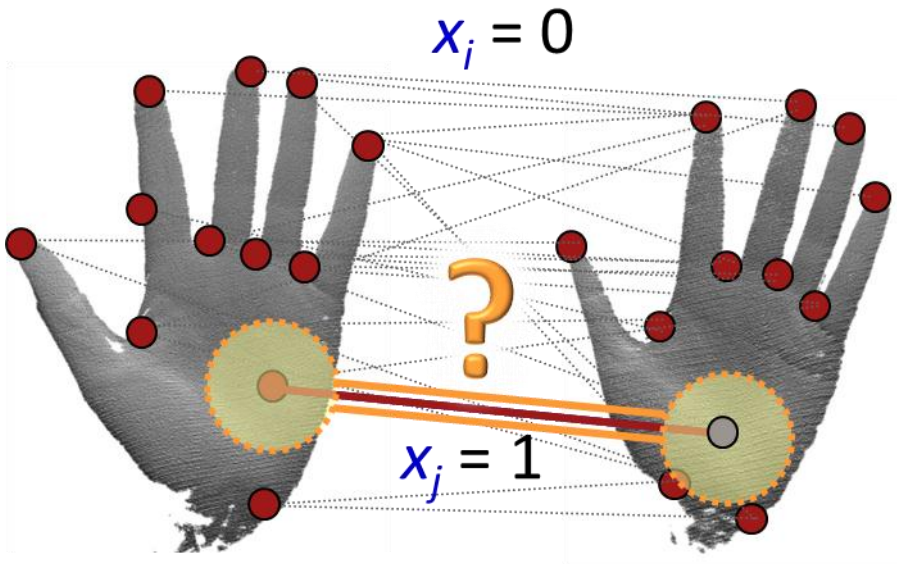
Global, Isometric, Pairwise:
Isometric Matching and Quadratic Assignment



Correspondence Problem Classification

How many meshes?

- Two: Pairwise registration
- More than two: multi-view registration

Initial registration available?

- Yes: Local optimization methods
- No: Global methods

Class of transformations?

- Rotation and translation: Rigid-body
- Non-rigid deformations

Overview and Motivation

Global Isometric Matching

Goal

- We want to compute correspondences between deformable shape
- *Global algorithm*, no initilization

Global Isometric Matching

Approach & Problems

- Consistency criterion: global isometry

Problem

- How to find globally consistent matches?

Model

- Quadratic assignment problem
  - General QA-problem is NP-hard
  - But it turns out: Isometric matching can be solved more efficiently

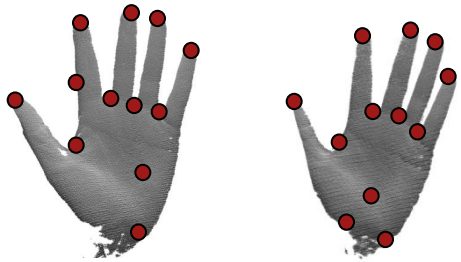
Feature Based Matching

Quadratic Assignment Model

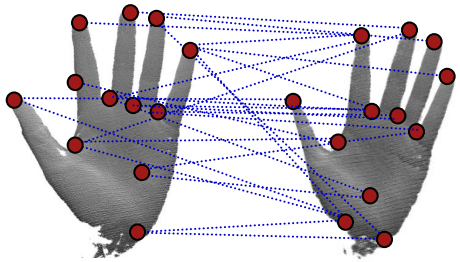
Algorithm

Feature-Matching

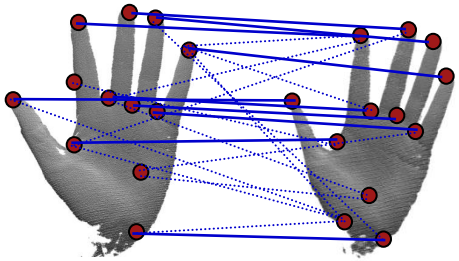
- Detect feature points



- Local matching: potential correspondences



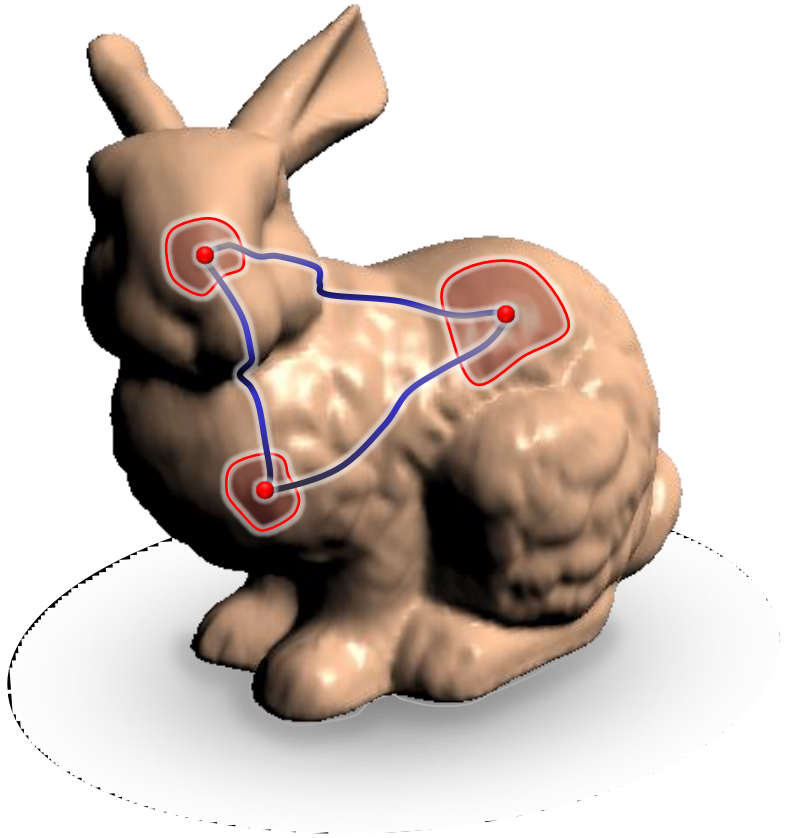
- Global filtering: correct subset



Global Matching Model

Matching model

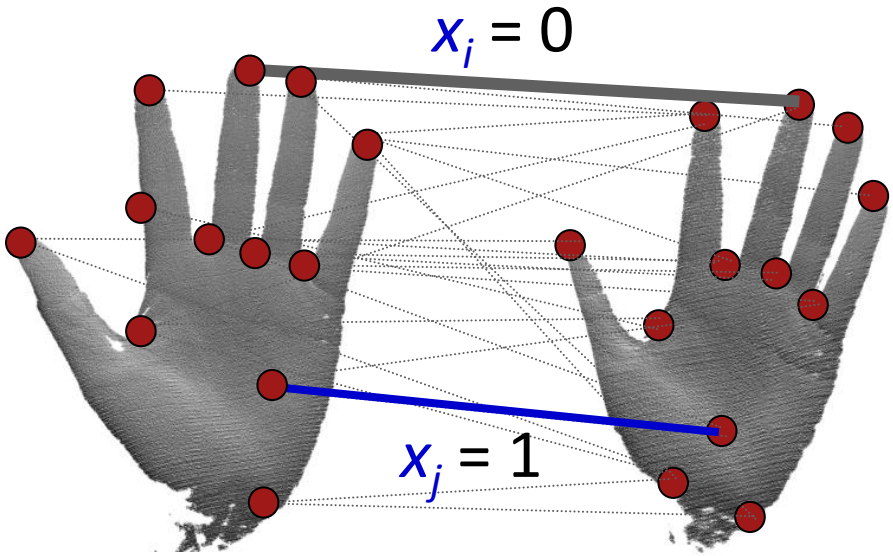
- Preserve *descriptors*
- Preserve *geometric relations*



