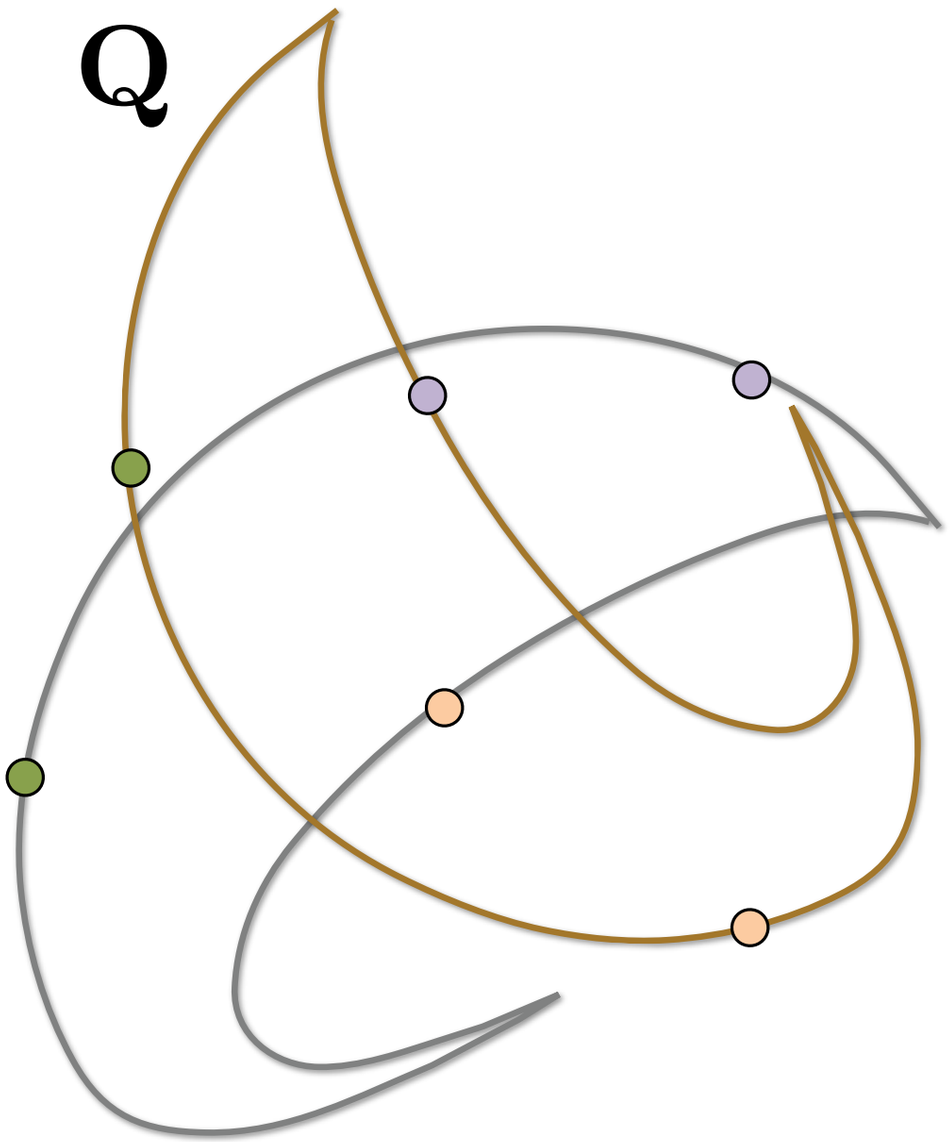
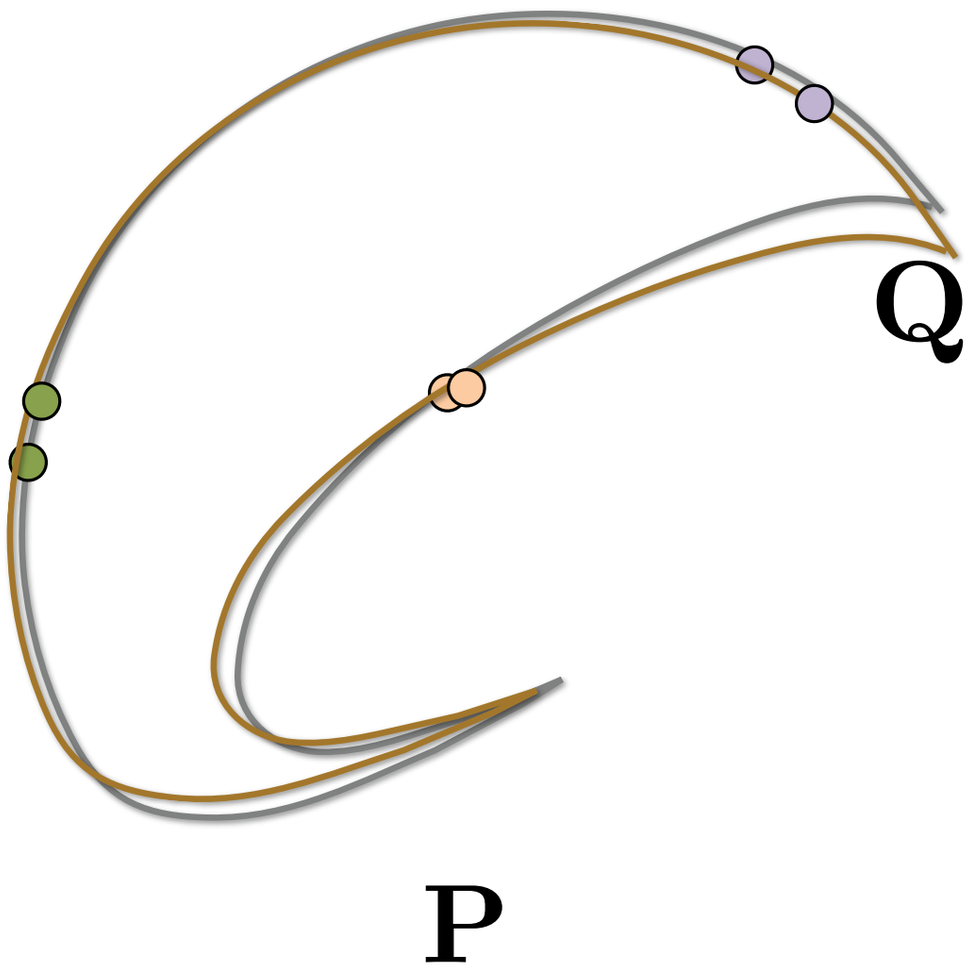


Alignment Approach

Alignment Approach



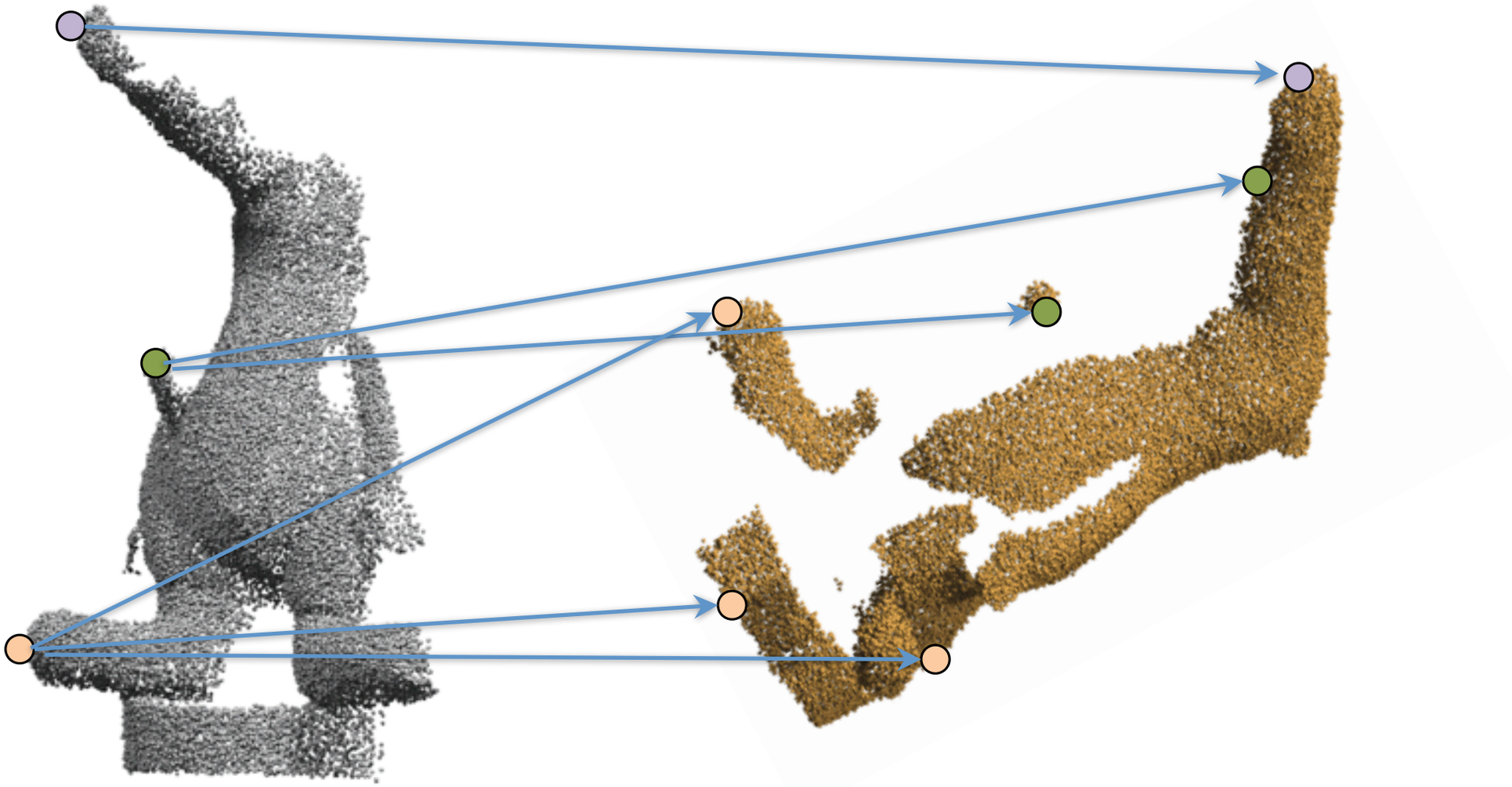
guess and verify

Partial Matching

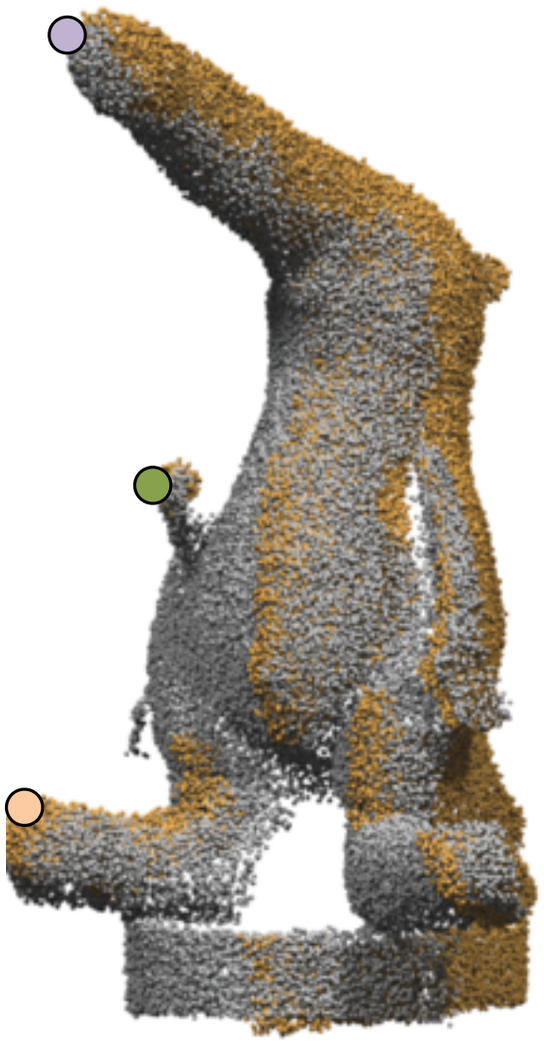
P **Q**

Aligning with Feature Points (FP)

Aligning with Feature Points

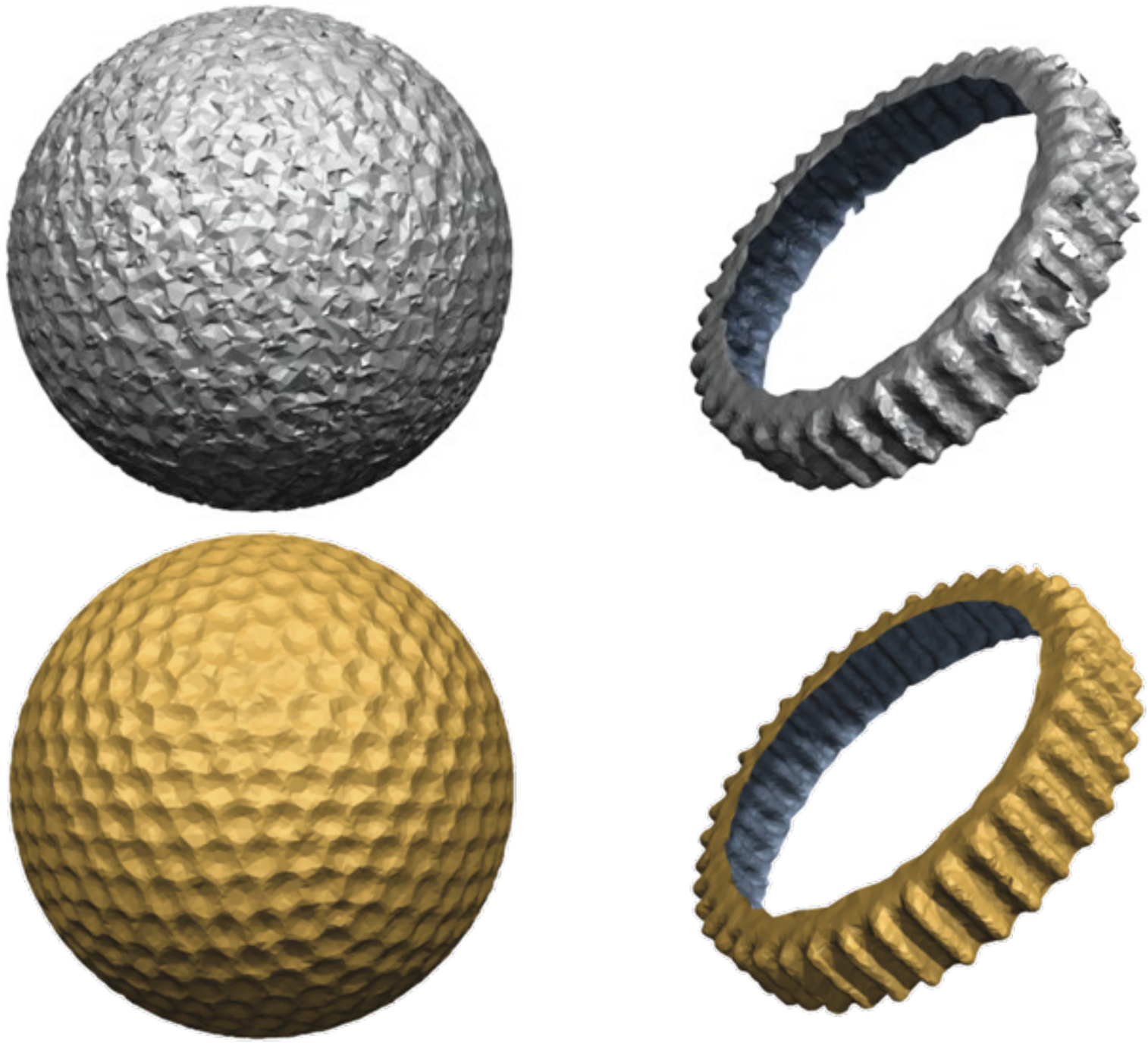


Aligning with Feature Points



Why not Denoise Scans?

de-noise,
compute FP-s,
align

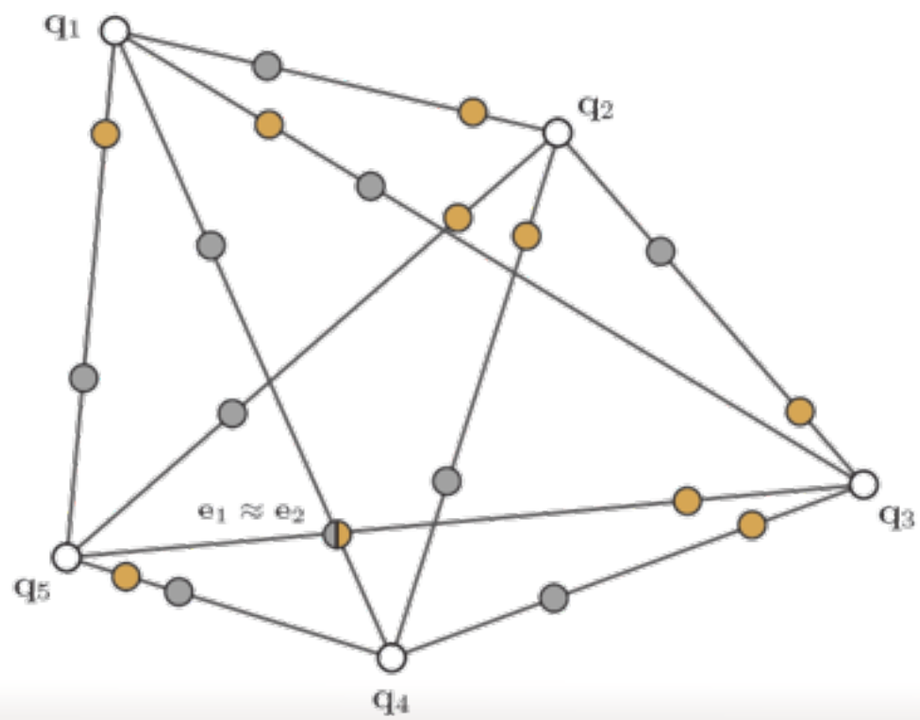


align with 4PCS,
de-noise

Key Observation

A pair of triples (from *P* and Q) is enough to uniquely define a *rigid transform* $O(n^3)$

Surprisingly, a *special* set of 4-points, *congruent sets*, makes the problem simpler $O(n^2)$



