Computing Correspondences in Geometric Datasets
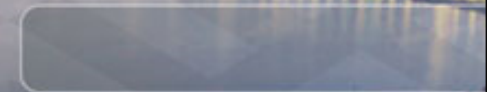
4.2 Symmetry & Symmetrization

Symmetry

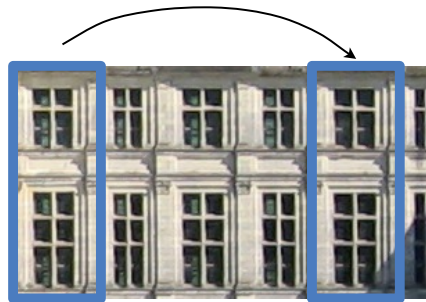- Invariance under a class of transformations
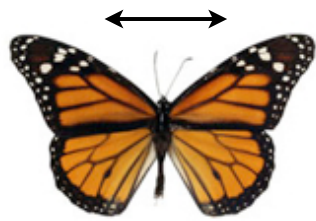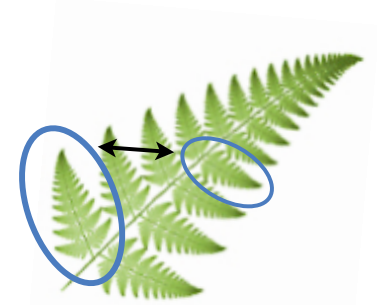

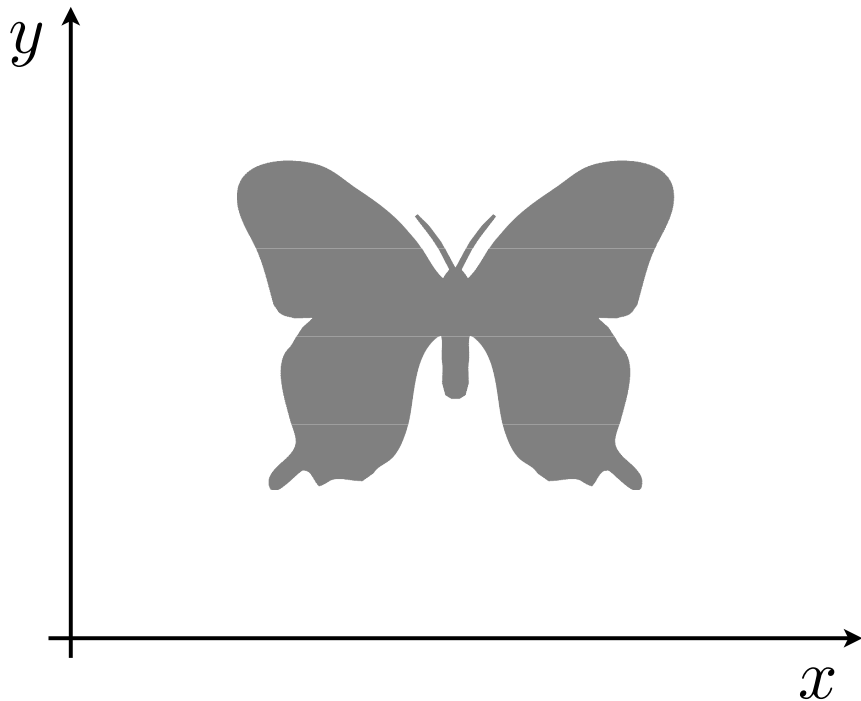Reflection Translation Rotation Reflection + Translation + Rotation + Scaling