Quadratic Assignment Model

Quadratic Assignment

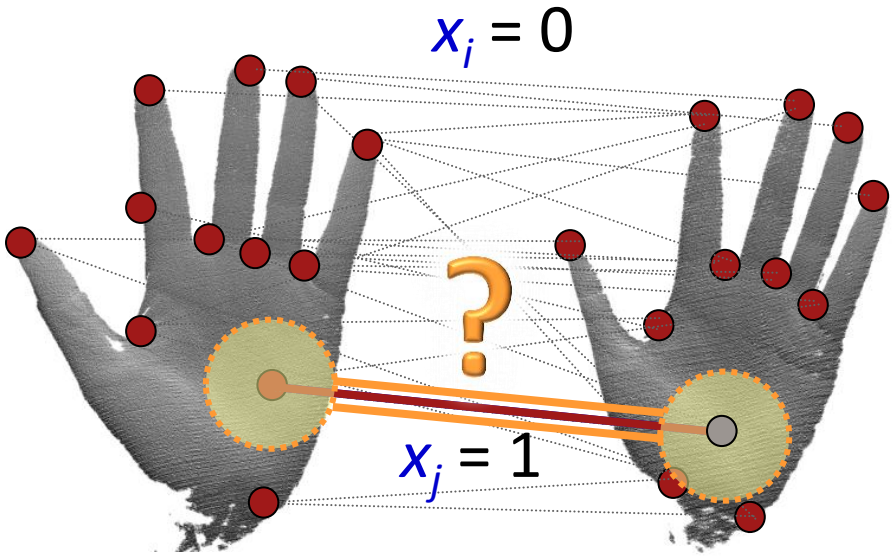
- n potential correspondences
- Each one can be turned on or off
- Label x_i



Quadratic Assignment Model

Quadratic Assignment

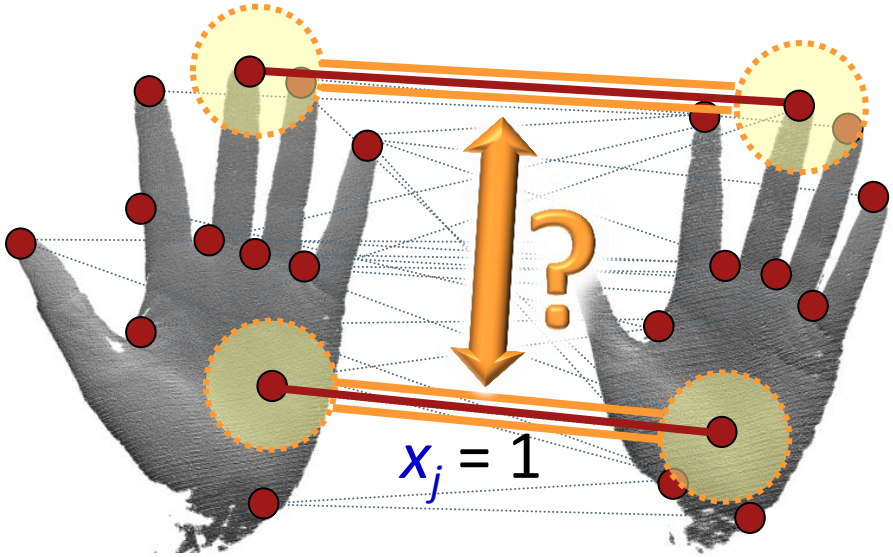
- Compatibility score:
  - **Singeltons:**
    Descriptor match



Quadratic Assignment Model

Quadratic Assignment

- Compatibility score:
  - **Singeltons:**
    Descriptor match
  - **Doubles:**
    Compatibility



Quadratic assignment problem

Quadratic Error Score:

$$E(x_1, \ldots, x_n) = \mathbf{xs} + \mathbf{x}^{\mathrm{T}}\mathbf{Dx}$$

- Pairwise scores are encoded in Matrix **D**
- Linear scores are encoded in Vector **s**
- Task: find optimal binary vector **x**

Trade Off:

- Maximize number of matches

Randomized Matching

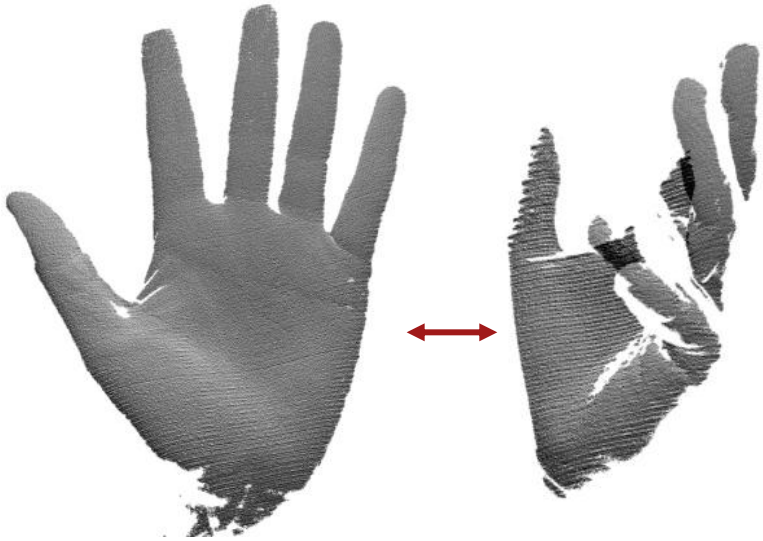
Problem Statement

Deformable Matching

- Two shapes: original, deformed
- How to establish correspondences?
- Looking for global optimum
  - Arbitrary pose

Assumption

- Approximately isometric deformation

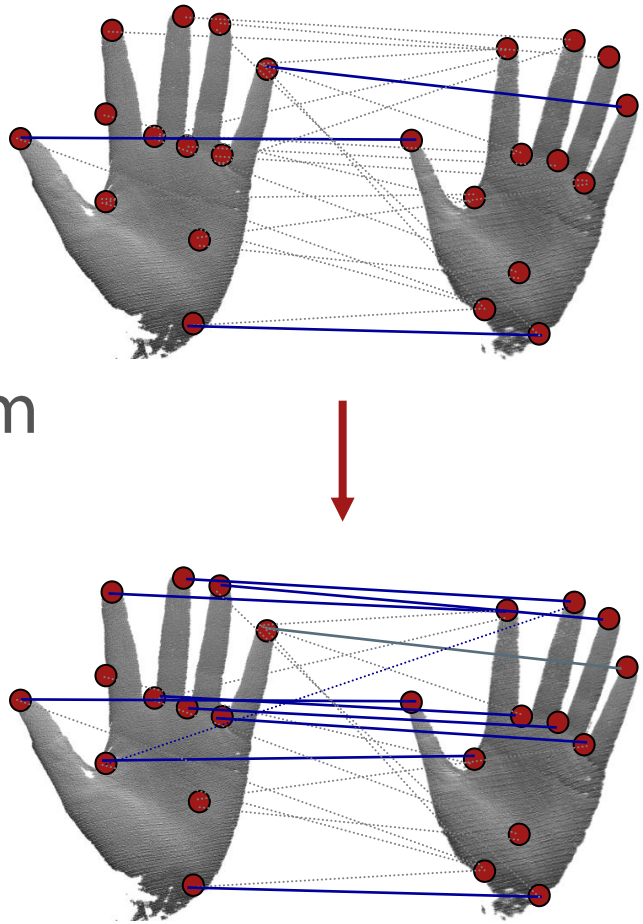


[data set: S. König, TU Dresden]