4-Points Congruent Sets

Few matches → **output sensitive algorithm**

Can be efficiently extracted

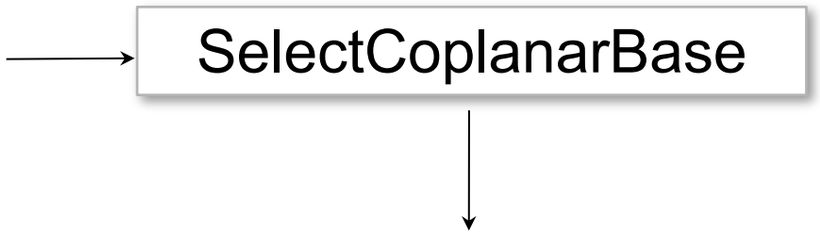
4PCS Algorithm

P

Q

4PCS Algorithm

P

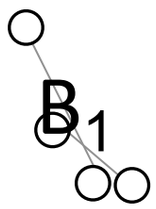


Q

4PCS Algorithm

P

SelectCoplanarBase



B1

Q

4PCS Algorithm

P

SelectCoplanarBase

B_1

Q

FindCongruent

$U = \{U_1, U_2, \ldots\}$

4PCS Algorithm

P

SelectCoplanarBase

B_1

Q

FindCongruent

U_1

4PCS Algorithm

P

SelectCoplanarBase

B_1

Q

FindCongruent

U_2

4PCS Algorithm

P

SelectCoplanarBase

B_1

Q

FindCongruent

U_3

4PCS Algorithm

P

SelectCoplanarBase

B_1

Q

FindCongruent

U_4

4PCS Algorithm

P

SelectCoplanarBase

B_1

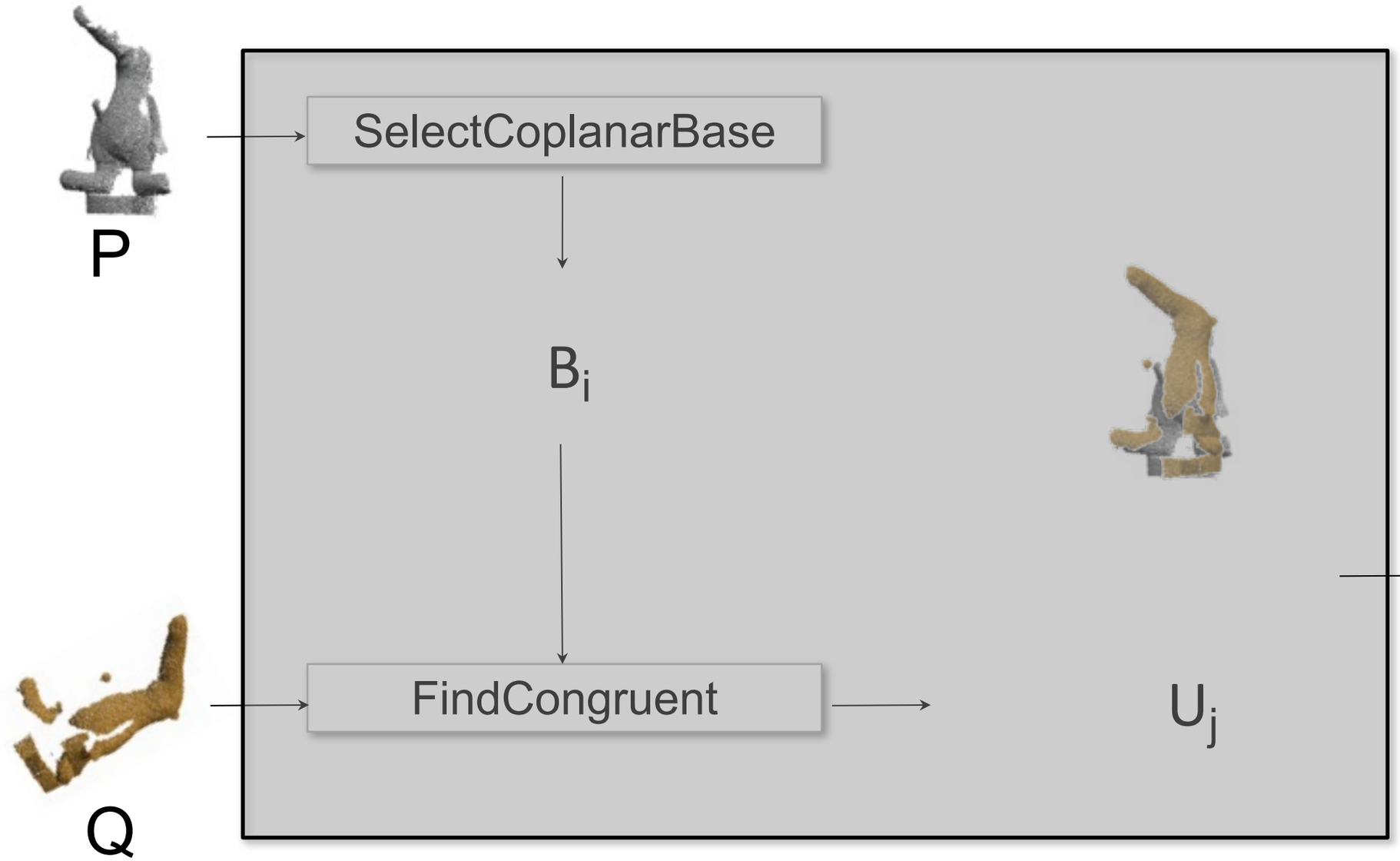
U_j

retain the best transform (T_1)

Q

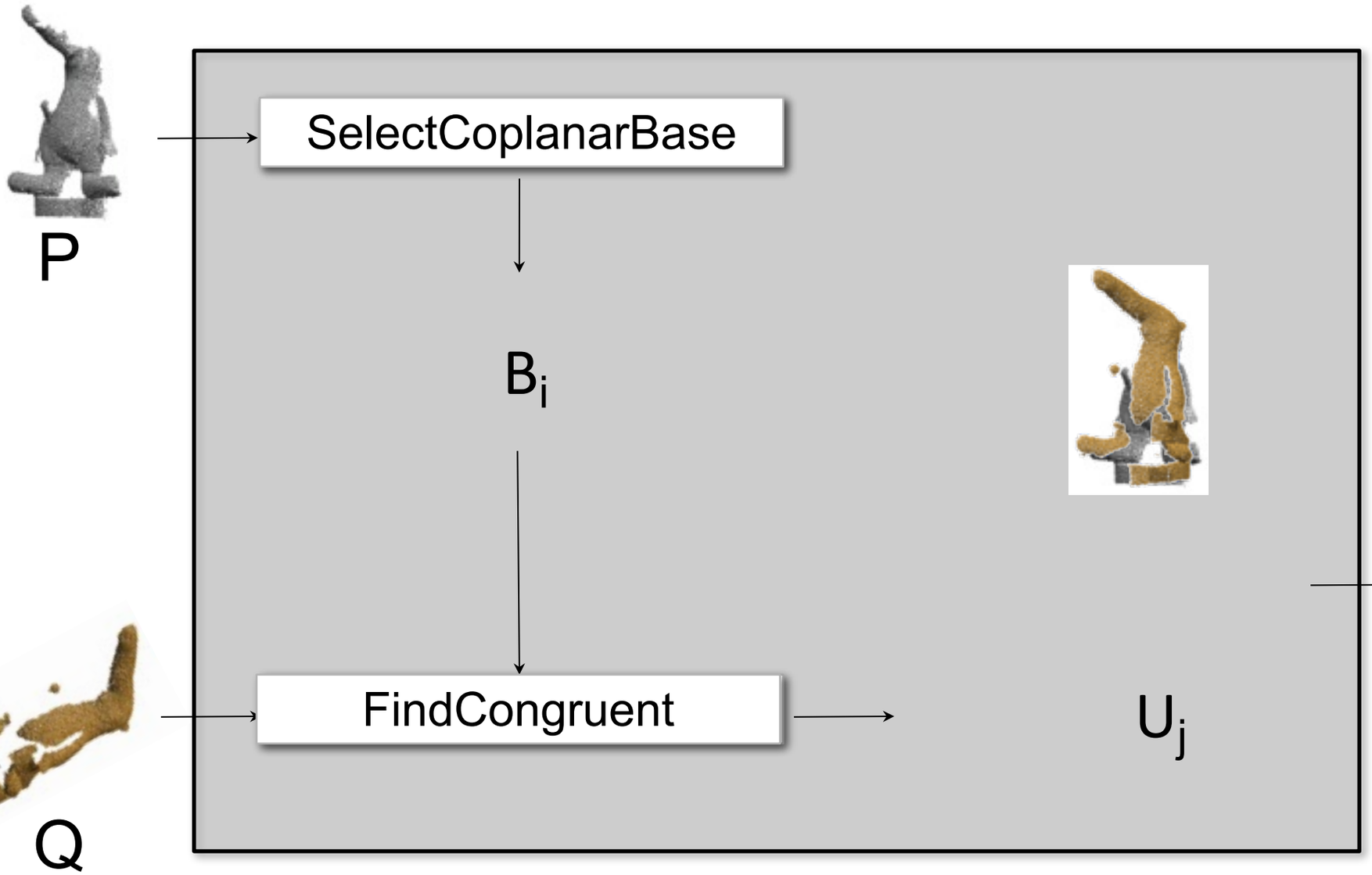
FindCongruent

4PCS Algorithm



RANSAC iterations

4PCS Algorithm



RANSAC iterations

Random Sampling

P

Subsampling

Q

SelectCoplanarBase

B_i

FindCongruent

U_j

verification

T_i

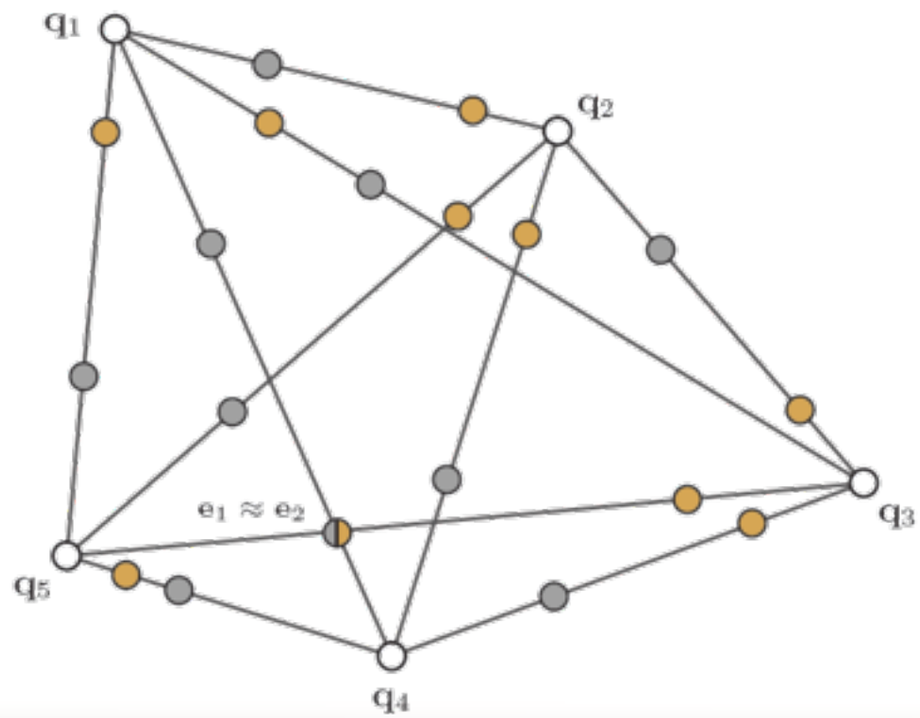
RANSAC iterations

FindCongruent

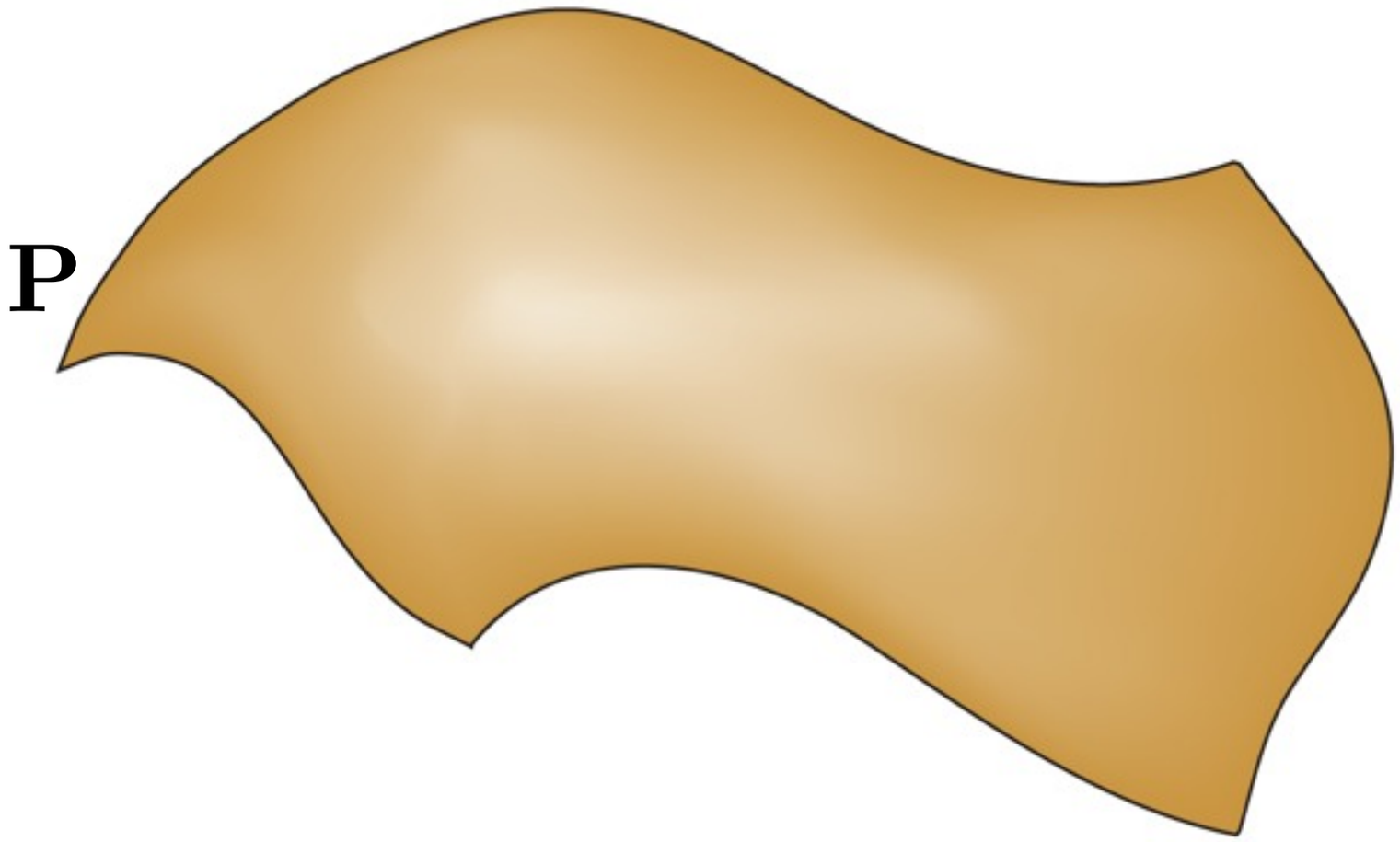
Key Observation

A pair of triples (from *P* and Q) is enough to uniquely define a *rigid transform* $O(n^3)$

Surprisingly, a *special* set of 4-points, *congruent sets*, makes the problem simpler $O(n^2)$



Affine Invariance



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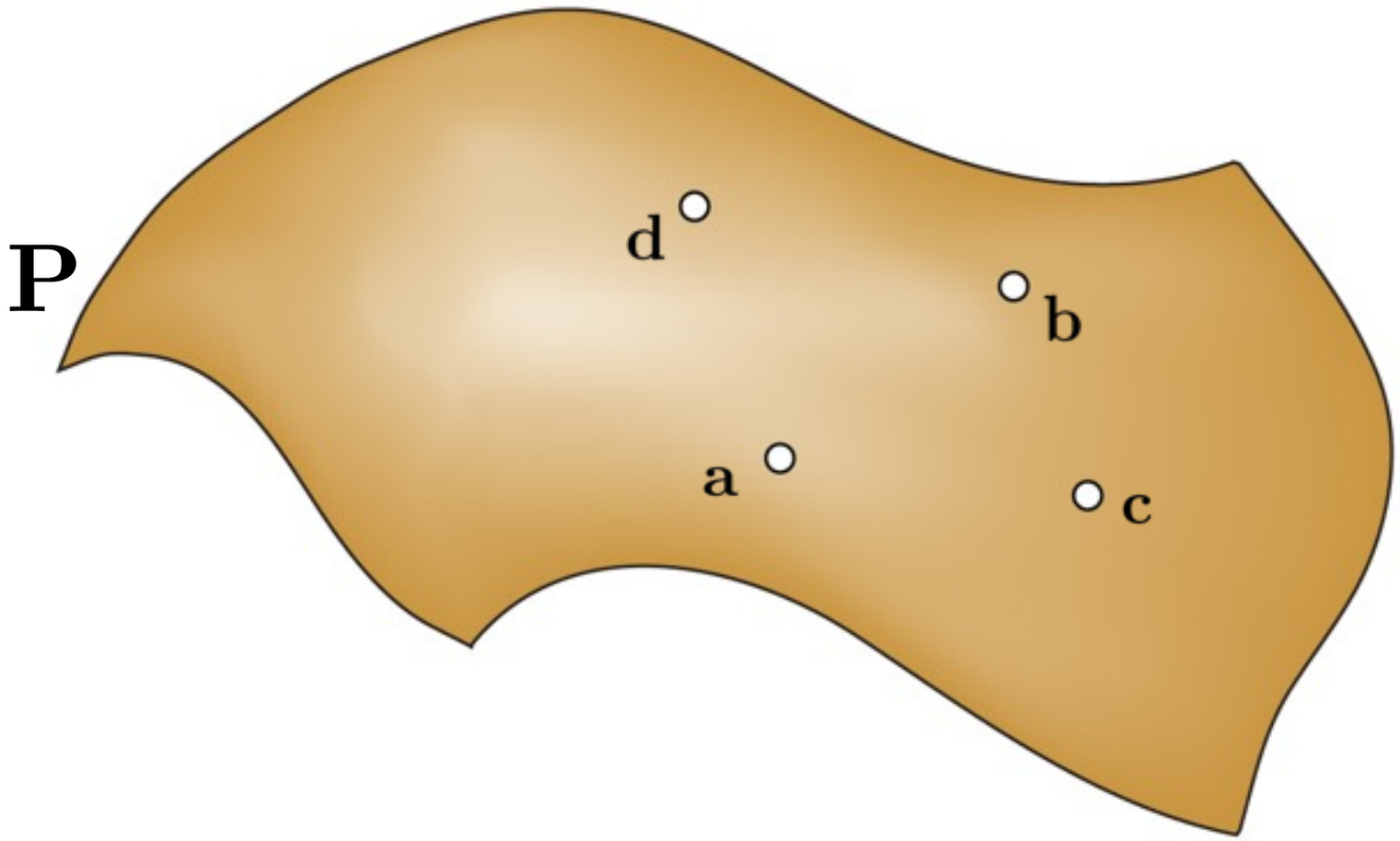