- – global vs. partial
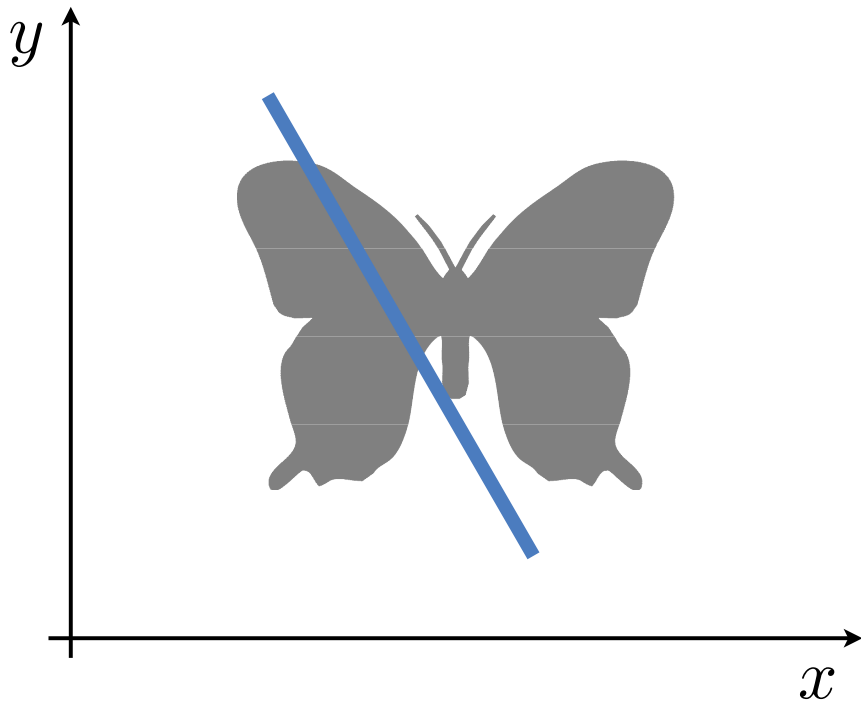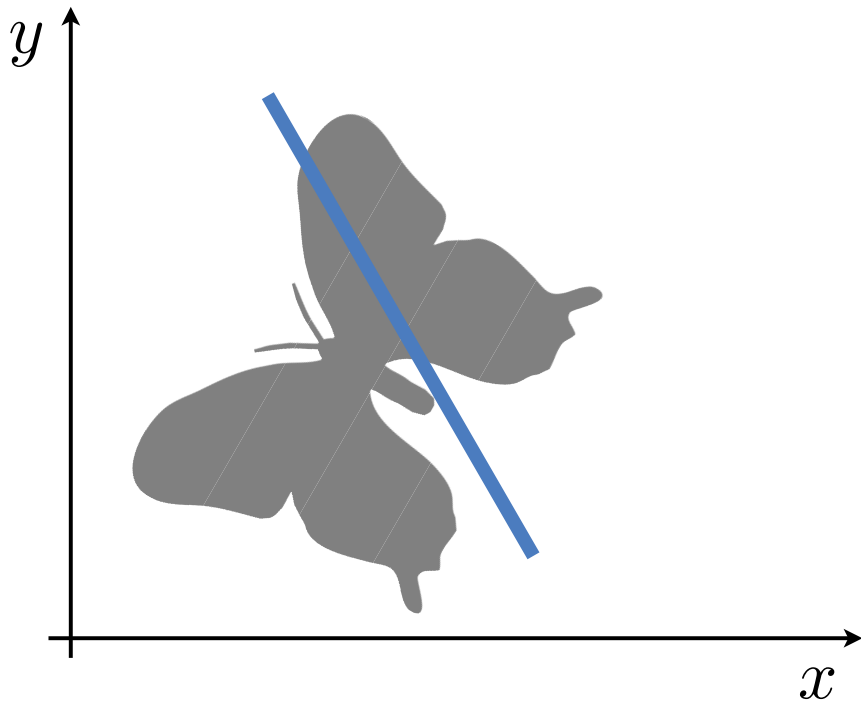