RANSAC/FWS Algorithm

Idea

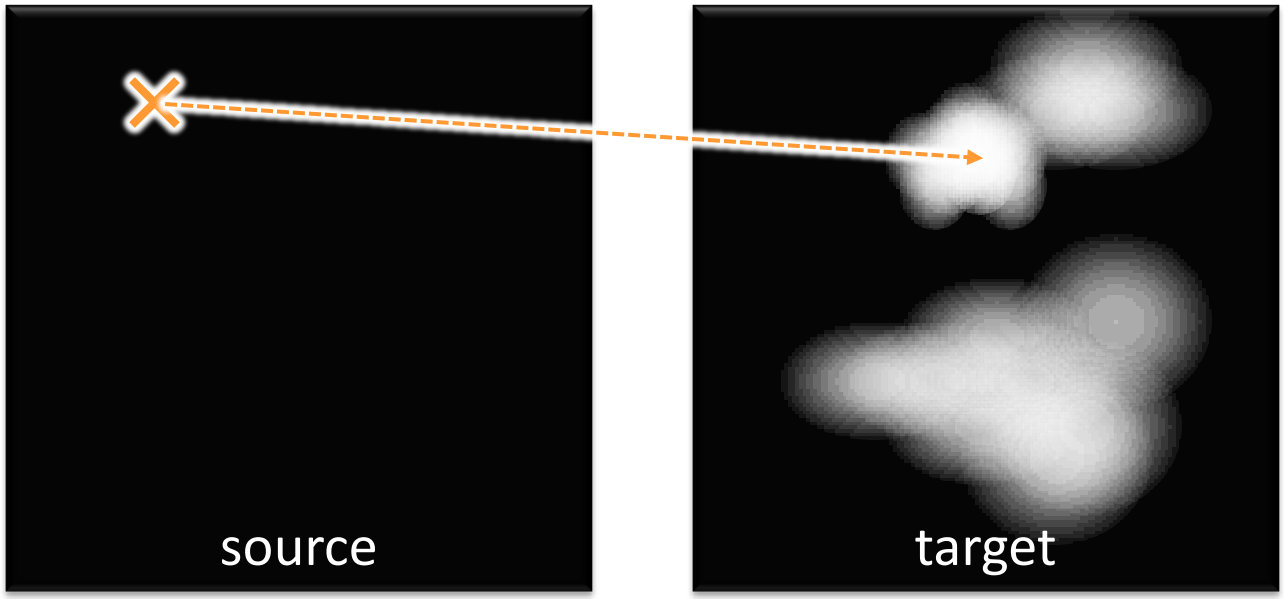
- Starting correspondence
- Add more that are consistent
  - Preserve intrinsic distances
- Importance sampling algorithm



Advantages

- Efficient (small initial set)
- General (arbitrary criteria)

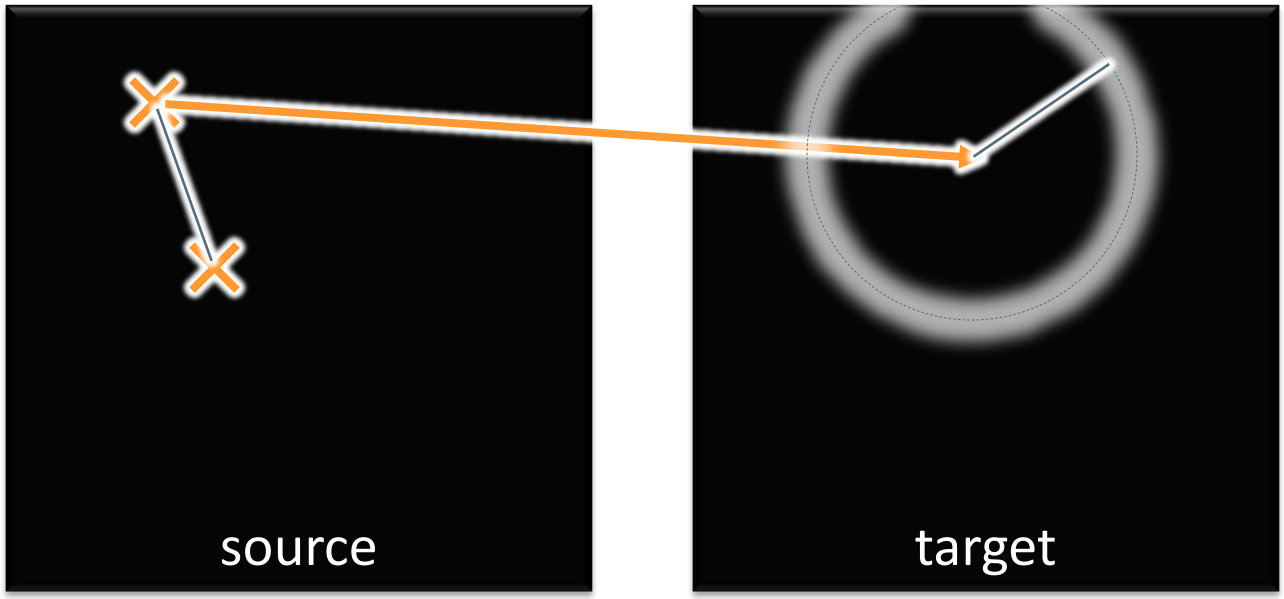
Foreward Search Algorithm



Step 1: Start with one correspondence

- Importance sampling:
  prefer good descriptor matches

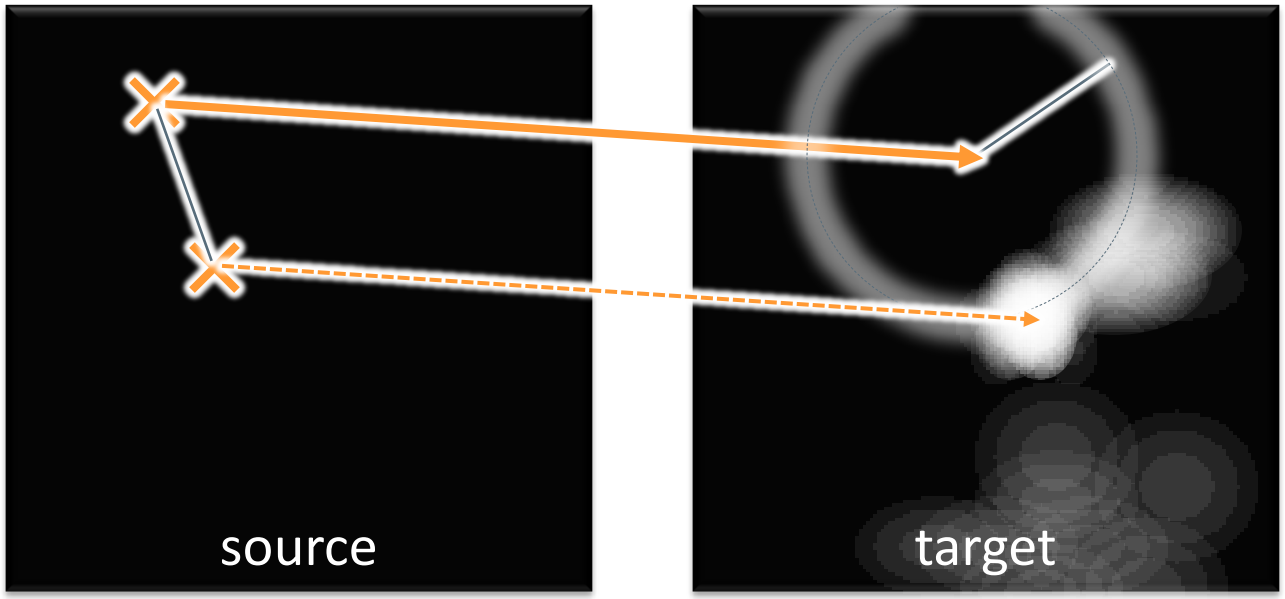
Foreward Search Algorithm



Step 2: „Posterior“ using geodesics

- Importance sampling:
  sample according to descriptor match × distance score

Foreward Search Algorithm



posterior (distance & descriptors)