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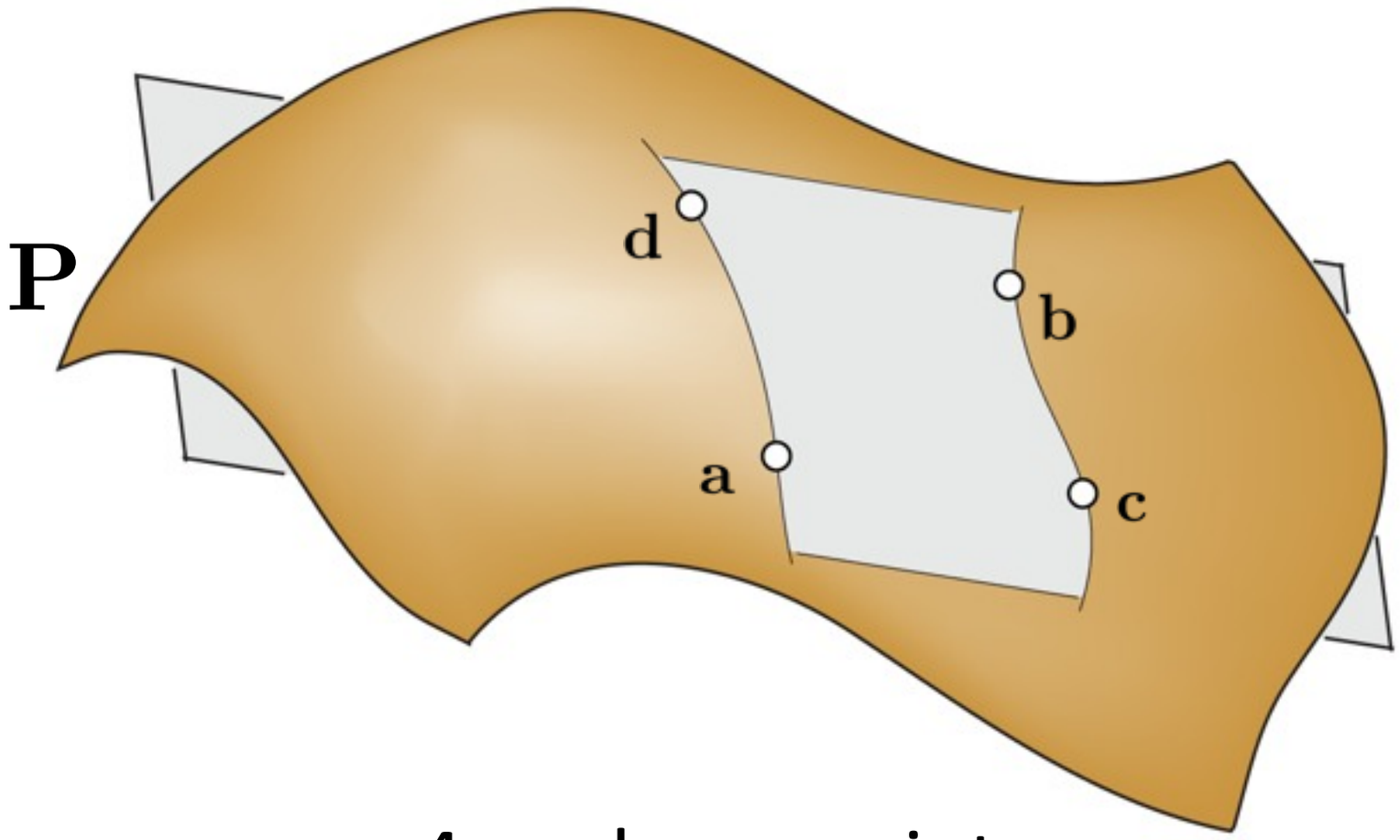
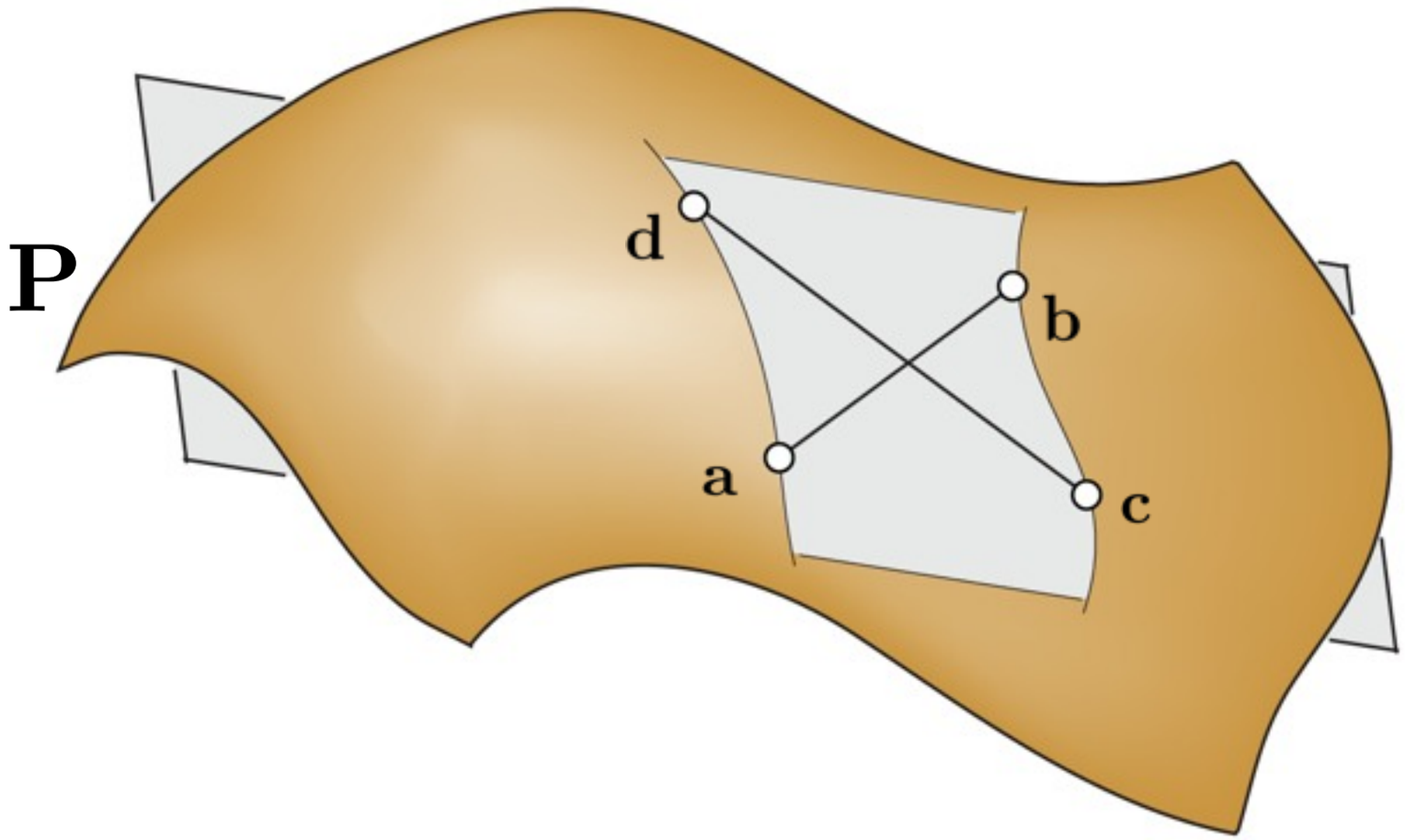
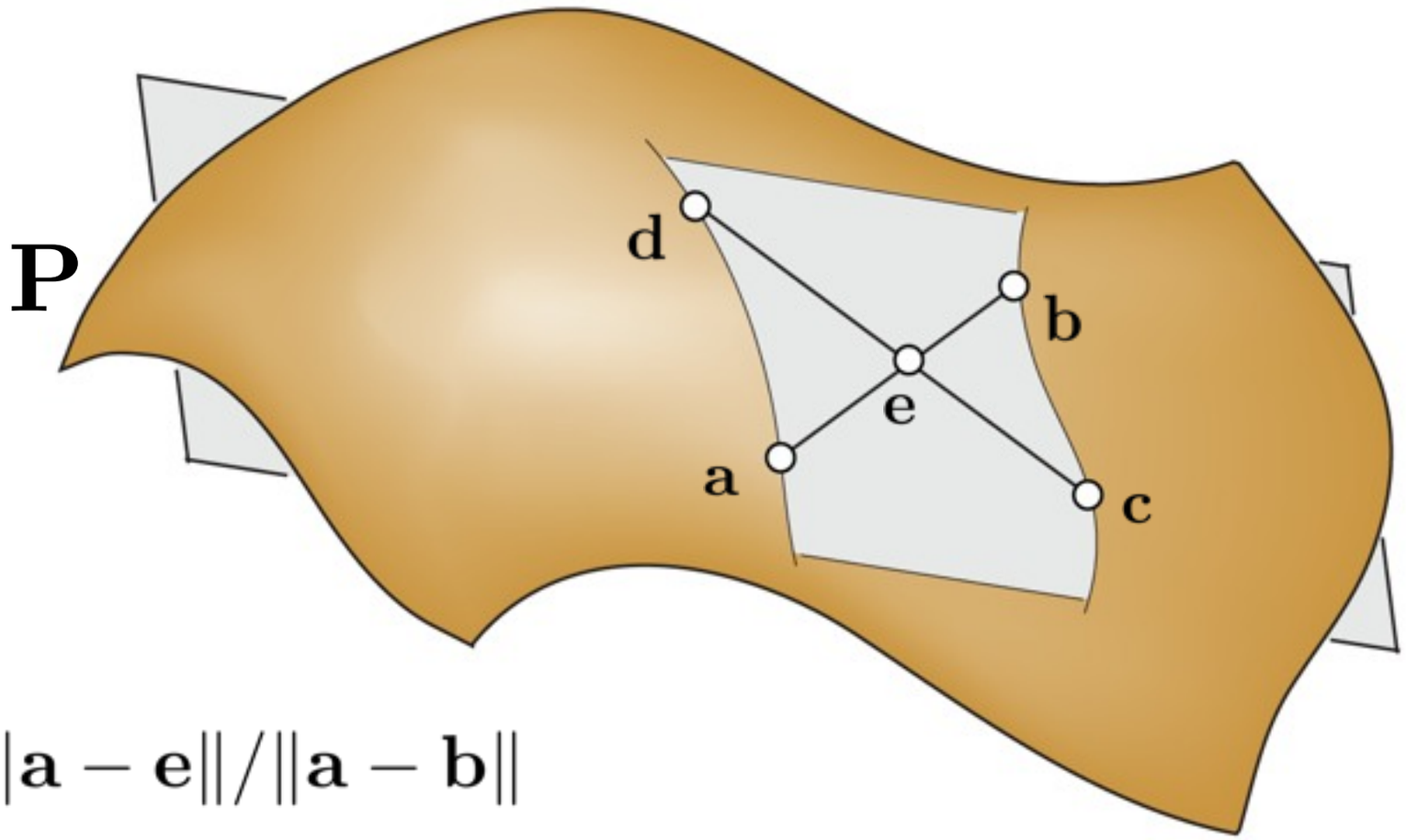


4 coplanar points

Affine Invariance



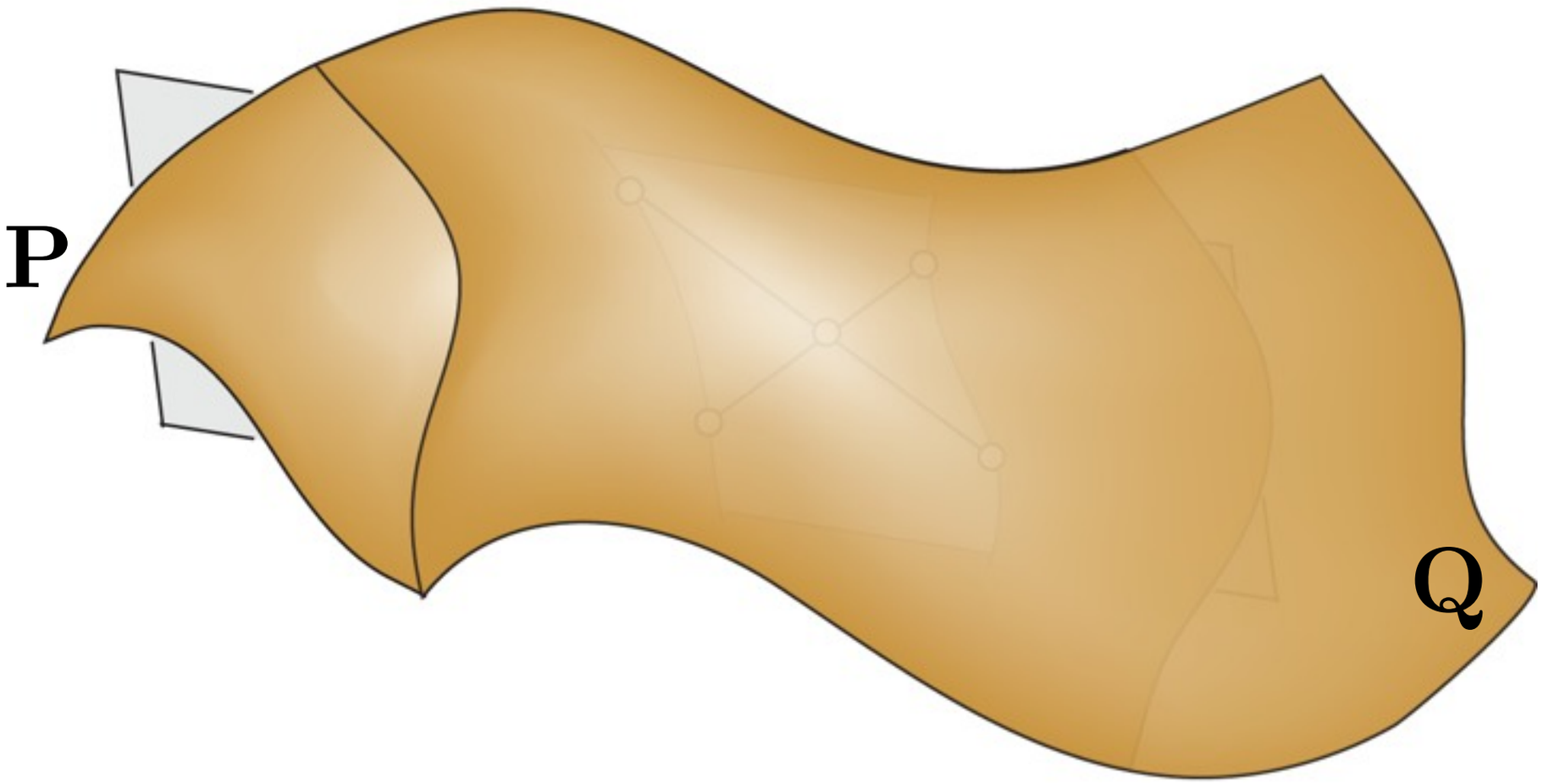
Affine Invariance



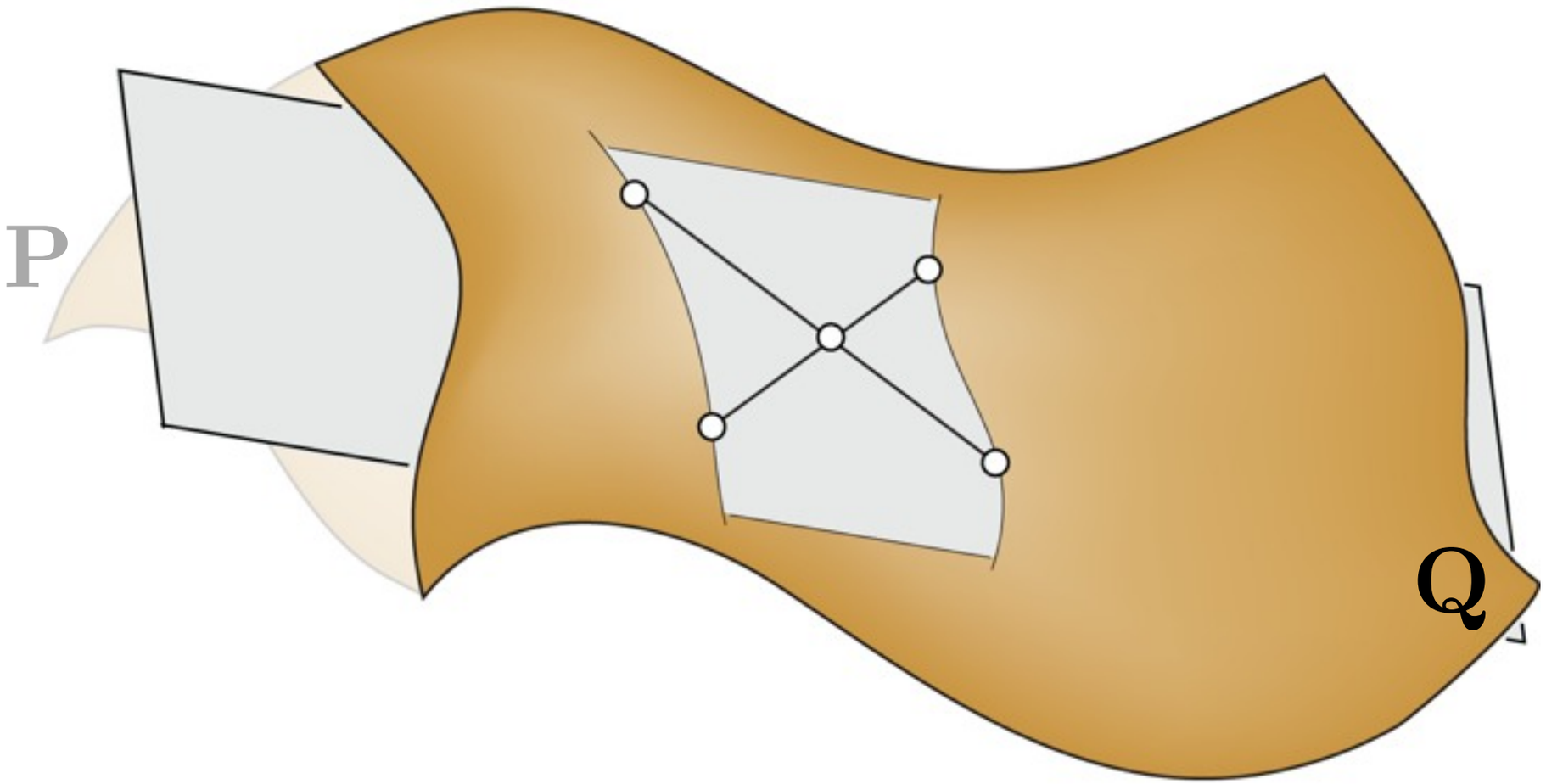
$$r_1 = \|\mathbf{a} - \mathbf{e}\| / \|\mathbf{a} - \mathbf{b}\|$$