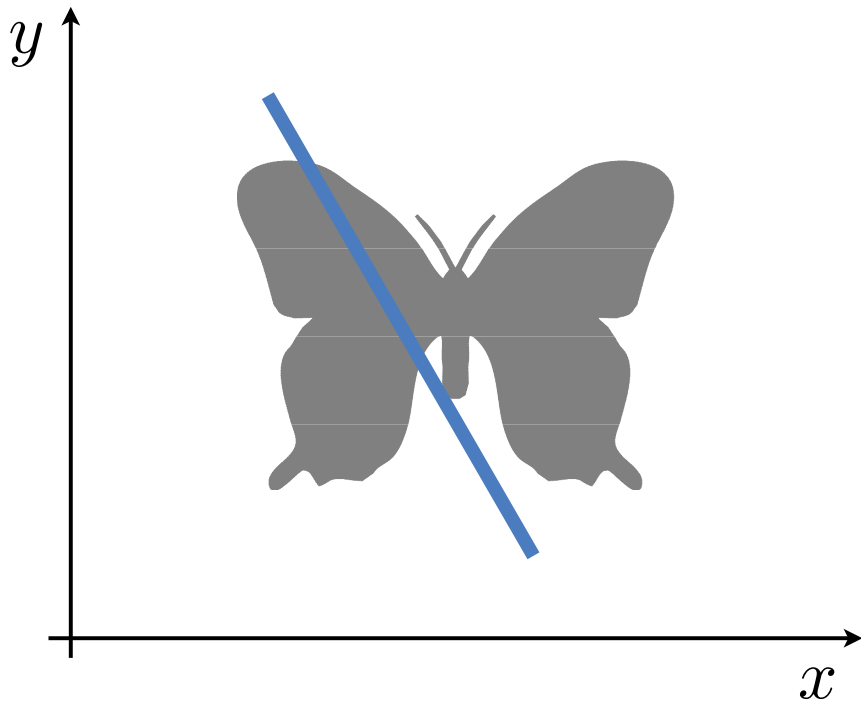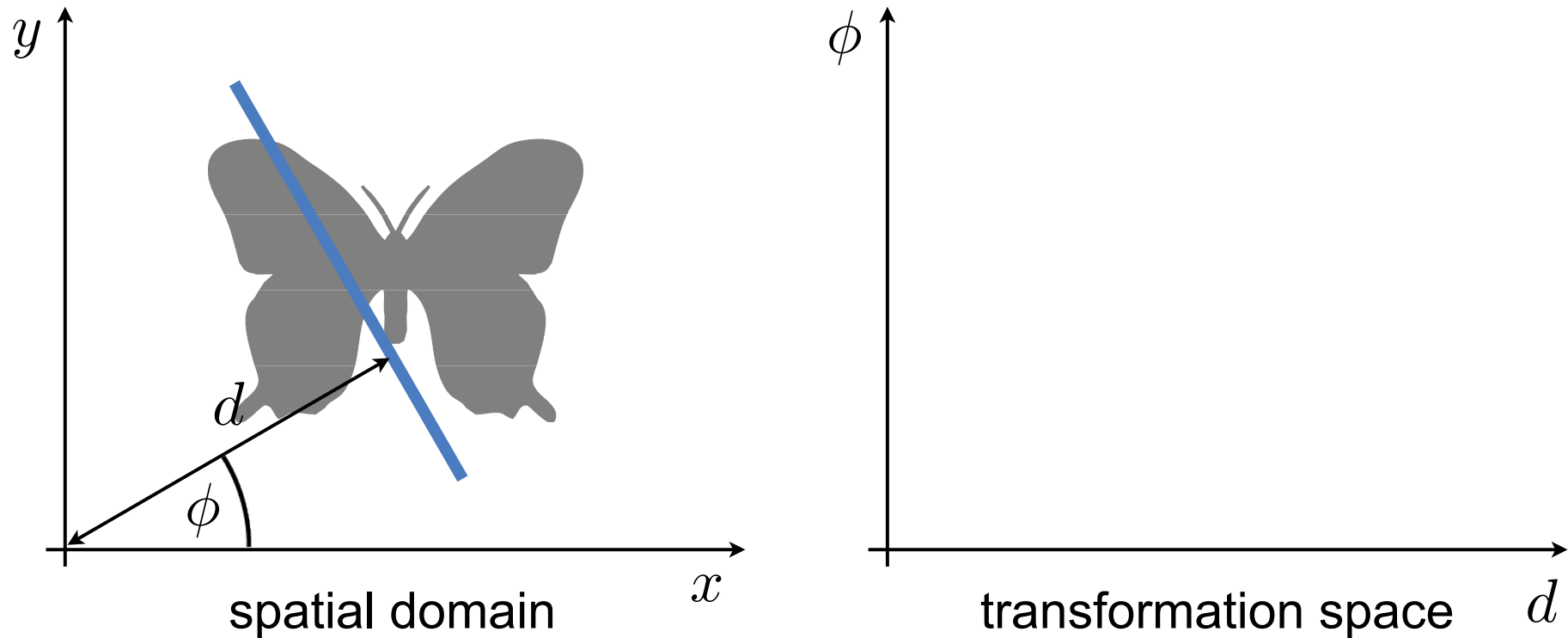