Step 2: „Posterior“ using geodesics

- Importance sampling:
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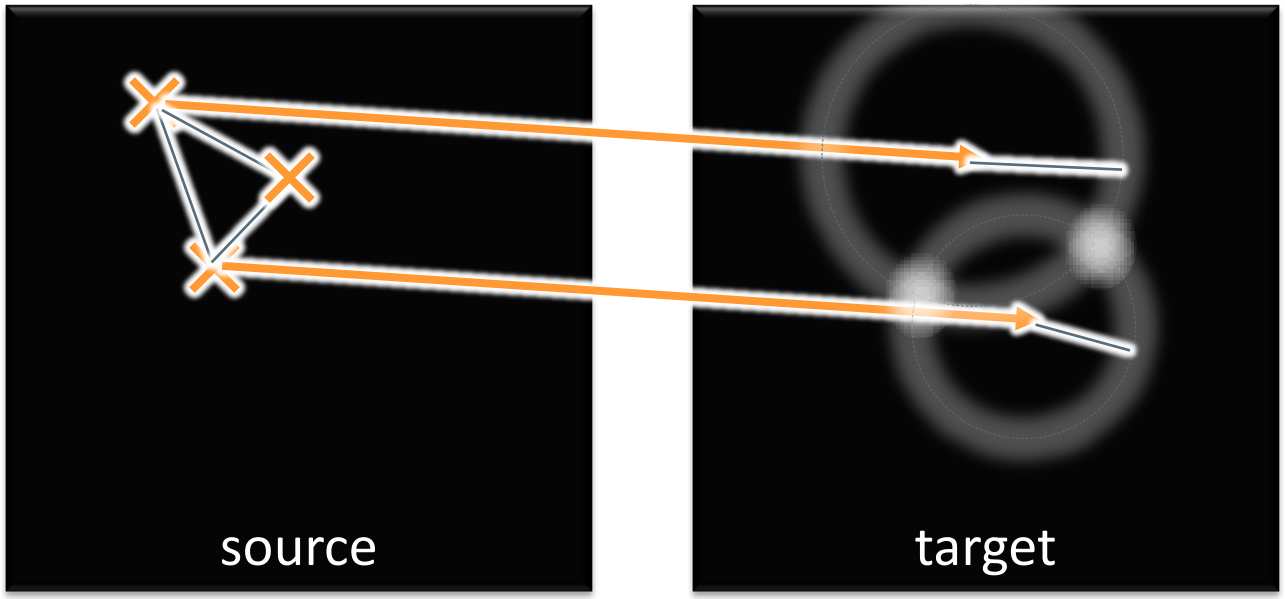
Foreward Search Algorithm

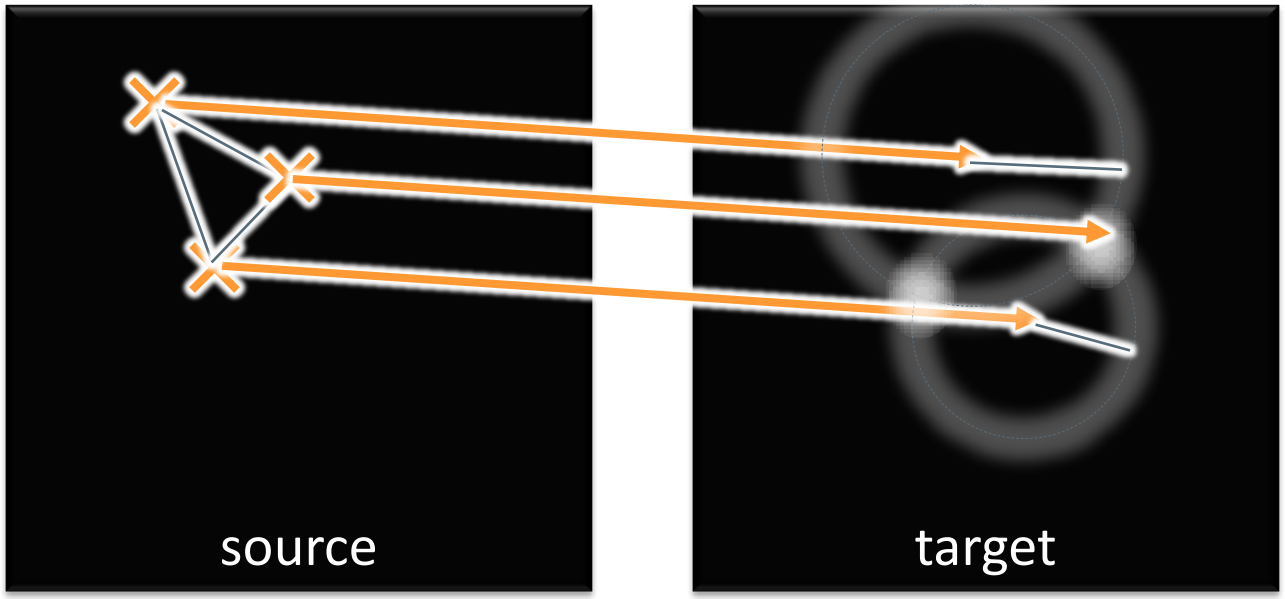


posterior (distance & descriptors)

Step 3:

- Same as step 2, continue sampling...