$$r_2 = \|\mathbf{c} - \mathbf{e}\| / \|\mathbf{c} - \mathbf{d}\|$$

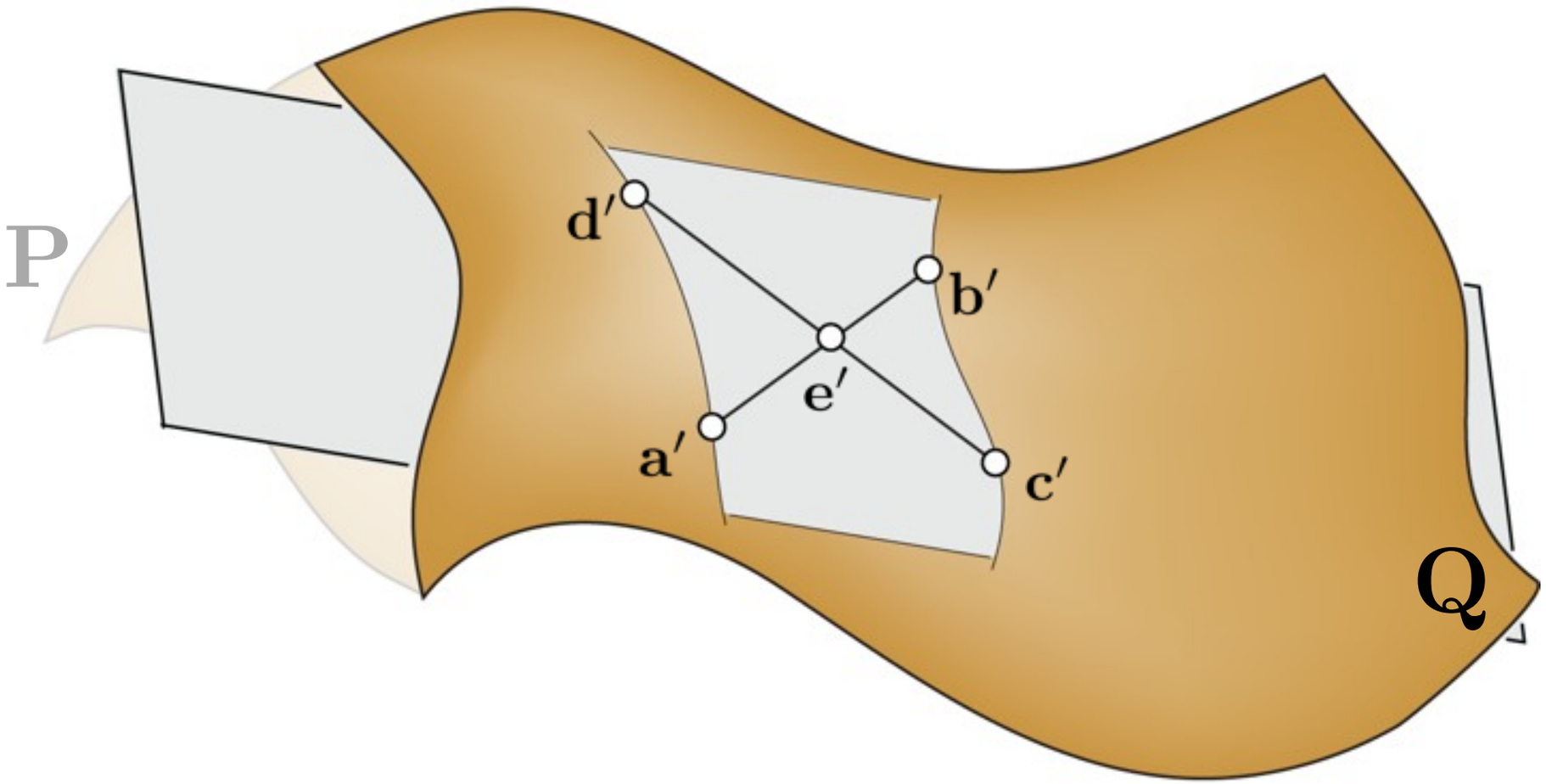
Affine Invariance



Affine Invariance



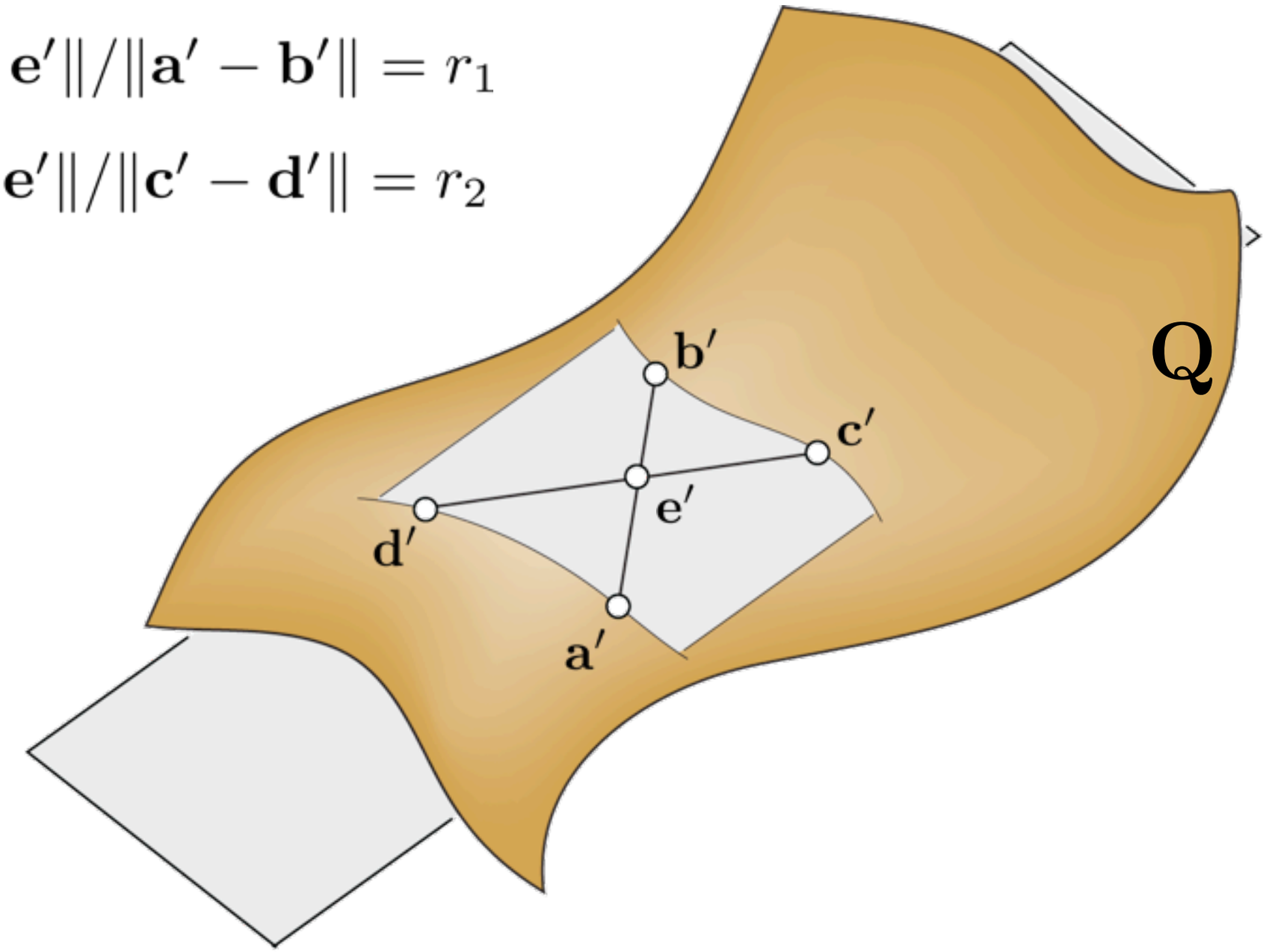
Affine Invariance



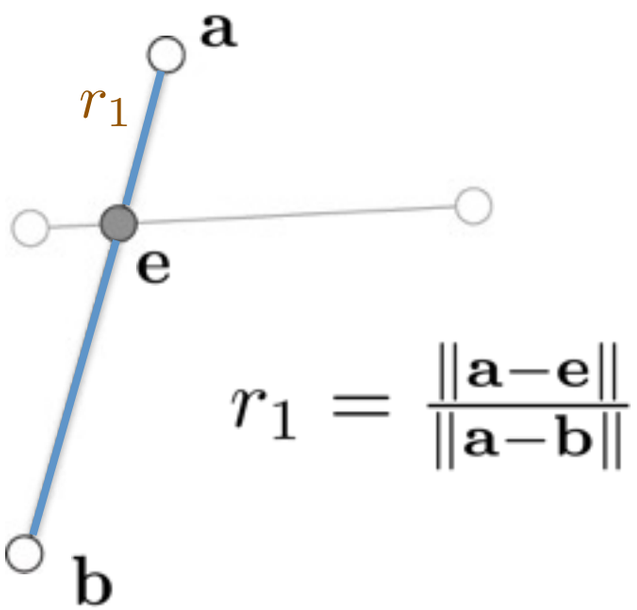
Affine Invariance

$$\|\mathbf{a}' - \mathbf{e}'\| / \|\mathbf{a}' - \mathbf{b}'\| = r_1$$

$$\|\mathbf{c}' - \mathbf{e}'\| / \|\mathbf{c}' - \mathbf{d}'\| = r_2$$



Extracting Congruent 4-points

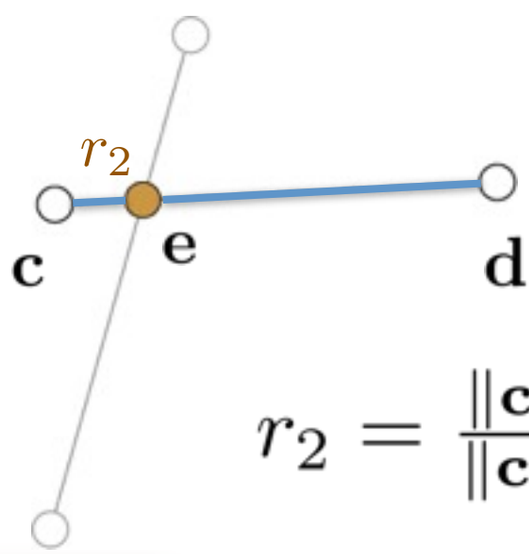


$$r_1 = \frac{\|\mathbf{a}-\mathbf{e}\|}{\|\mathbf{a}-\mathbf{b}\|}$$

$$\{\mathbf{a}, \mathbf{b}, \mathbf{c}, \mathbf{d}\}$$

$$\rightarrow \mathbf{e}, r_1, r_2$$

P



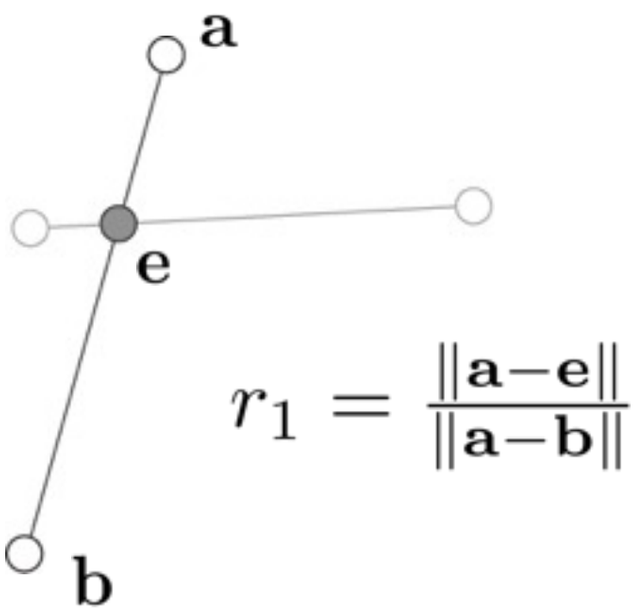
$$r_2 = \frac{\|\mathbf{c}-\mathbf{e}\|}{\|\mathbf{c}-\mathbf{d}\|}$$

$$\mathbf{a}', \mathbf{b}', r_1 \rightarrow \mathbf{e}'$$

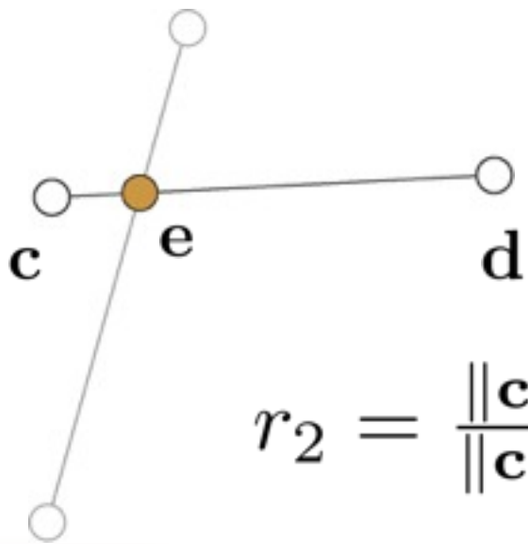
$$r_1 = \frac{\|\mathbf{a}'-\mathbf{e}'\|}{\|\mathbf{a}'-\mathbf{b}'\|}$$

Q

Extracting Congruent 4-points



$$r_1 = \frac{\|\mathbf{a}-\mathbf{e}\|}{\|\mathbf{a}-\mathbf{b}\|}$$



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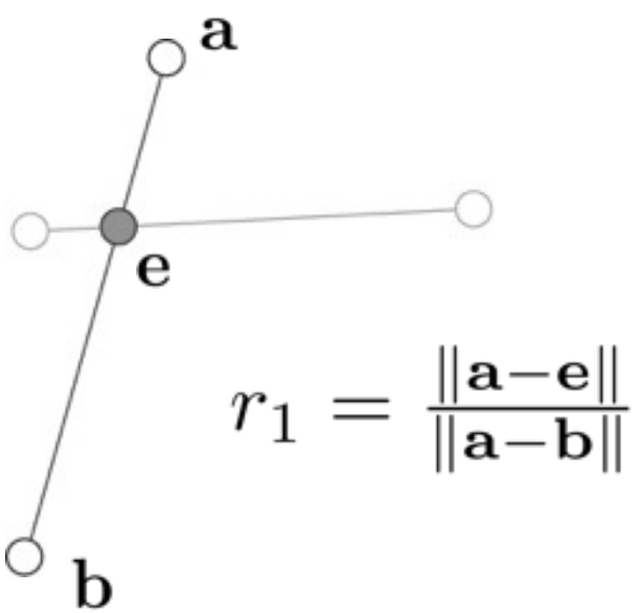

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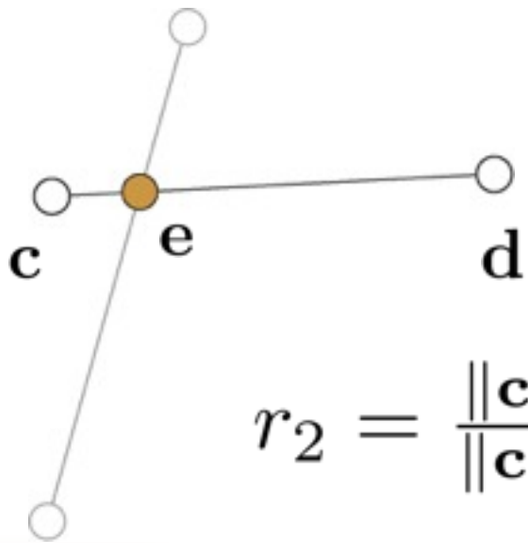

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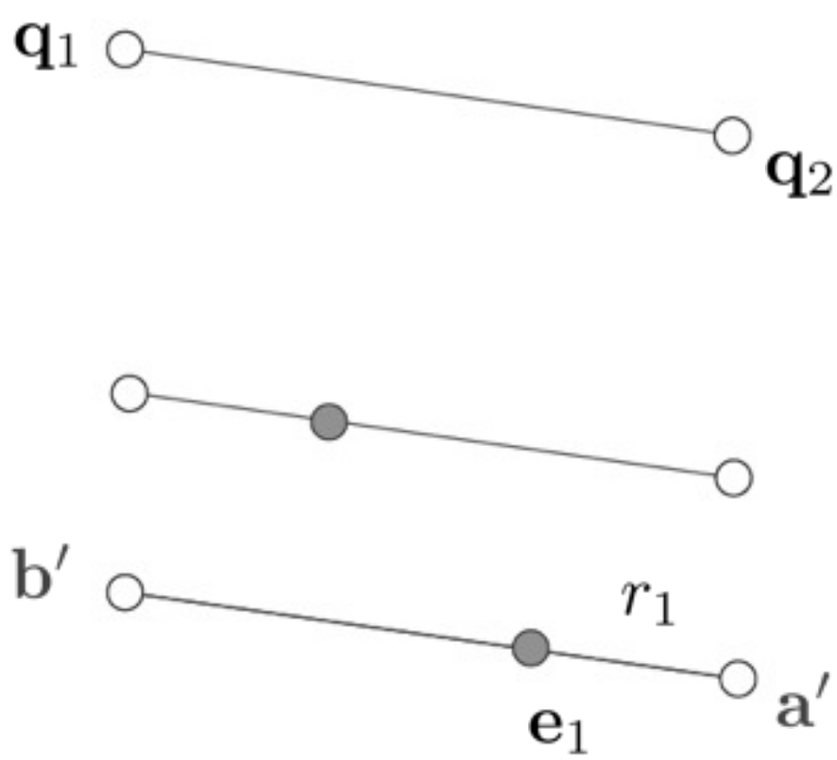