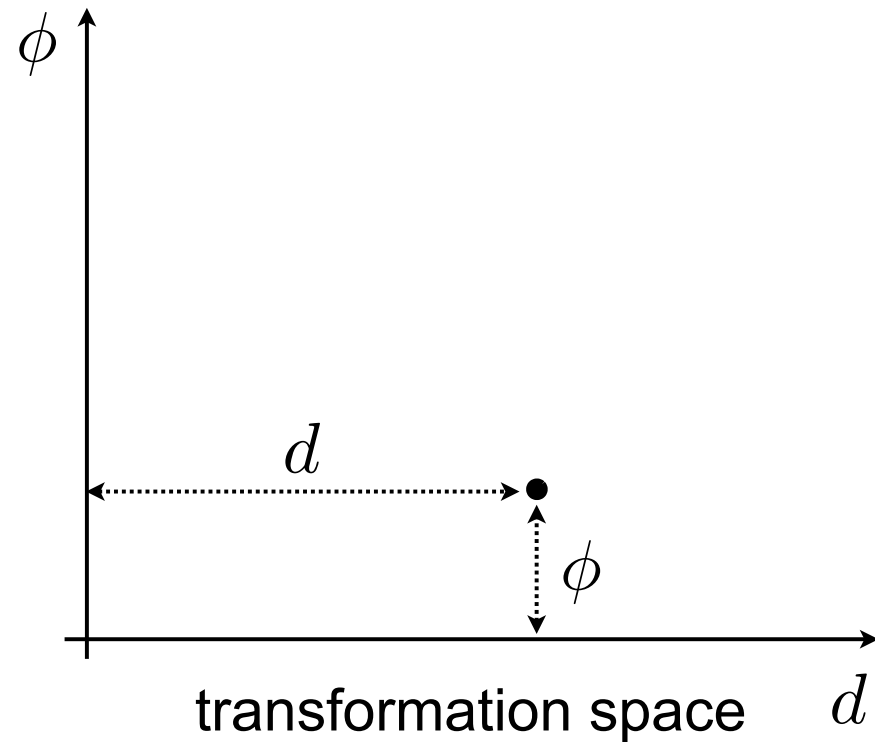- – exact vs. approximative