Foreward Search Algorithm

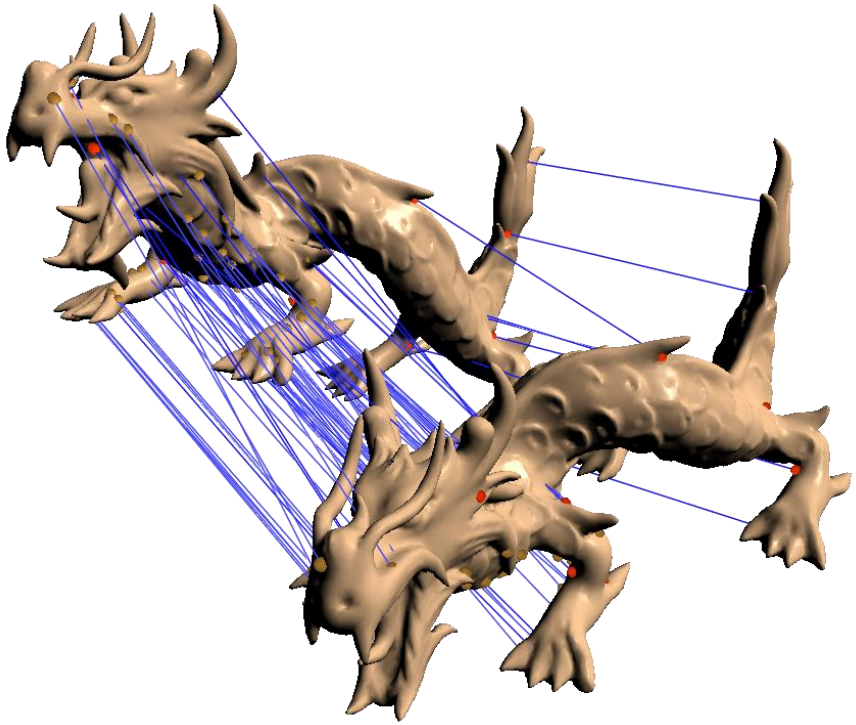


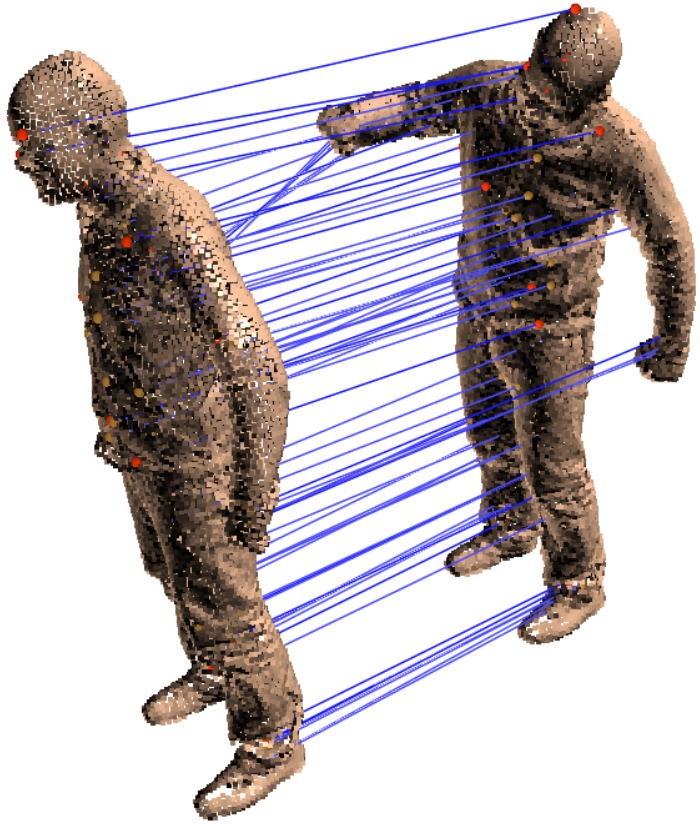
posterior (distance & descriptors)

Step 3:

- Same as step 2, continue sampling...

Results

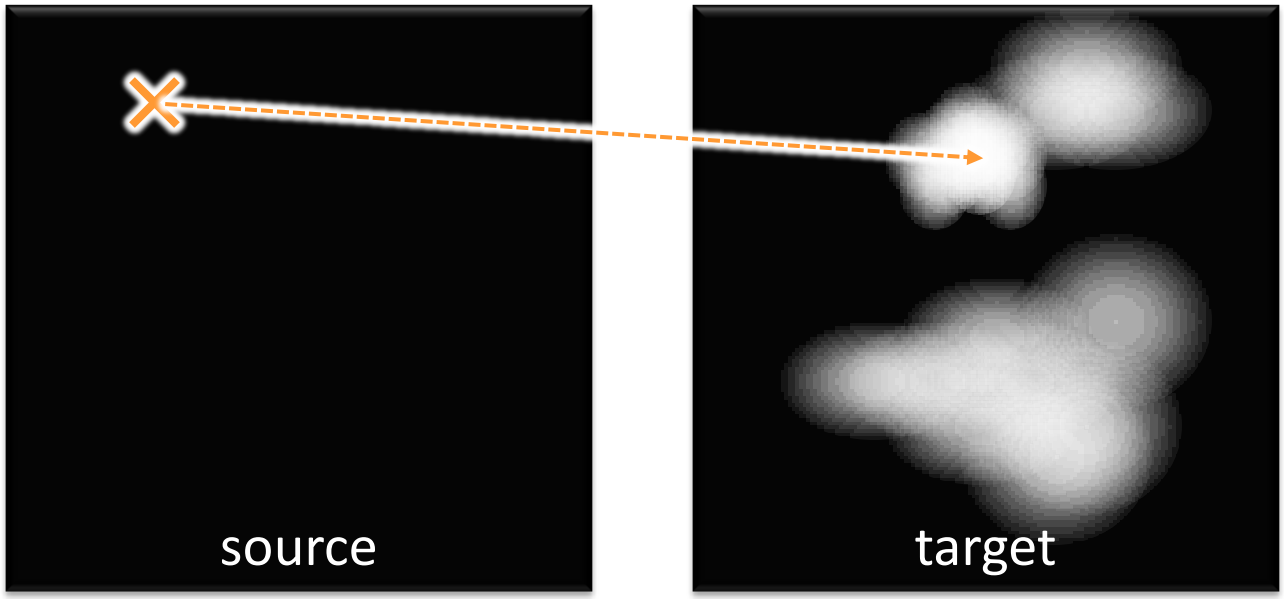




Typically: 100 random trials required

[data sets: Stanford 3D Scanning Repository / Carsten Stoll]

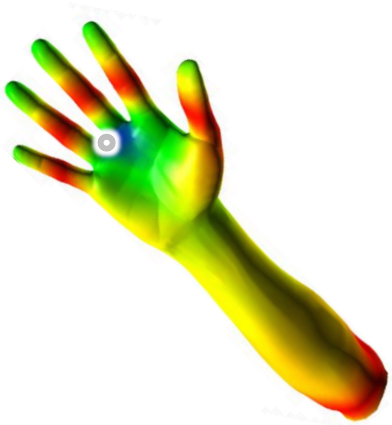
Foreward Search Algorithm



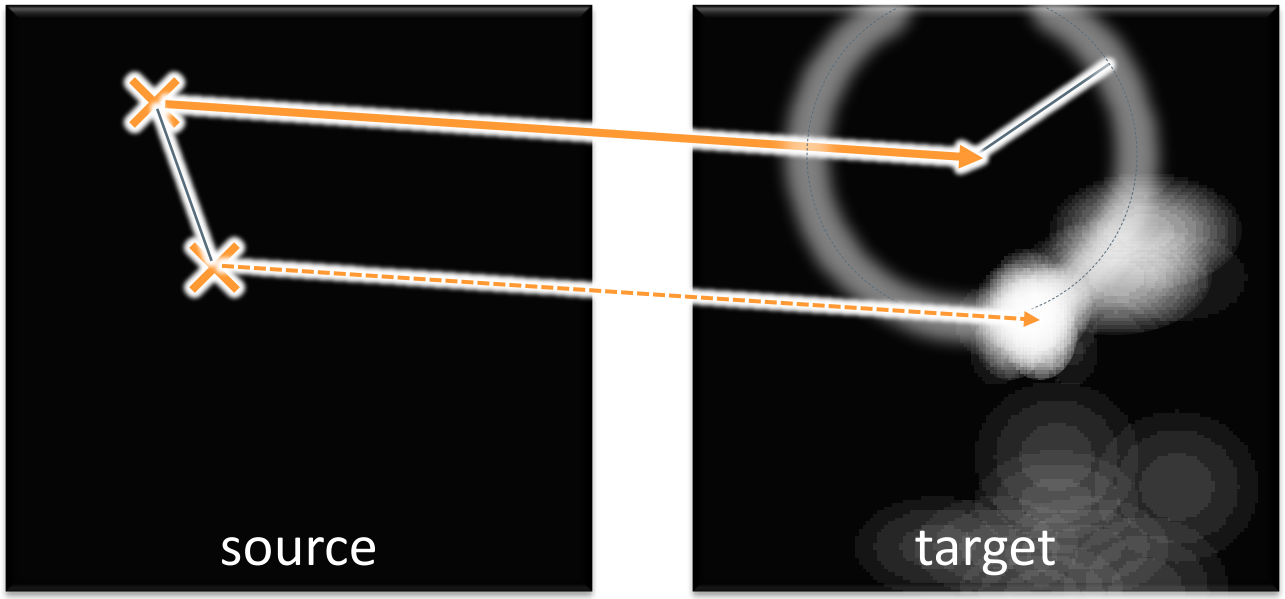
Descriptor matching scores

Step 1: Start with one correspondence

- Target side importance sampling: prefer good descriptor matches
- Source side importance sampling: descriptor entropy