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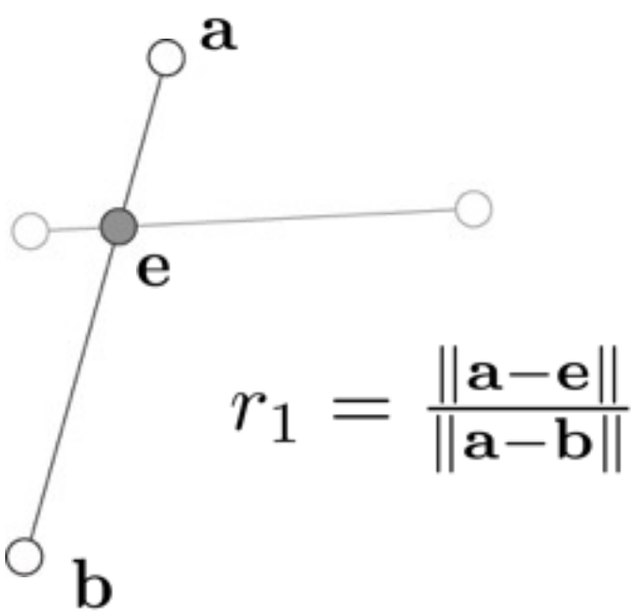

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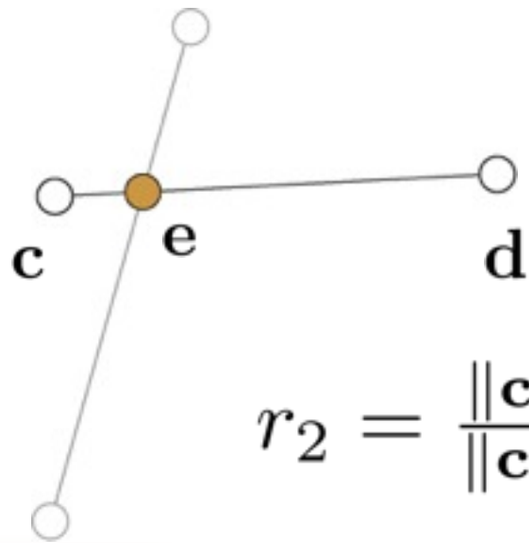

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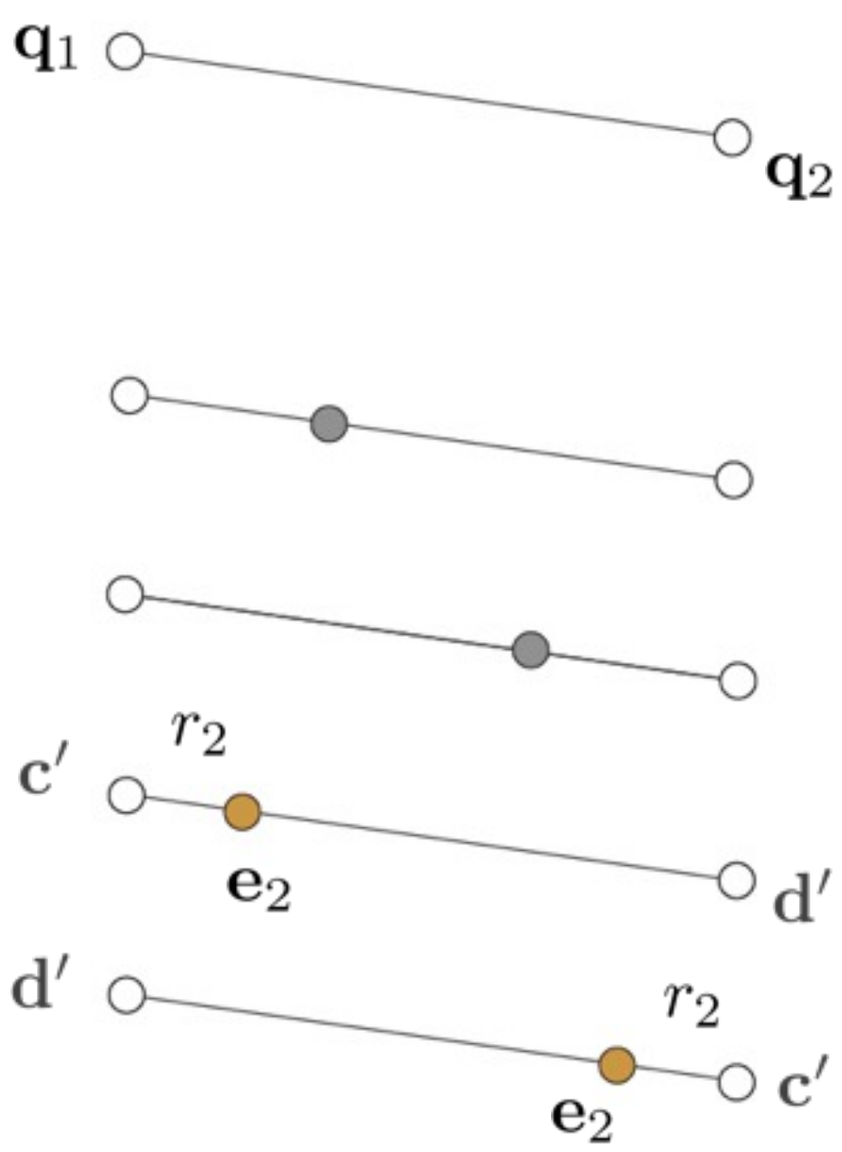

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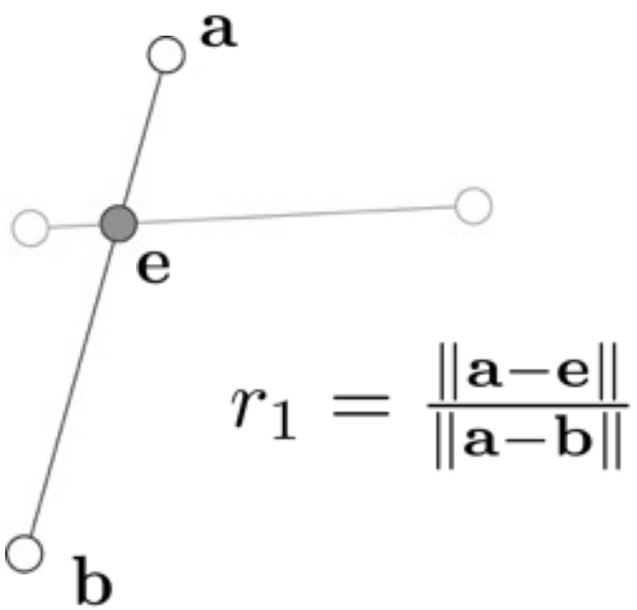

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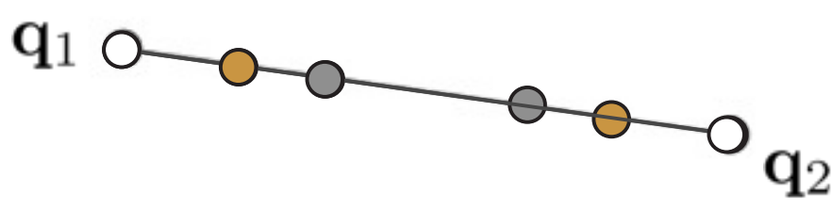
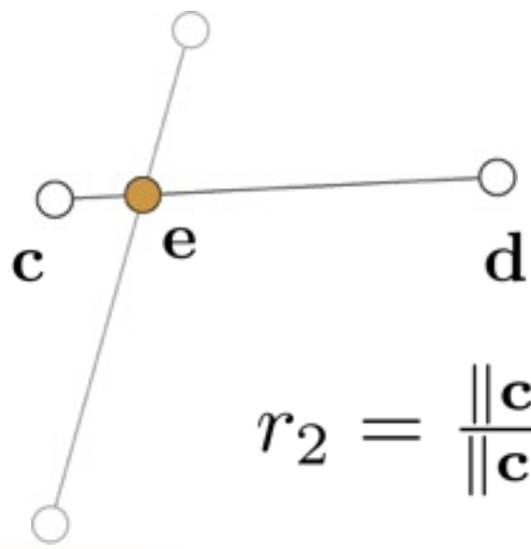
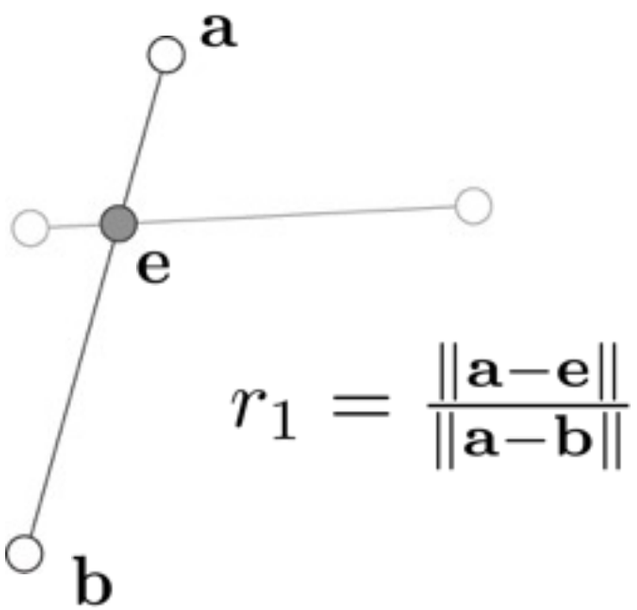


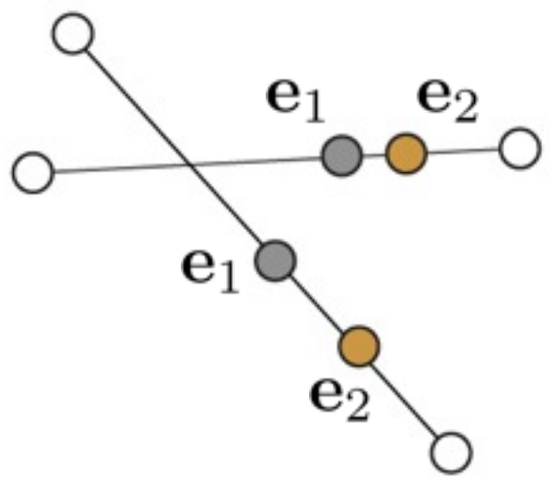


$$r_2 = \frac{\|\mathbf{c}-\mathbf{e}\|}{\|\mathbf{c}-\mathbf{d}\|}$$

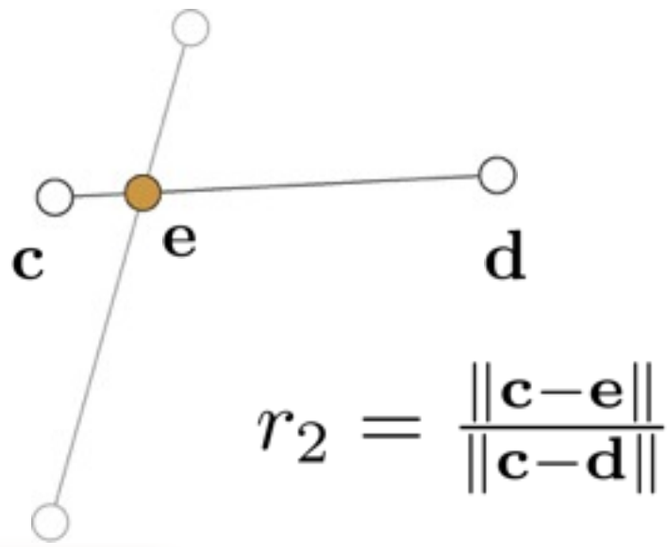
What if $\mathbf{e}_1 \neq \mathbf{e}_2$?



$$r_1 = \frac{\|\mathbf{a}-\mathbf{e}\|}{\|\mathbf{a}-\mathbf{b}\|}$$

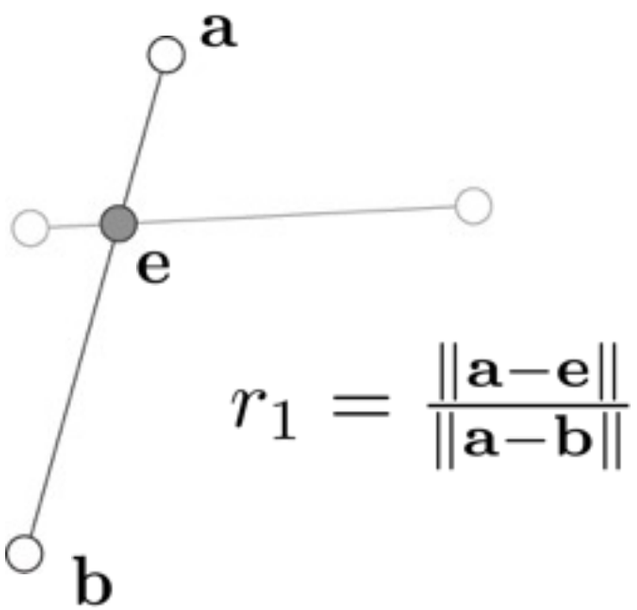


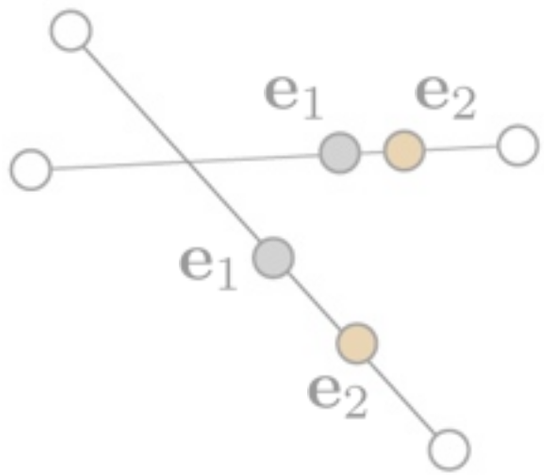
typical scenario



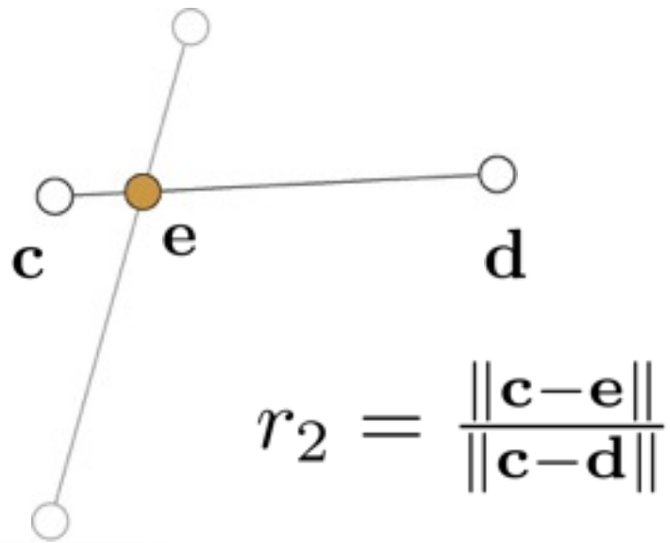
$$r_2 = \frac{\|\mathbf{c}-\mathbf{e}\|}{\|\mathbf{c}-\mathbf{d}\|}$$

What if $\mathbf{e}_1 = \mathbf{e}_2$?

