Global Shape Matching

Section 3.7: Spectral Matching and Applications

Quadratic Assignment Model

Quadratic Assignment

- Matrix notation:

$$P^{(match)}(x_1,\ldots,x_n) = \prod_{i=1}^{n} P_i^{(single)} \prod_{i,j=1}^{n} P_{i,j}^{(compatible)}$$

$$\log P^{(match)}(x_1,\ldots,x_n) = \sum_{i=1}^{n} \log P_i^{(single)} + \sum_{i,j=1}^{n} \log P_{i,j}^{(compatible)}$$

$$= \quad \mathbf{xs} \quad + \quad \mathbf{x}^T\mathbf{Dx}$$

- Quadratic scores are encoded in Matrix **D**
- Linear scores are encoded in Vector **s**
- Task: find optimal binary vector **x**

Spectral Matching

Simple & Effective Approximation:

- Spectral matching [Leordeanu & Hebert 05]
- Form compatibility matrix:

$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{21} & a_{31} & \cdots \\ a_{12} & a_{22} & & \\ a_{13} & & \ddots & \\ & & & \ddots \end{pmatrix}$$

Diagonal:
Descriptor match

Off-Diagonal:
Pairwise compatibility

All entries within [0..1]
= [no match...perfect match]

Spectral Matching

Approximate largest clique:

- **Compute eigenvector with largest eigenvalue**
- **Maximizes Rayleigh quotient:**

$$\arg\max \frac{\mathbf{x}^T \mathbf{A} \mathbf{x}}{\|\mathbf{x}\|^2}$$

- **“Best yield” for bounded norm**
  - The more consistent pairs (rows of 1s), the better
  - Approximates largest clique
- **Implementation**
  - For example: power iteration

Spectral Matching

Post-processing

- **Greedy quantization**
  - Select largest remaining entry, set it to 1
  - Set all entries to 0 that are not pairwise consistent with current set
  - Iterate until all entries are quantized

In practice...

- This algorithm turns out to work quite well.
- Very easy to implement
- Limited to (approx.) quadratic assignment model

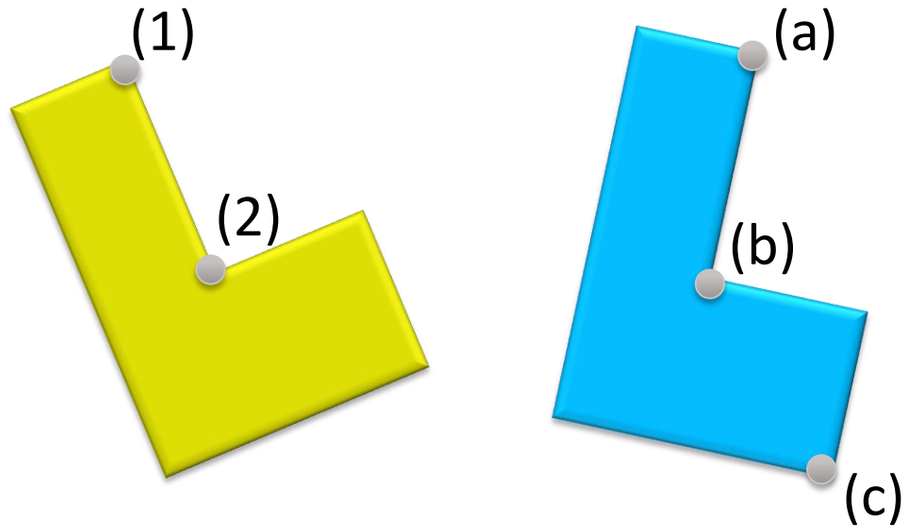
Simple Numerical Example

Consider matching some points two simple 2D shapes

1. Form compatibility matrix

	(1, a)	(1,b)	(1,c)	(2,a)	(2,b)	(2,c)
(1,a)						
(1,b)						
(1,c)						
(2,a)						
(2,b)						
(2,c)						

(Symmetric matrix)



On a rough scale of 0 to 100

- *0 == not compatible*
- *100 == very compatible*

Diagonal values:

- *Descriptor match score*

Off-diagonal values:

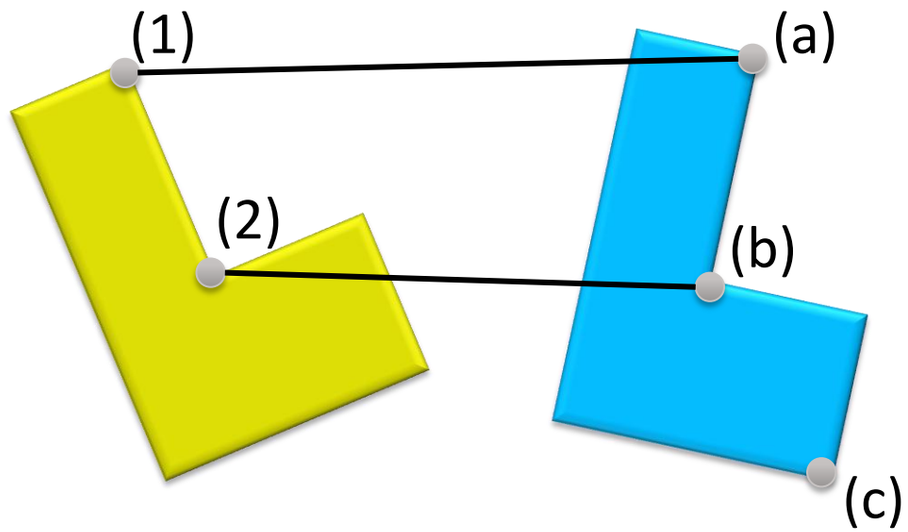
- *Pairwise compatibility*
- *ex) For (1, a) & (2,c) compare distance between 1—2 & a—c*

Simple Numerical Example

Consider matching some points two simple 2D shapes

2. Compute e-vals / e-vecs

```
%% Matlab spectral matching example
M = [ 0   0   0   0 100  50;
      0   0   0 100   0 100;
      0   0   0  50 100   0;
      0   0   0   0   0   0;
      0   0   0   0   0   0;
      0   0   0   0   0   0];
M = (M + M.') + diag([100 50 75 50 100 50])
[V,D] = eig(M); evals = diag(D); [val, idx] = max(evals);
Largest = evals(idx) * V(:,idx)
```



3. Pick best matches, remove constraint violators

- One-to-one mapping constraint

Match	Score
(1,a)	125.5
(1,b)	59.5
(1,c)	106.0
(2,a)	56.7
(2,b)	155.8
(2,c)	61.6

Result:

1. Take (2,b)

- Remove (1,b), (2,c), (2,a)

2. Take (1,a)

- Remove (1,c)

3. Done!

Spectral Matching Example

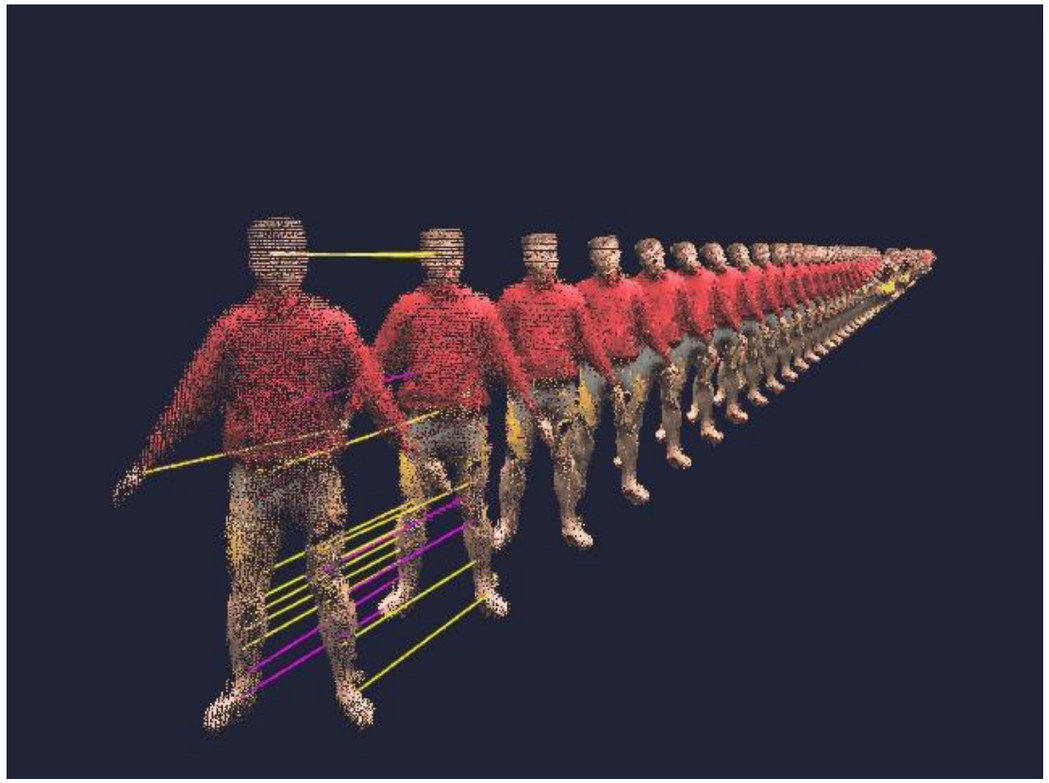
Application to Animations

- **Feature points:**
  Geometric MLS-SIFT features [Li et al. 2005]

- **Descriptors:**
  Curvature & color ring histograms

- **Global Filtering:**
  Spectral matching

- **Pairwise animation matching:**
  Low precision passive stereo data



Data courtesy of C. Theobald, MPI Informatik