Foreward Search Algorithm

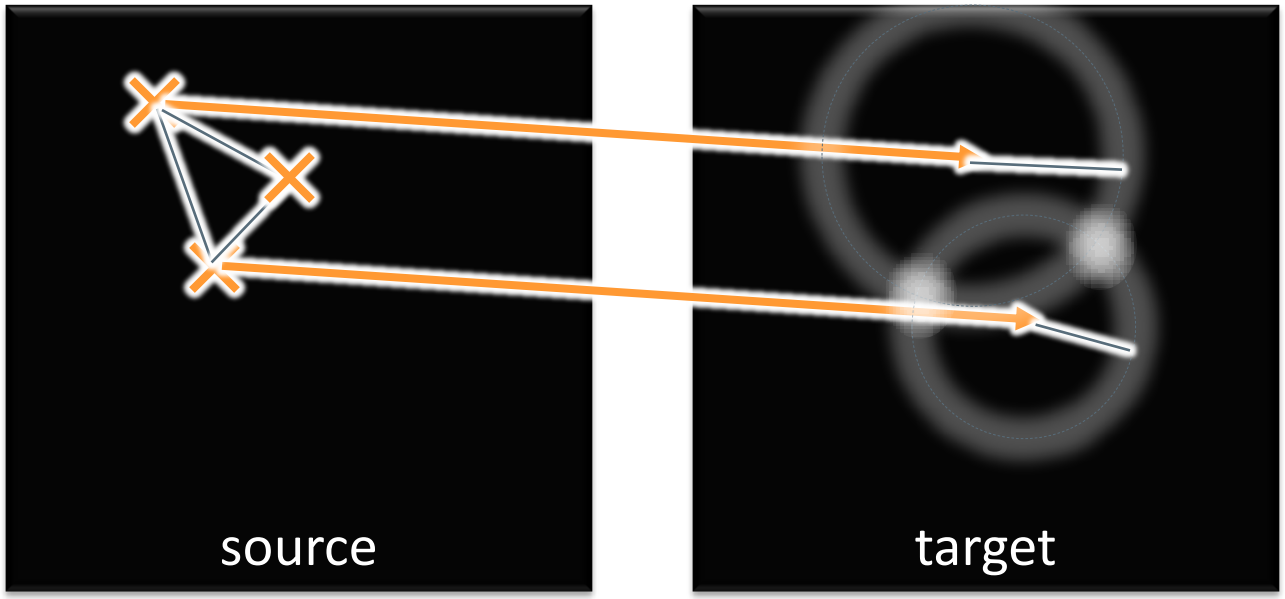


posterior (distance & descriptors)

Step 2: „Posterior“ using geodesics

- Target side importance sampling:
  sample according to descriptor match × distance score
- Source side importance sampling:
  minimize posterior marginals entropy

Foreward Search Algorithm

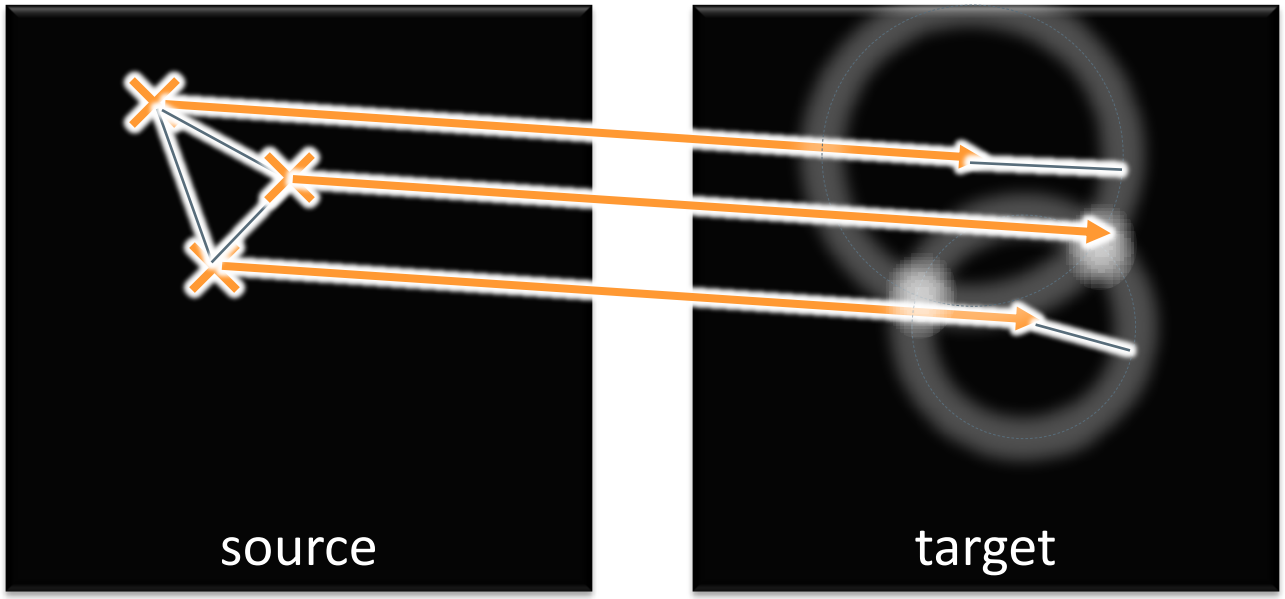


posterior (distance & descriptors)

Step 3:

- Same as step 2, continue sampling...

Foreward Search Algorithm



posterior (distance & descriptors)

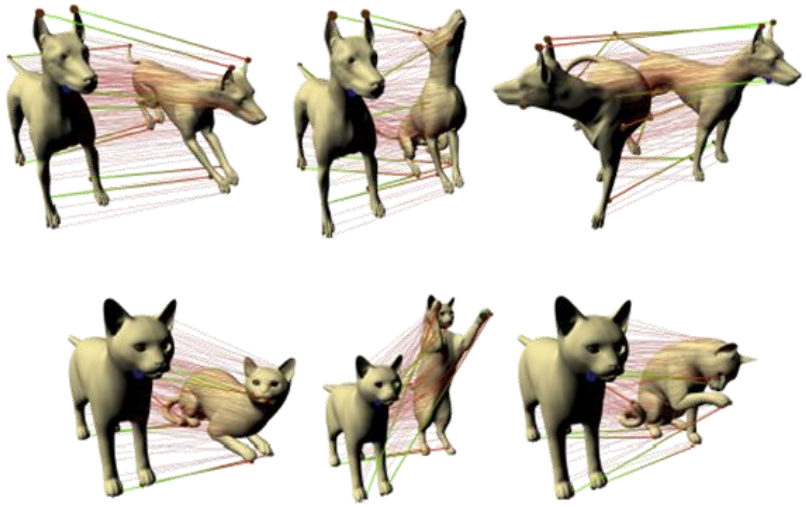
Step 3:

- Same as step 2, continue sampling...