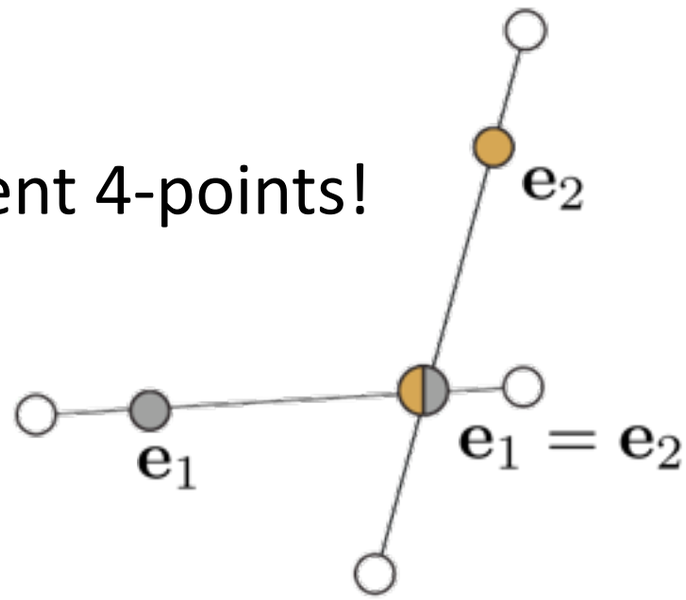




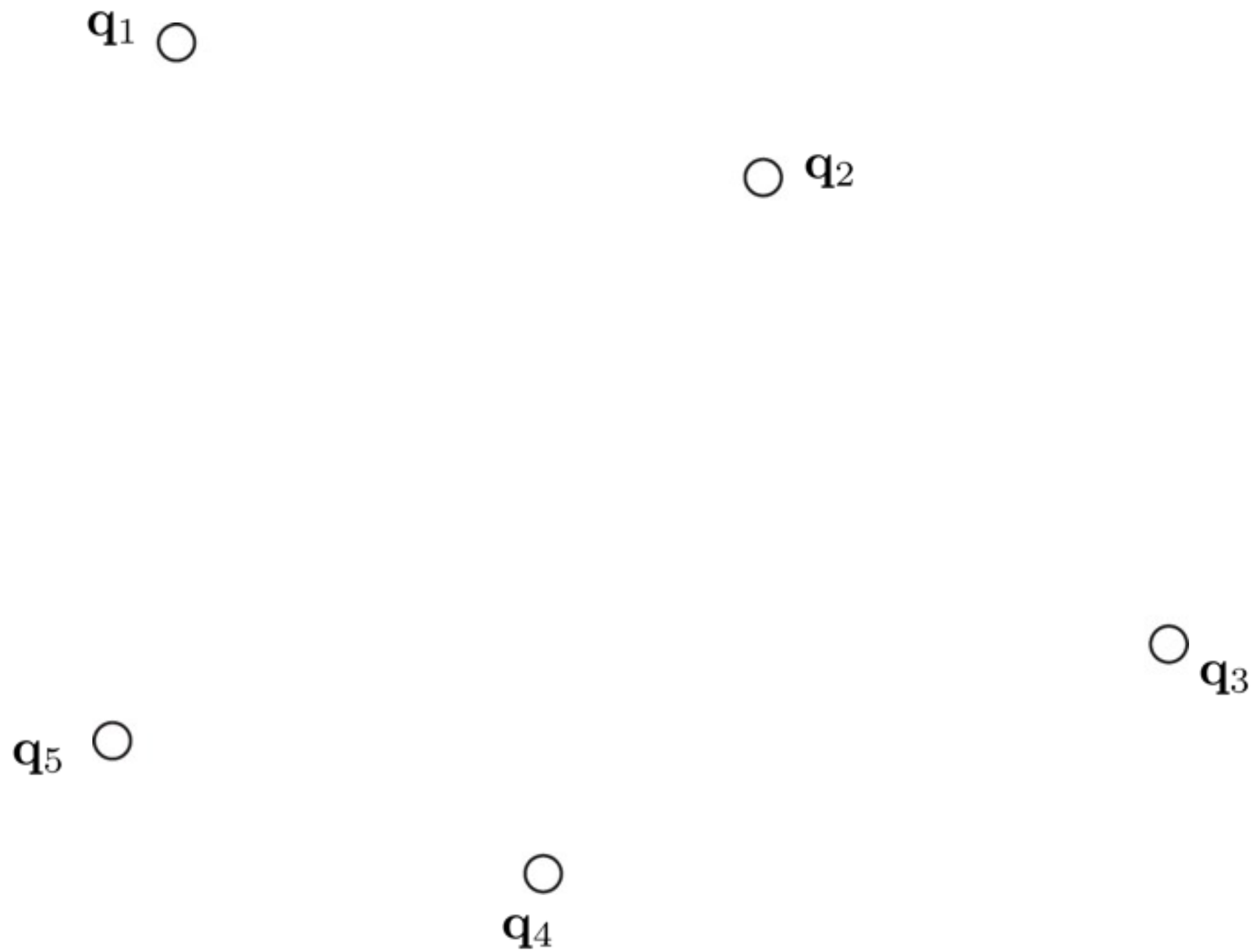
typical scenario



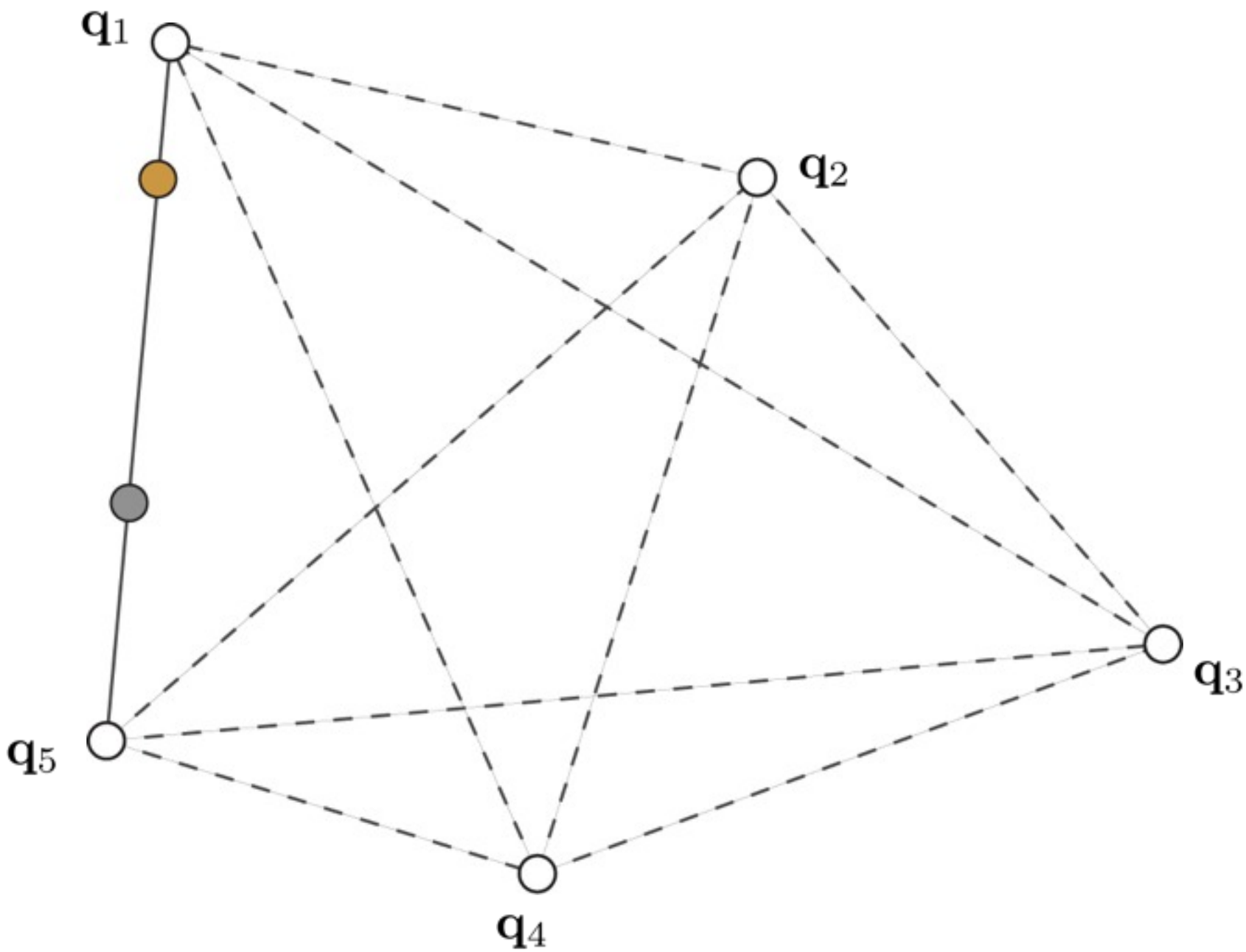
congruent 4-points!



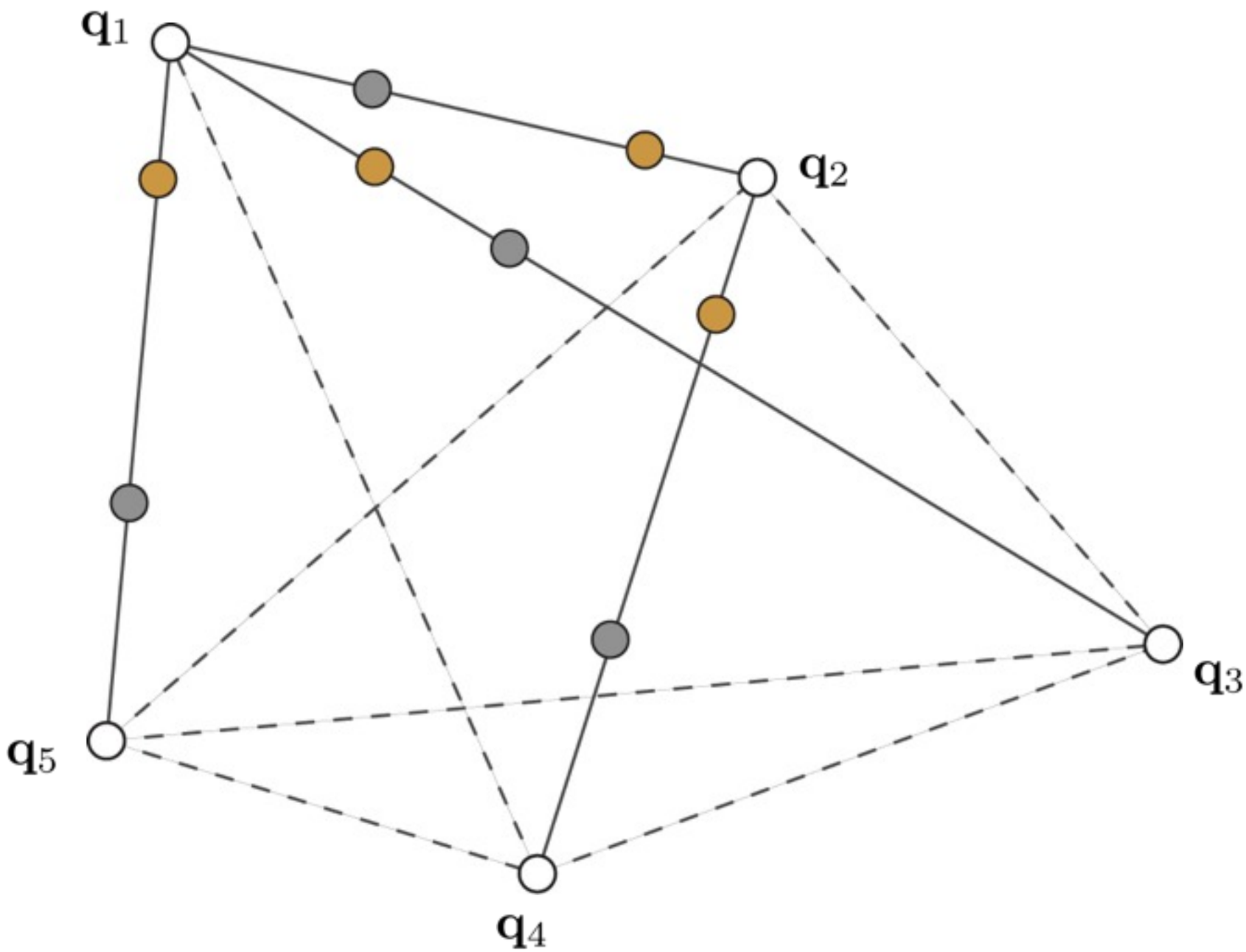
Extracting Congruent 4-points



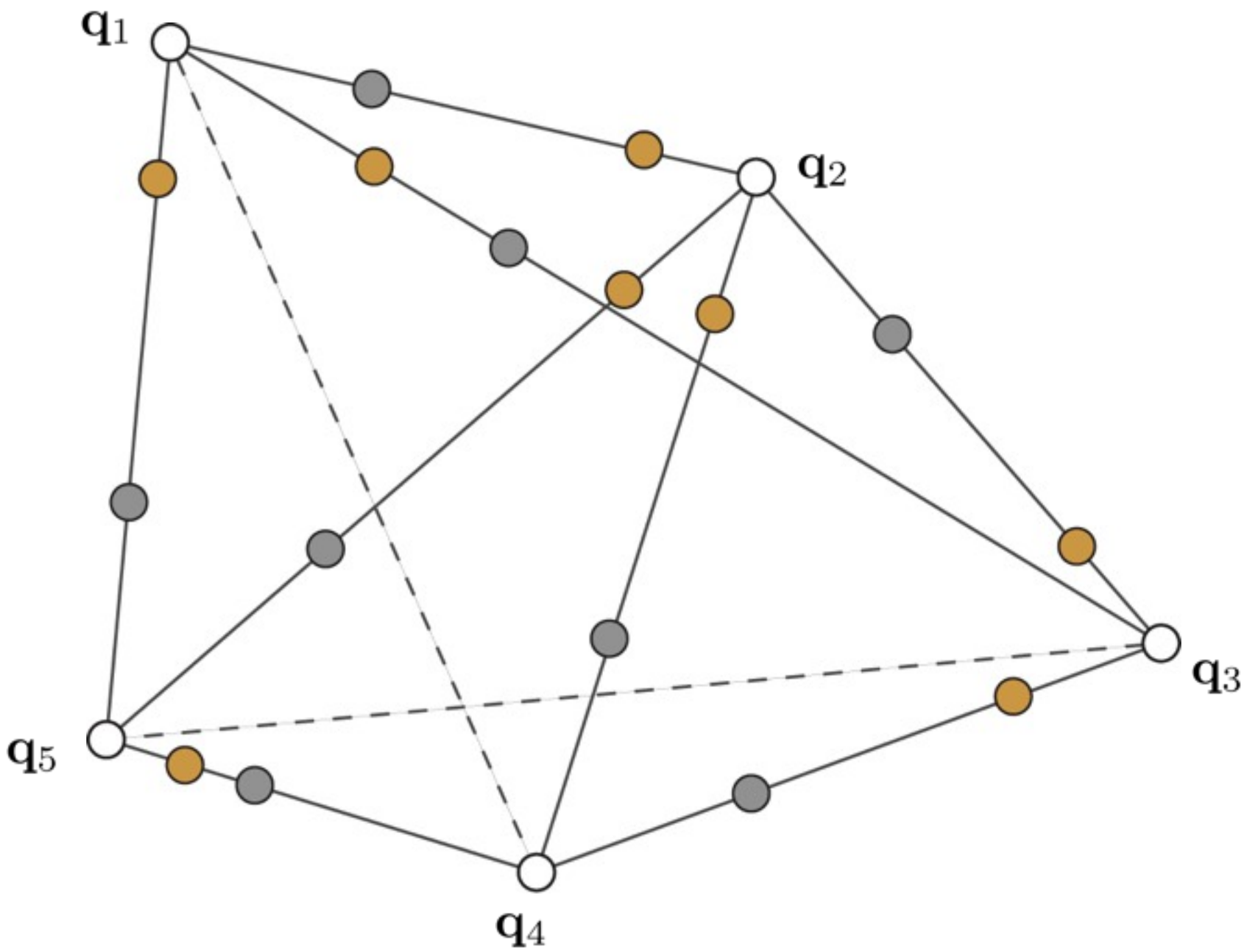
Extracting Congruent 4-points



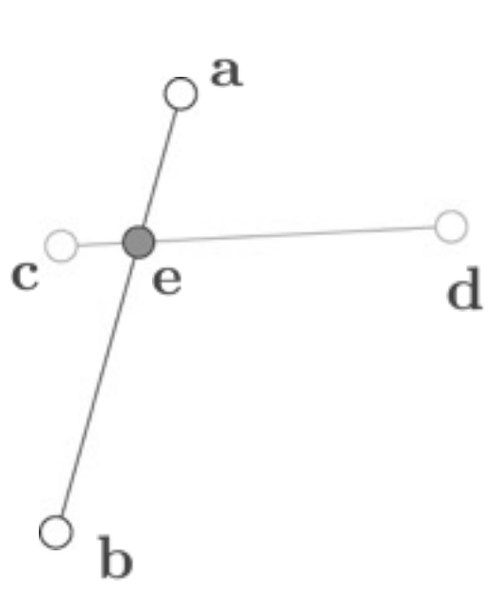
Extracting Congruent 4-points



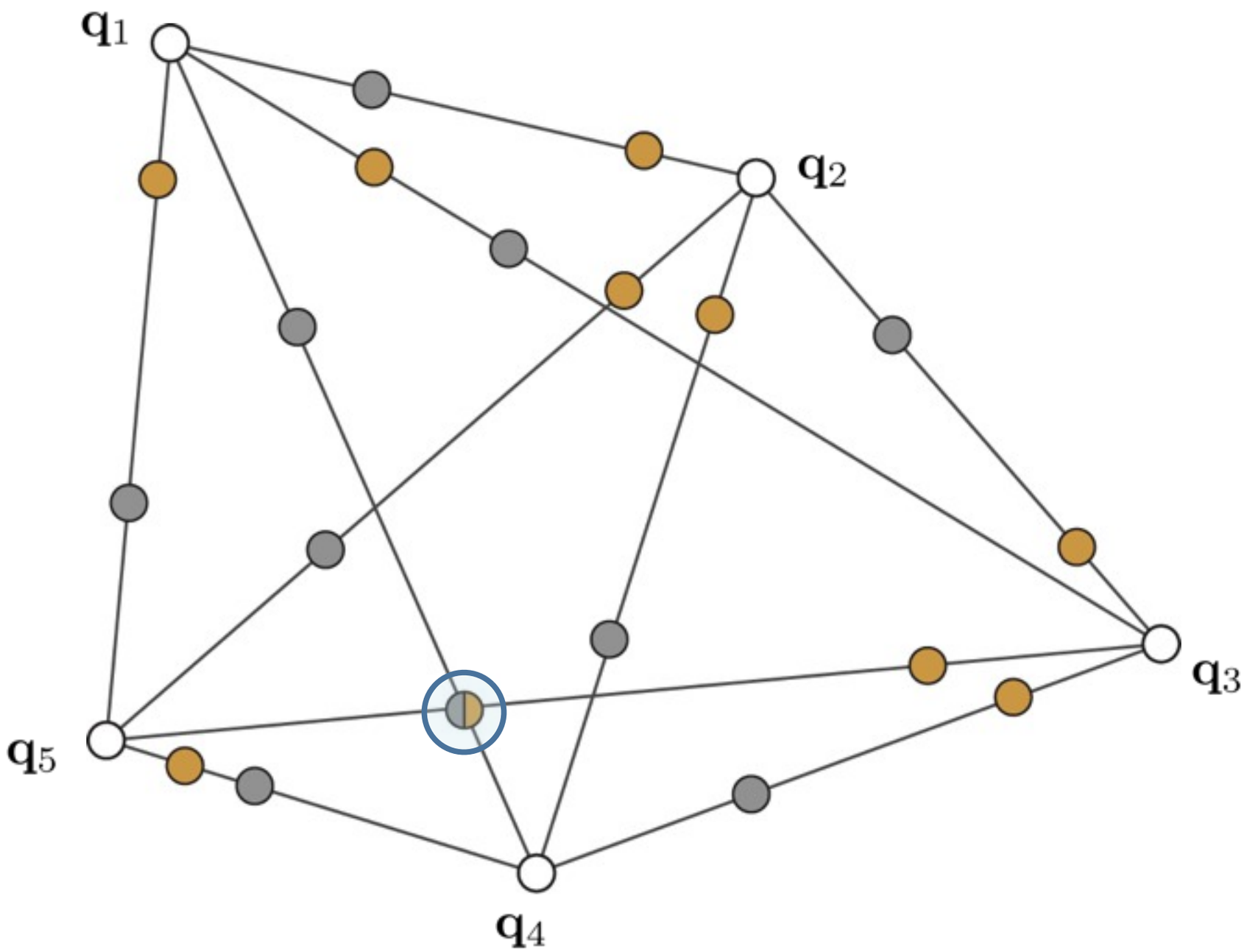
Extracting Congruent 4-points



Extracting Congruent 4-points



$$\{\mathbf{a}, \mathbf{b}, \mathbf{c}, \mathbf{d}\} \equiv$$
$$\{\mathbf{q}_1, \mathbf{q}_2, \mathbf{q}_3, \mathbf{q}_4\}$$



FindCongruent

For all the points arising using r_1 build an *approximate range-tree* (ANN)

For all the points due to r_2, *quickly* lookup for neighbors using range-tree

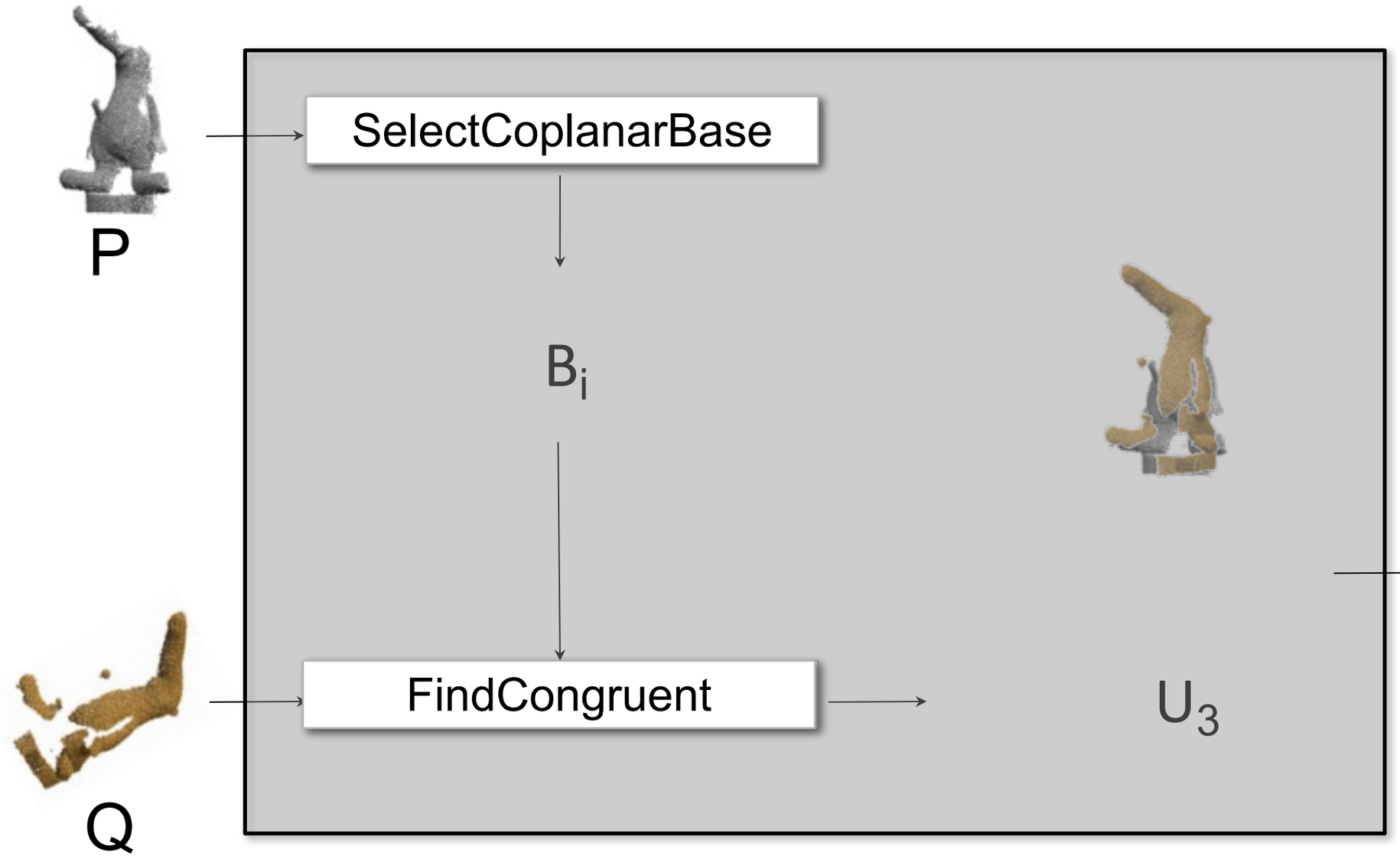
Rigid Transformation

Euclidean distances are preserved

$$(\mathbf{q}_1, \mathbf{q}_2) \longrightarrow \|\mathbf{q}_1 - \mathbf{q}_2\| \approx \|\mathbf{a} - \mathbf{b}\|$$

SelectCoplanarBase

4PCS Algorithm



RANSAC iterations

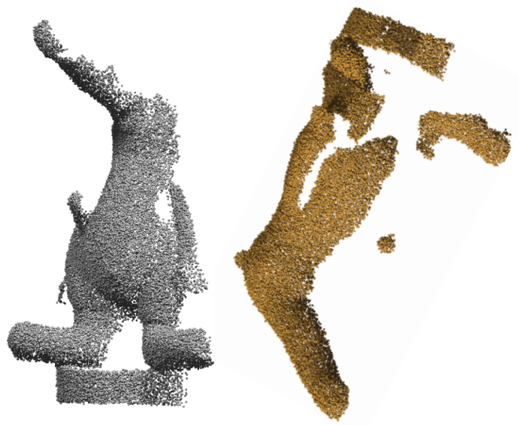
SelectCoplanarBase

Select 3 points (from P) at random →
4^{th} point to ensure (approx) *coplanarity*

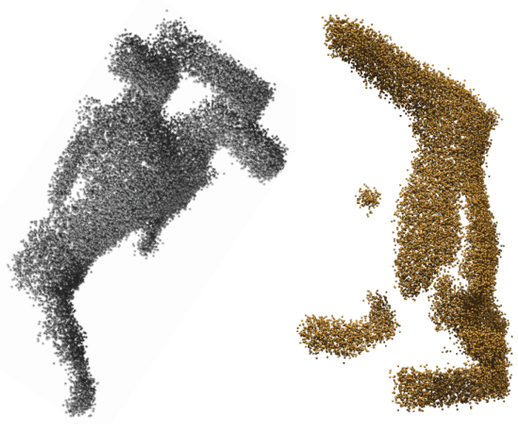
Overlap amount →
decreasing guesses f = 1, 0.5, 0.25, …

Results

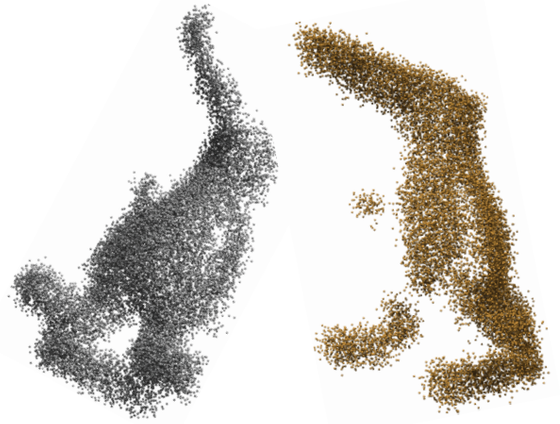
Effect of Noise



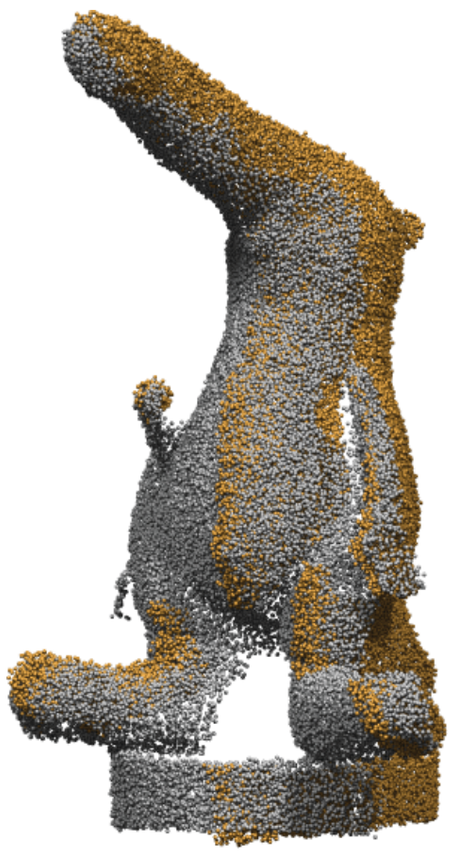
$\sigma = 0.5$



$\sigma = 2.0$

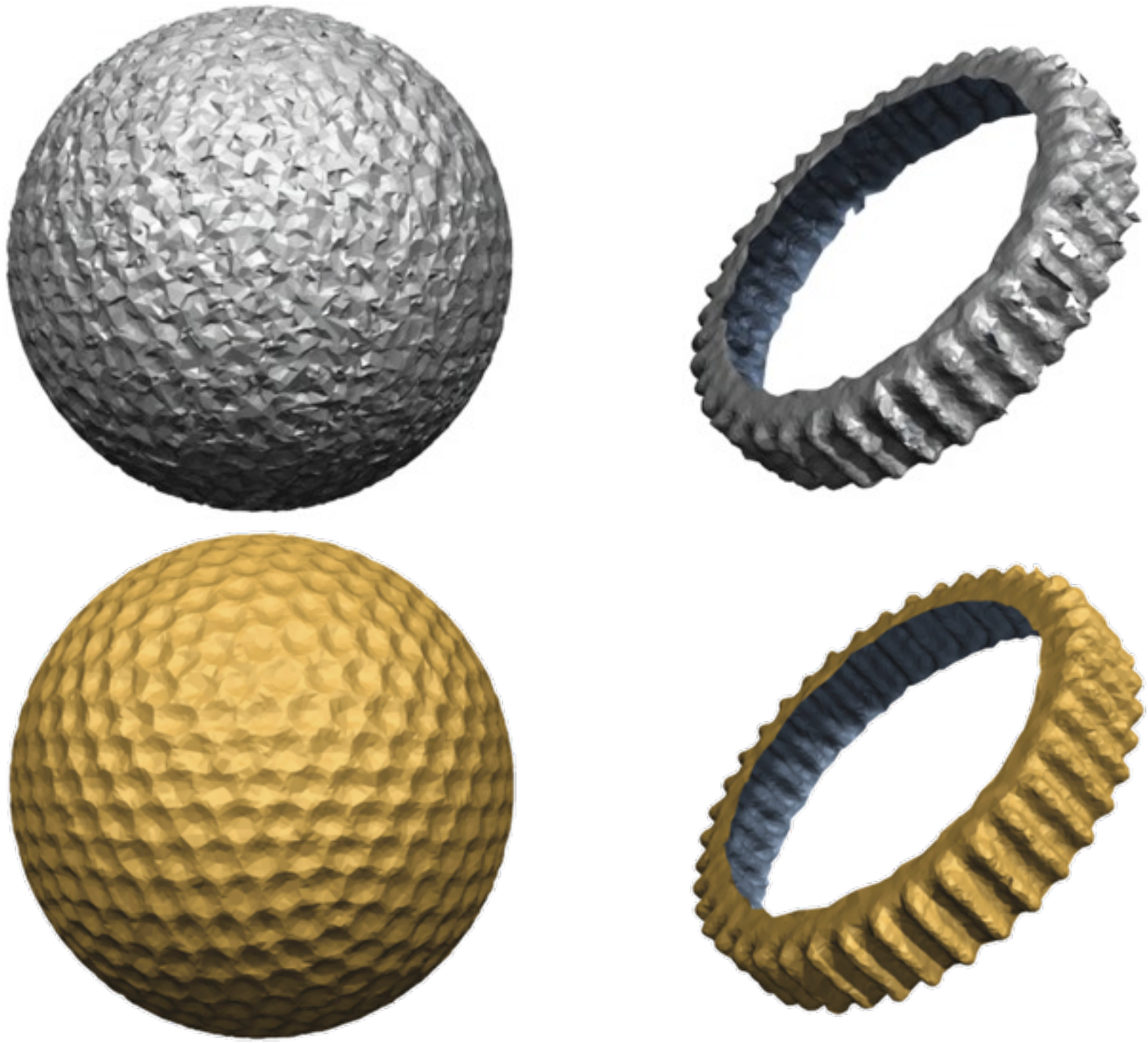


$\sigma = 4.0$



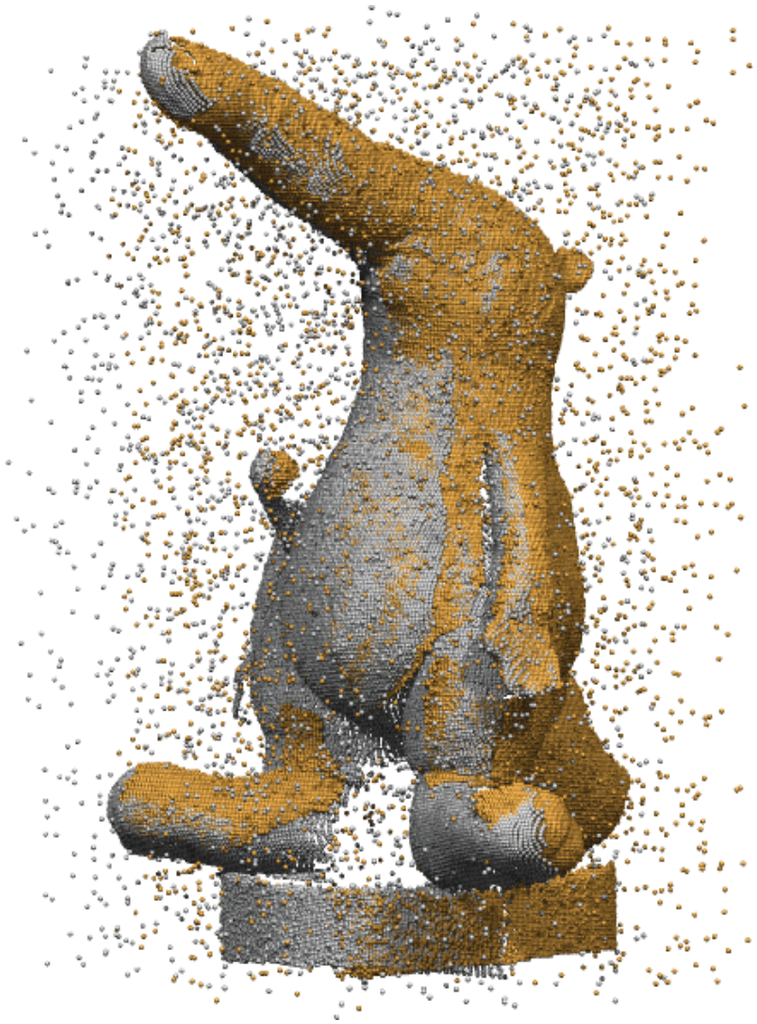
Why not Denoise Scans?