Matching Algorithm

Entropy-based planning

- More efficient
- Typically 1-15 trials

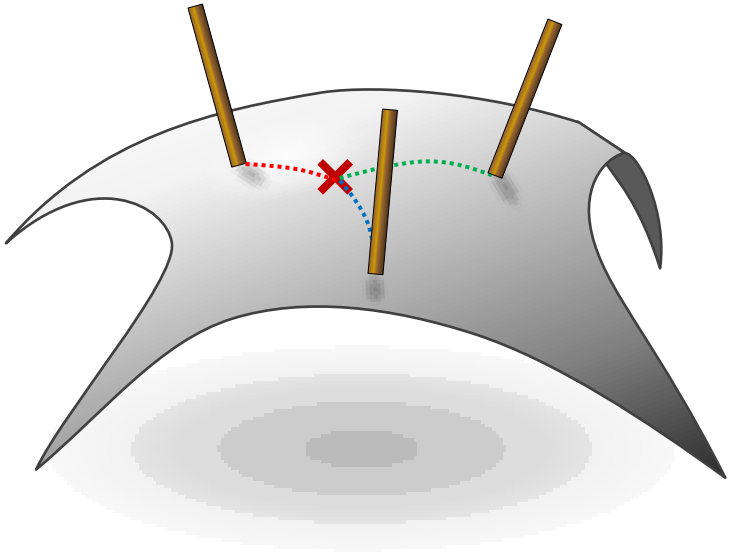


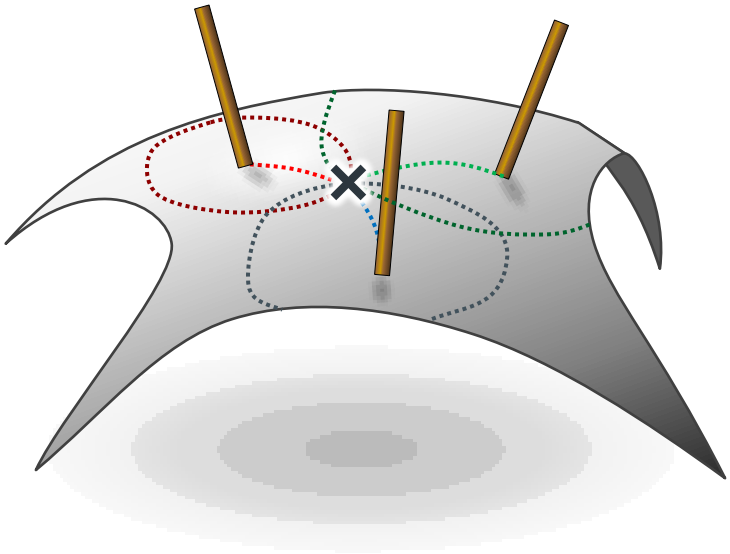
Another View

Landmark Coordinates

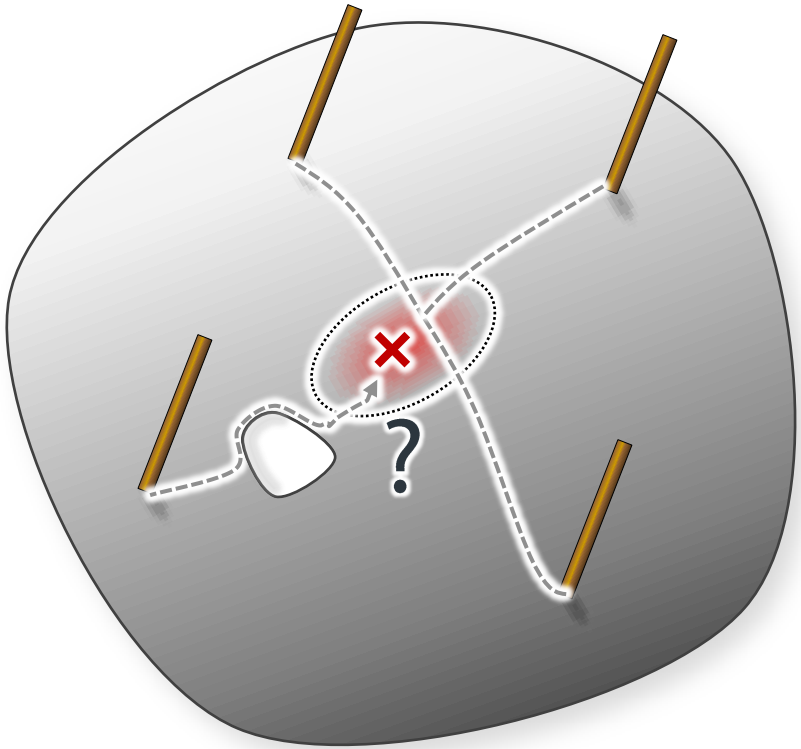
- Distance to already established points give a charting of the manifold