de-noise,
compute FP-s,
align



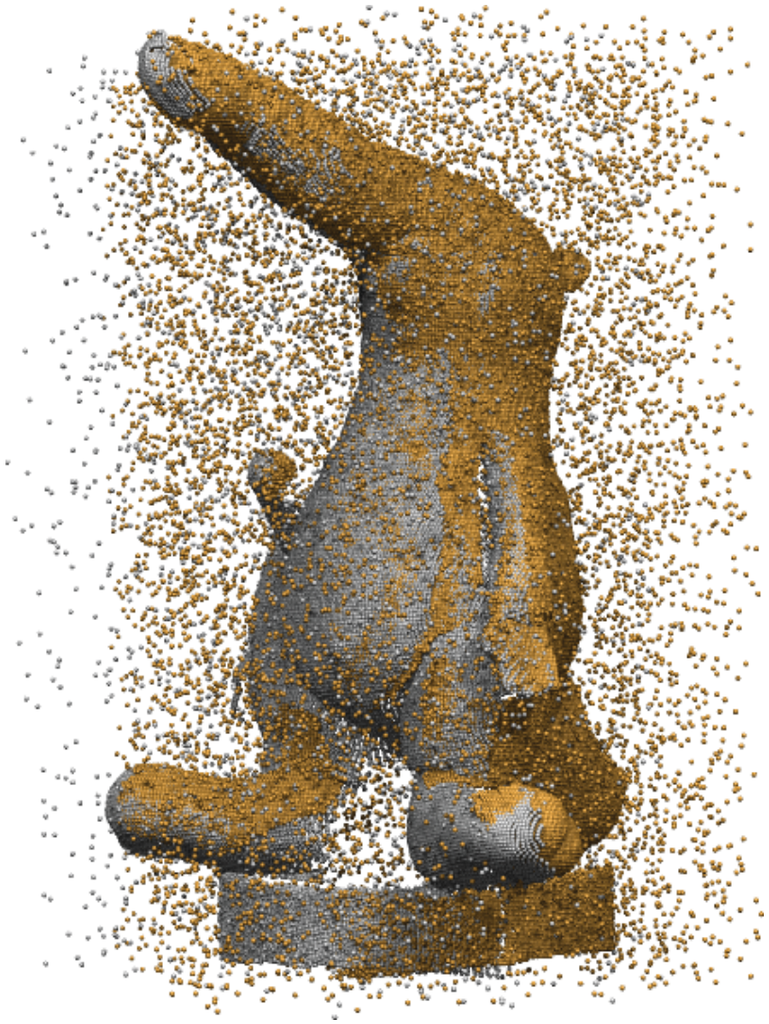
align with 4PCS,
de-noise

Effect of Outliers



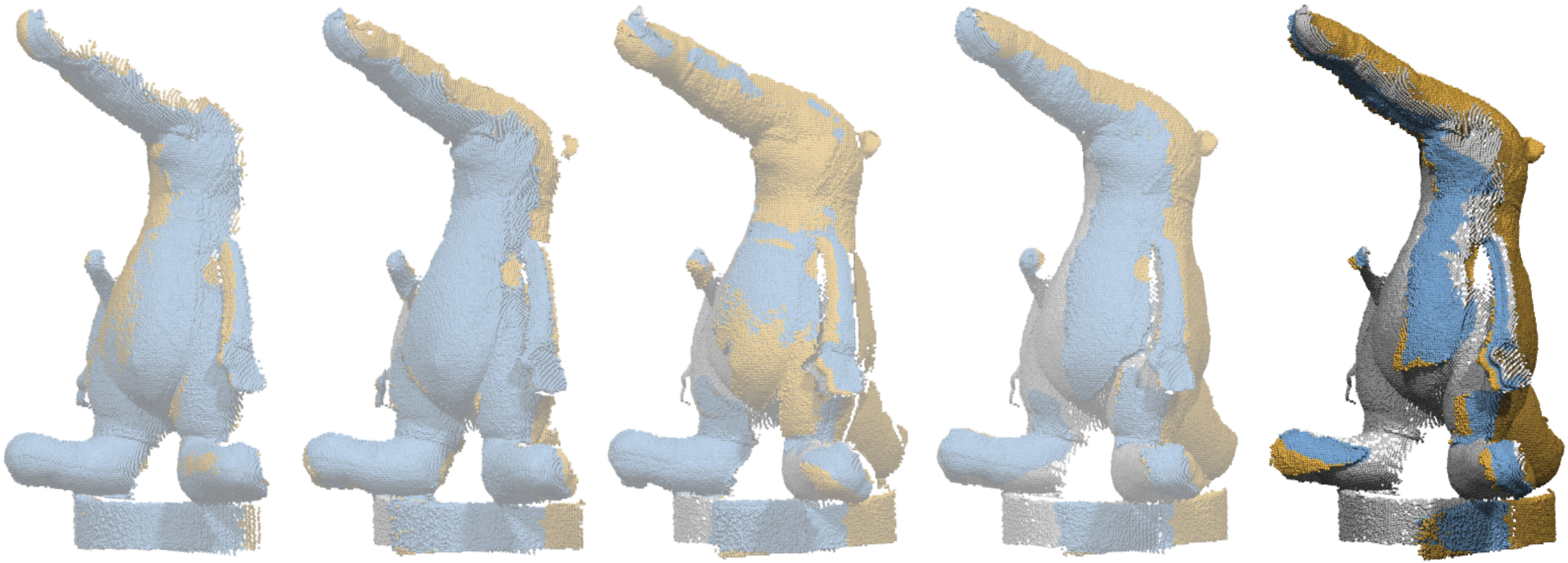
10 %

20 %



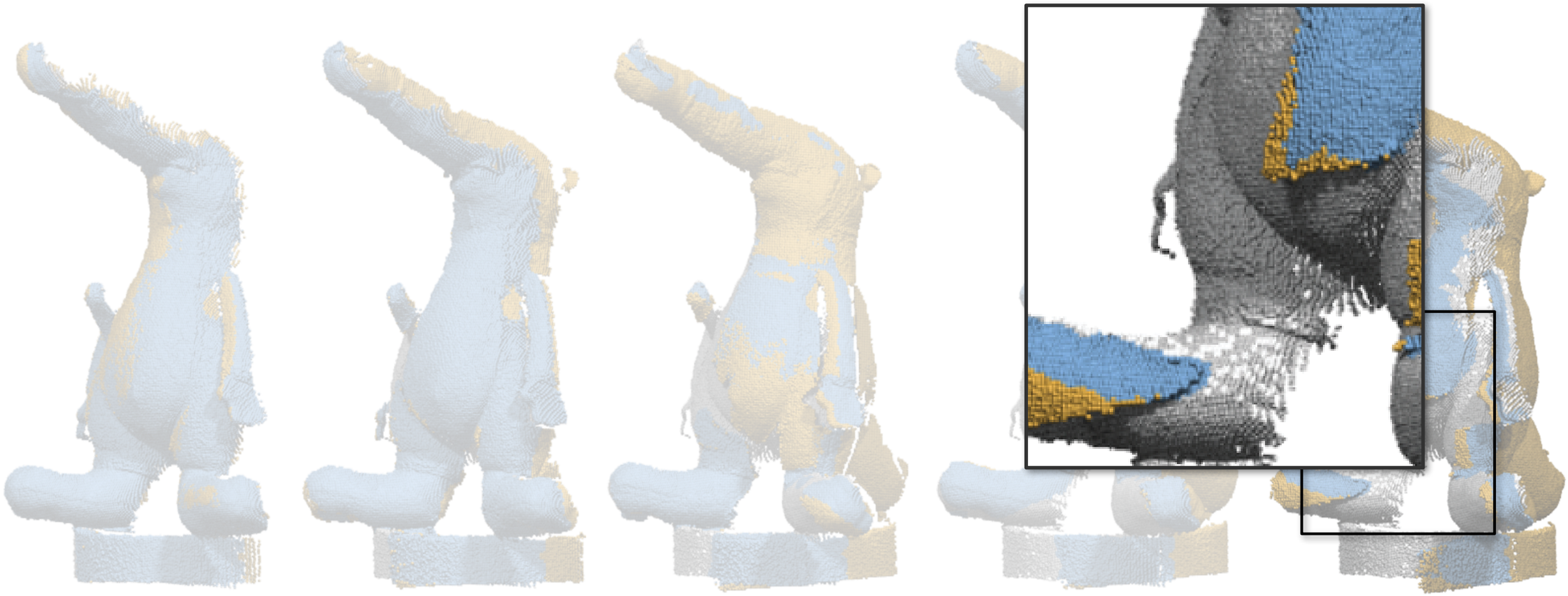
40 %

Varying Overlap



80 % 70 % 60 % 50 % 40 %

Varying Overlap



80 %

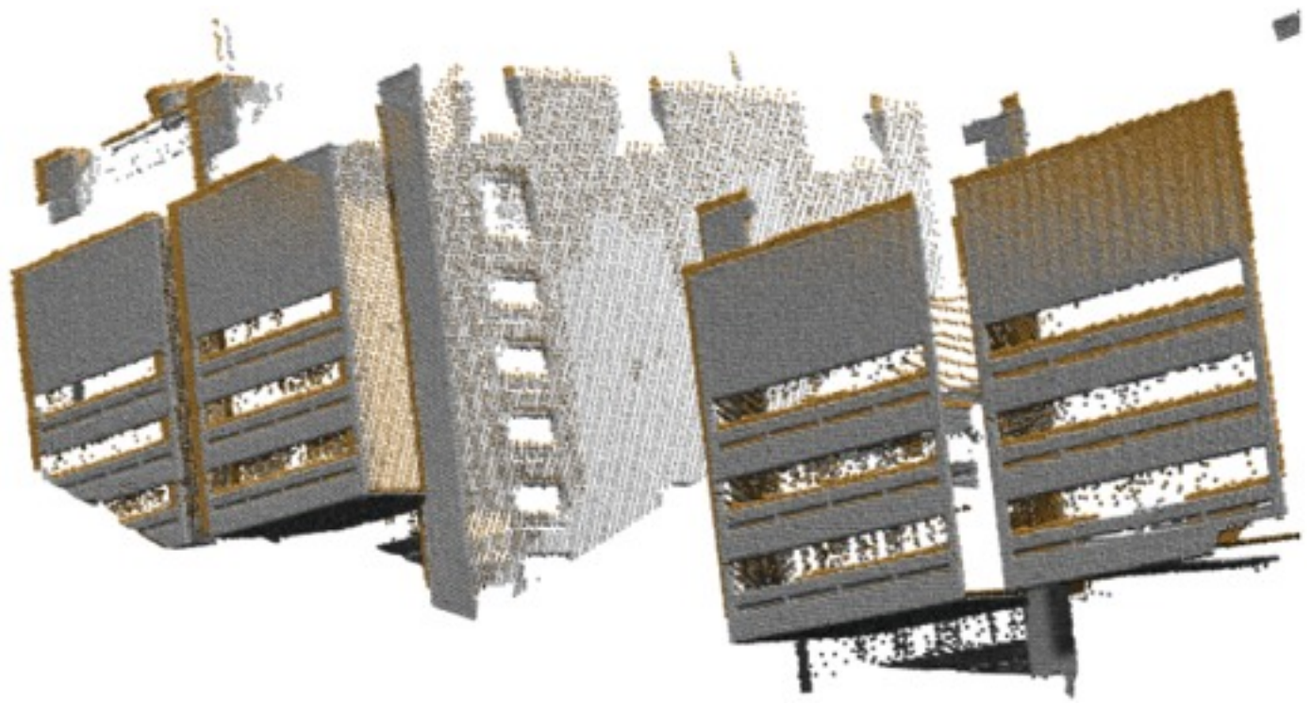
70 %

60 %

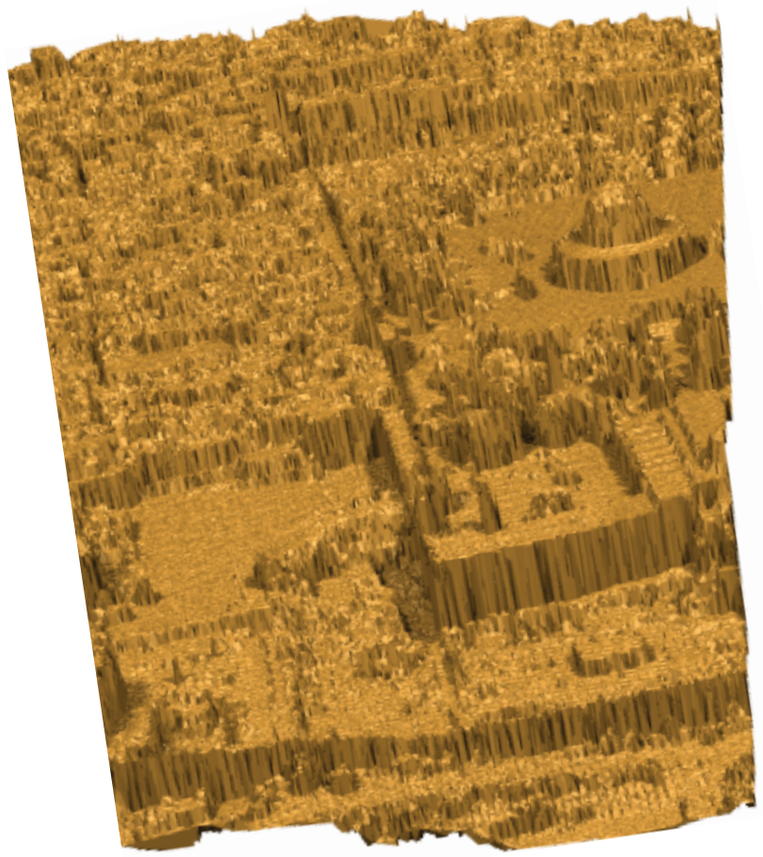
50 %

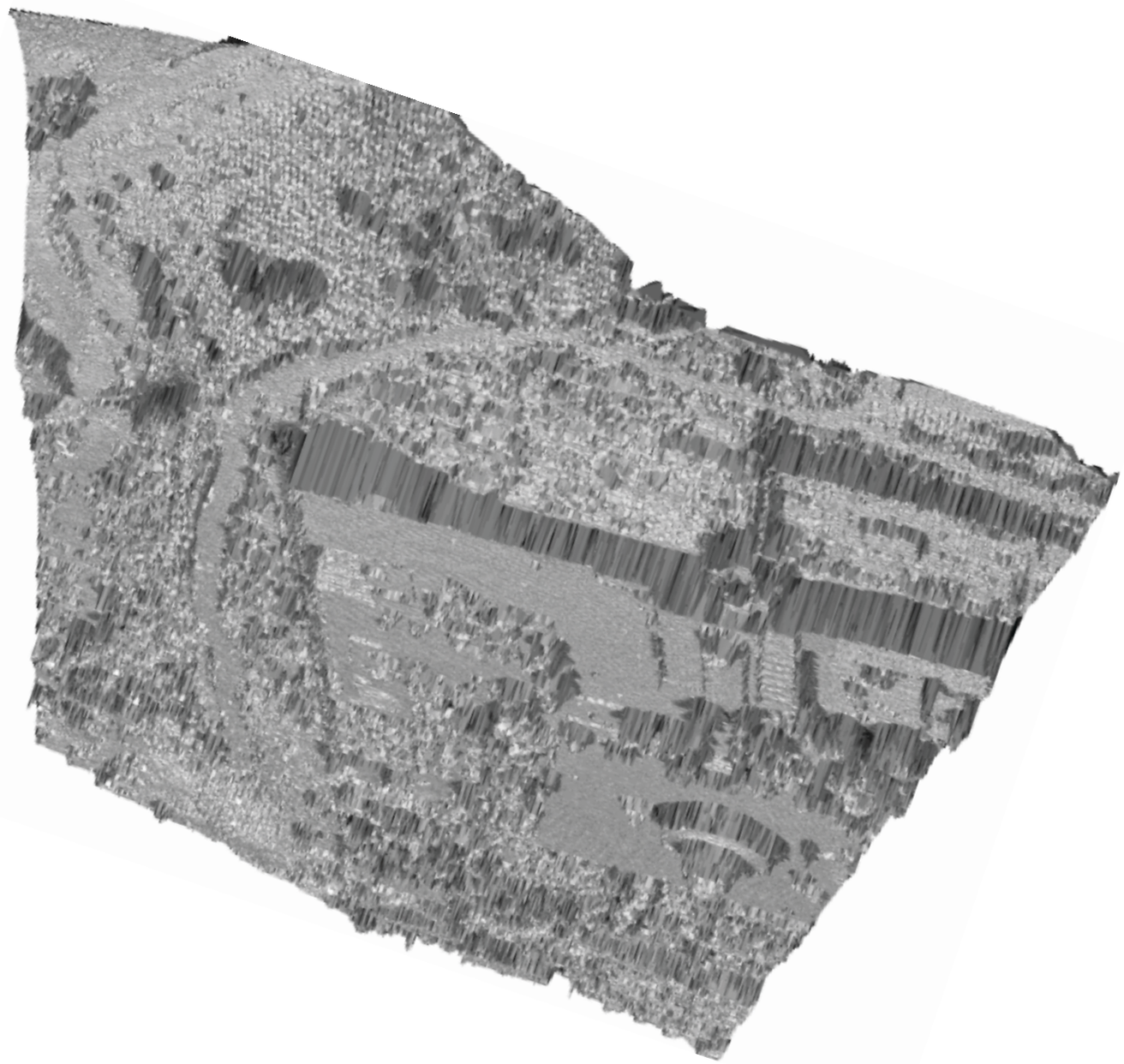
40 %

Building Facade

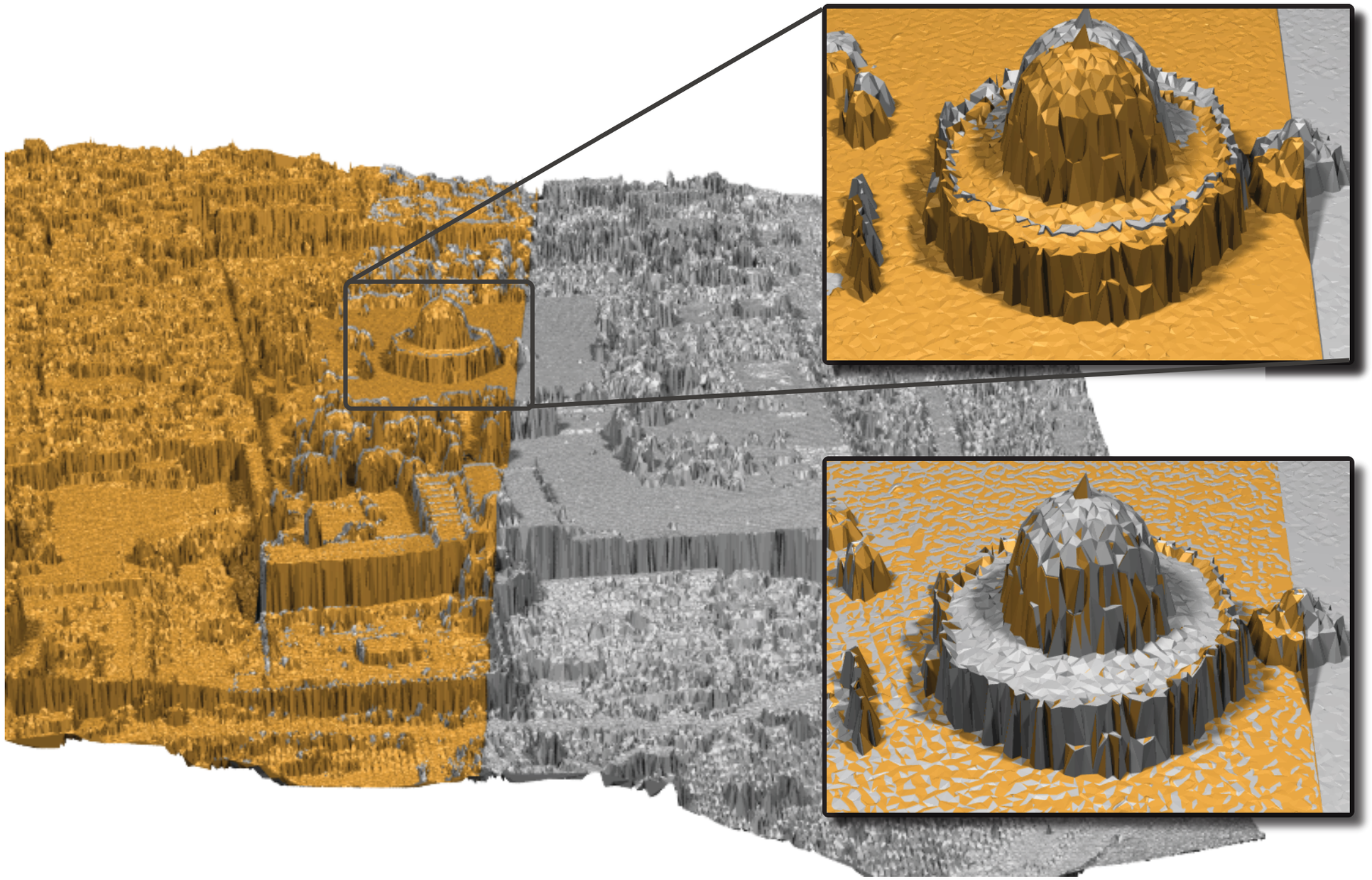


Jerusalem Scan

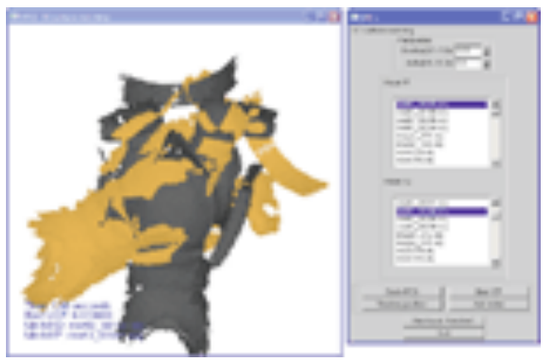




Jerusalem Scan



Try It

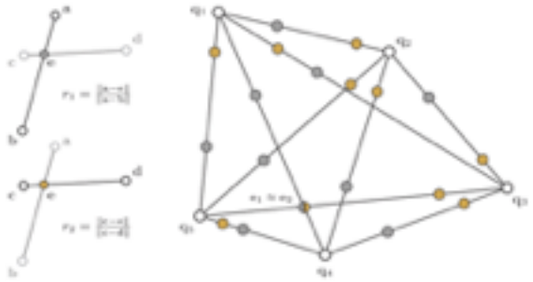


application

code

http://graphics.stanford.edu/~niloy/research/fpcs/fpcs_sig_08.html

Reference



4-points Congruent Sets for Robust Surface Registration,
Dror Aiger, Niloy J. Mitra, Daniel Cohen-Or,
SIGGRAPH 2008.