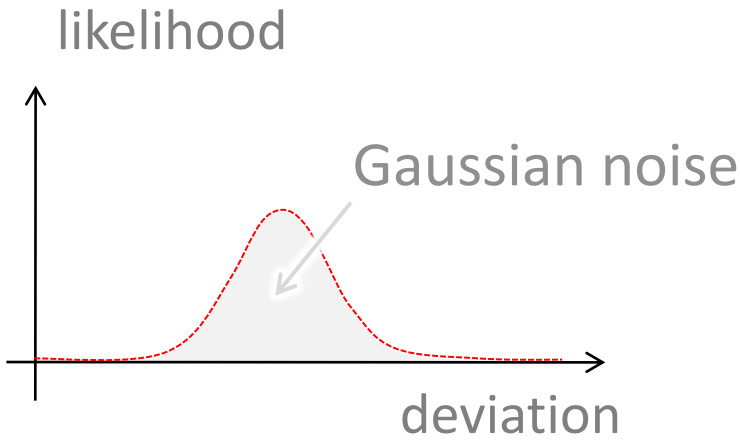
Landmark Distances



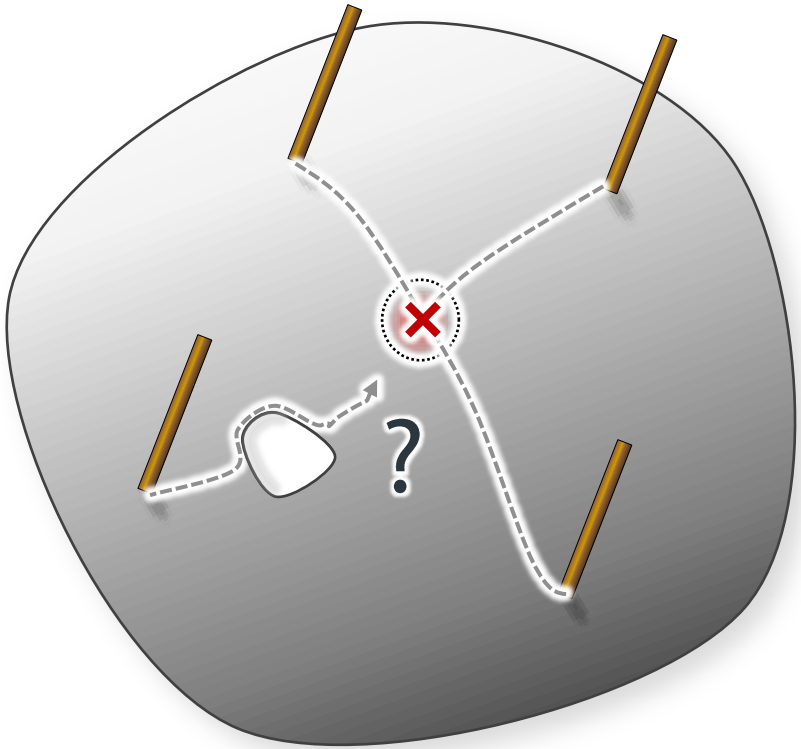


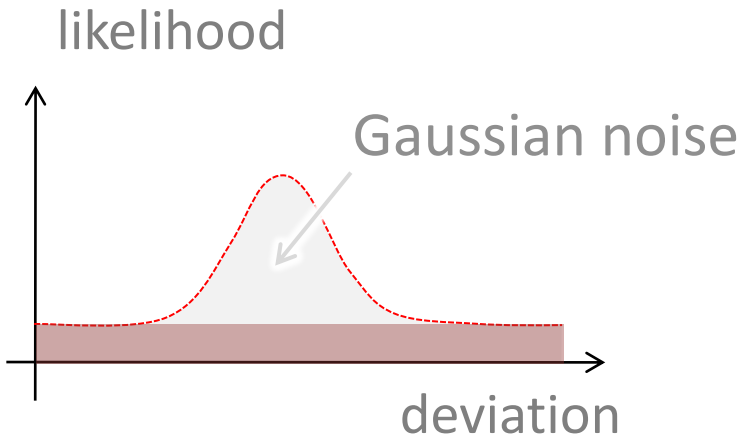
Topological Noise





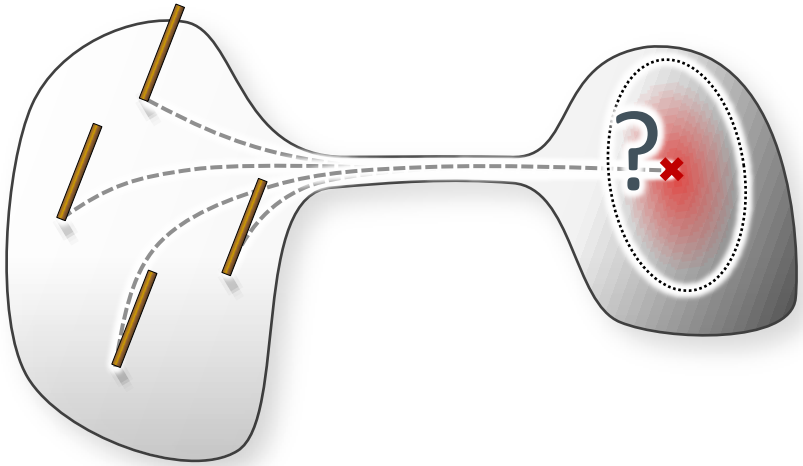
Robust Landmark Distances



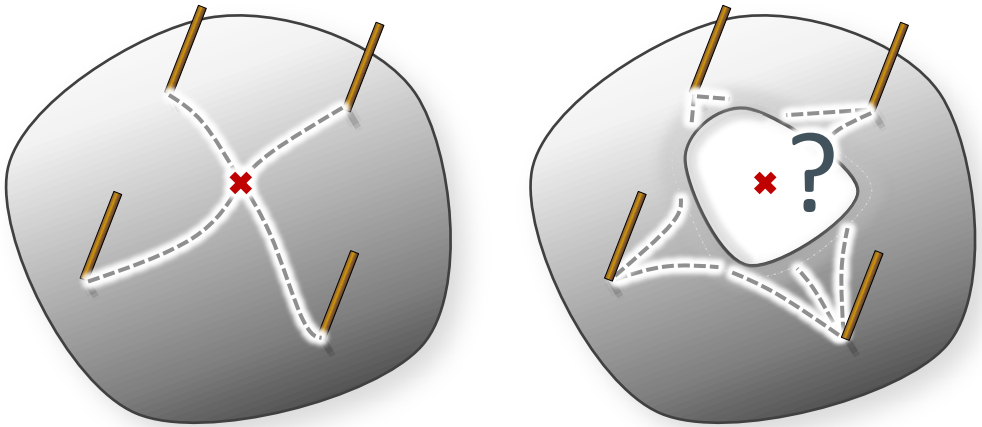


Variance Provides Additinoal Cues

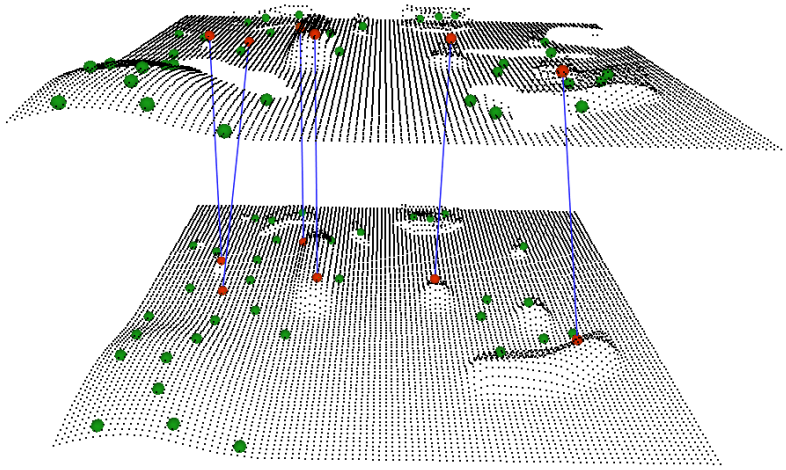
Numerical Instability:



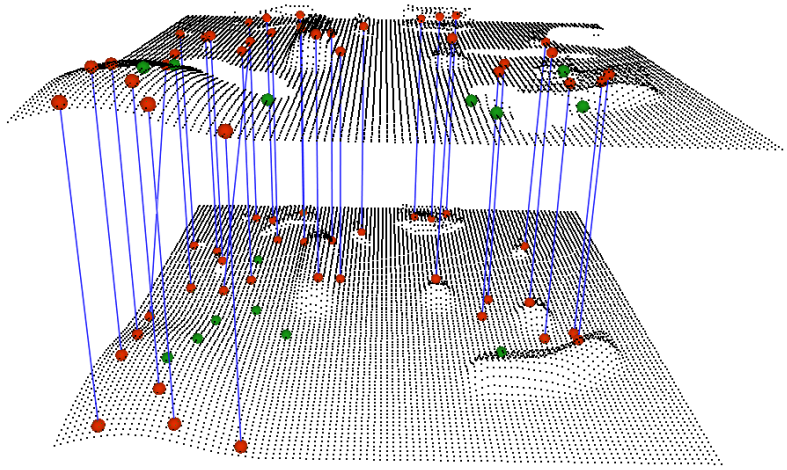
Missing Geometry:



Results: Topological Noise

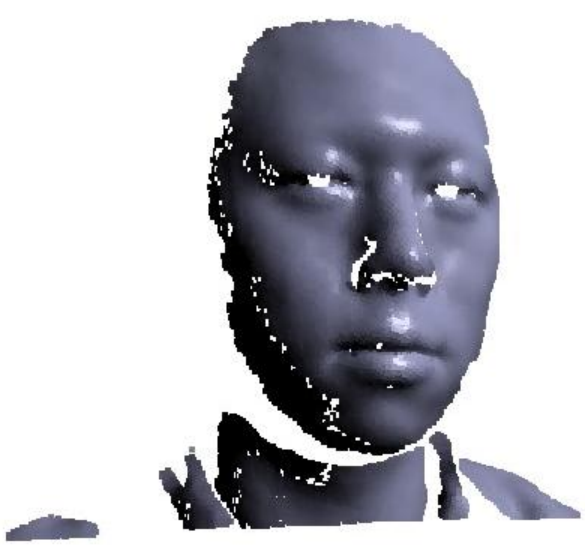


Spectral Quadratic Assignment
[Leordeanu et al. 05]

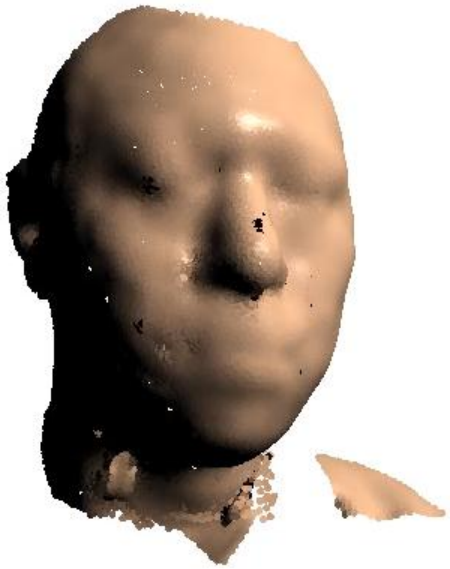


Ransac Algorithm
[Tevs et al. 09]

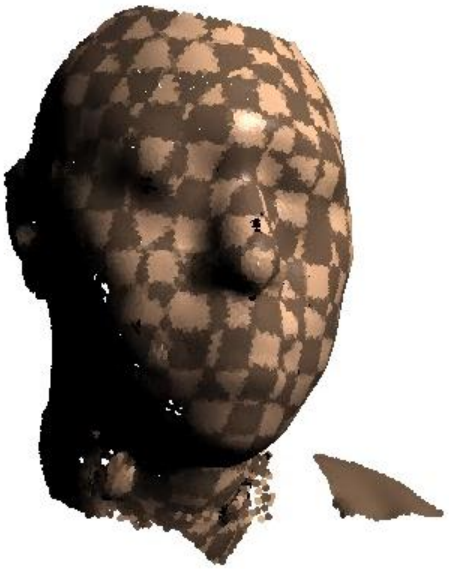
Global Animation Reconstruction



input data



reconstruction



correspondences

[Data set: Hao Li, ETH Zürich]