Symmetry

- Example: Reflection in 2D

Symmetry

- Example: Reflection in 2D

Symmetry

- Example: Reflection in 2D

Symmetry

- Example: Reflection in 2D

Symmetry

- Example: Reflection in 2D

spatial domain

transformation space

Symmetry

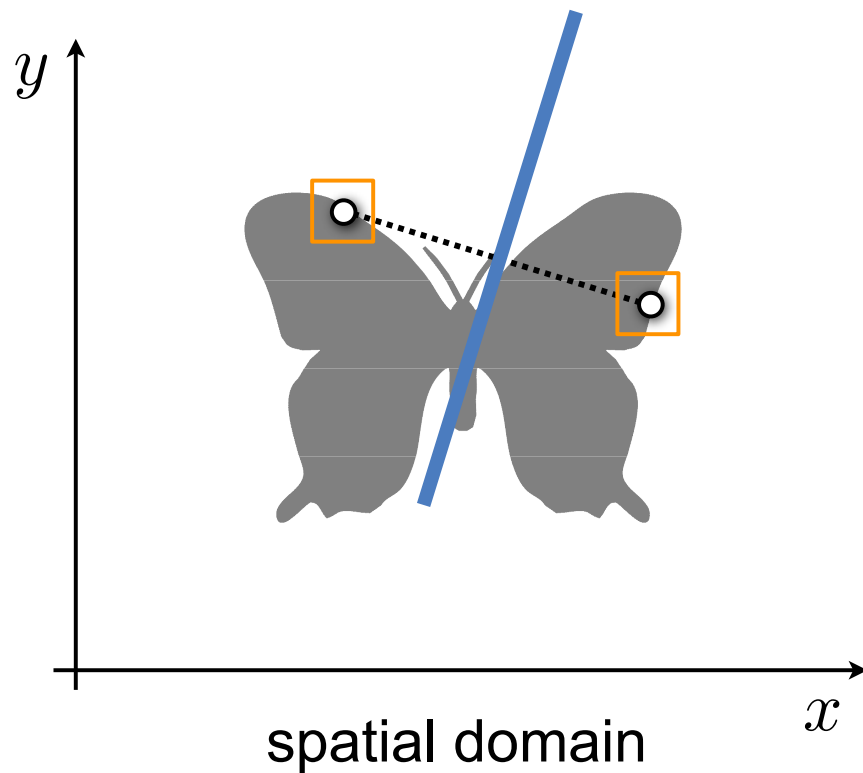- Example: Reflection in 2D

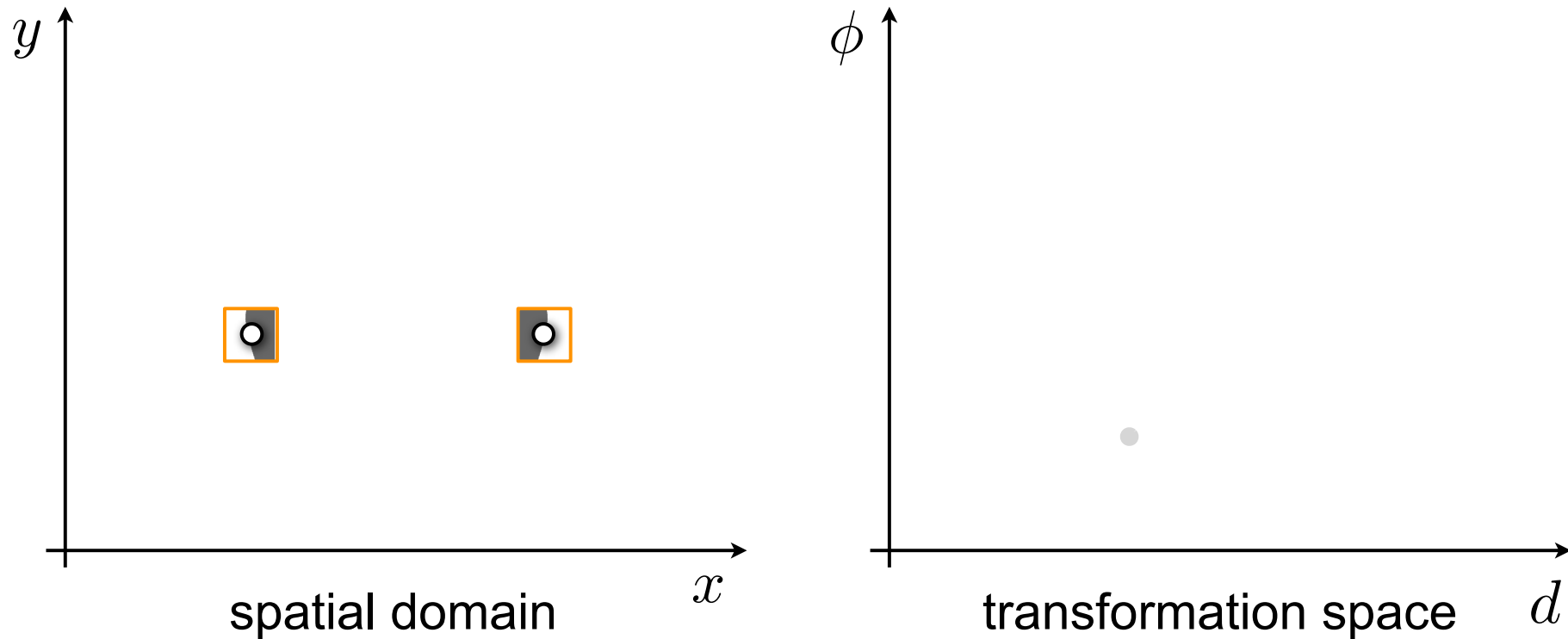spatial domain
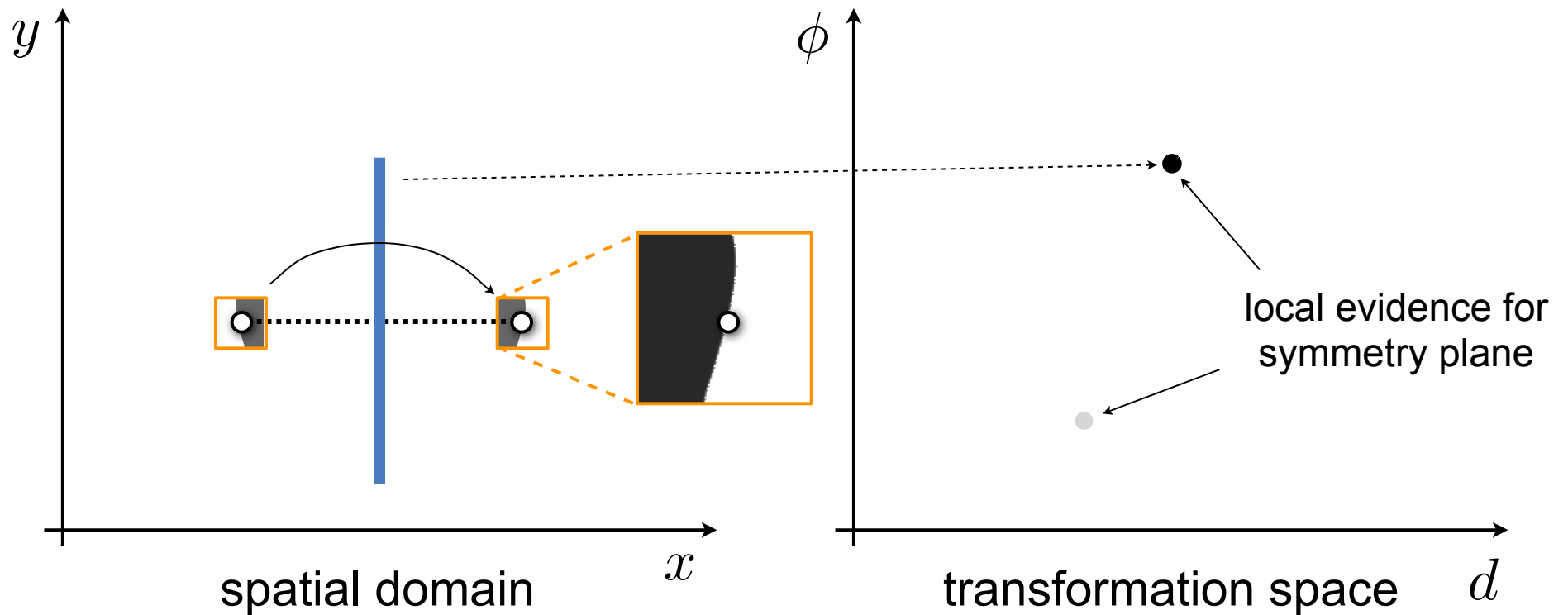
transformation space

Symmetry

- Local analysis: Symmetry as a pair relation

spatial domain

Symmetry

- Local analysis: Symmetry as a pair relation

spatial domain

transformation space

Symmetry

- Local analysis: Symmetry as a pair relation

spatial domain

transformation space

Symmetry

- Local analysis: Symmetry as a pair relation

spatial domain

transformation space

Symmetry

- Local analysis: Symmetry as a pair relation

spatial domain

transformation space

local evidence for
symmetry plane

Symmetry

- Accumulation of local evidence

local evidence for symmetry plane

spatial domain

transformation space
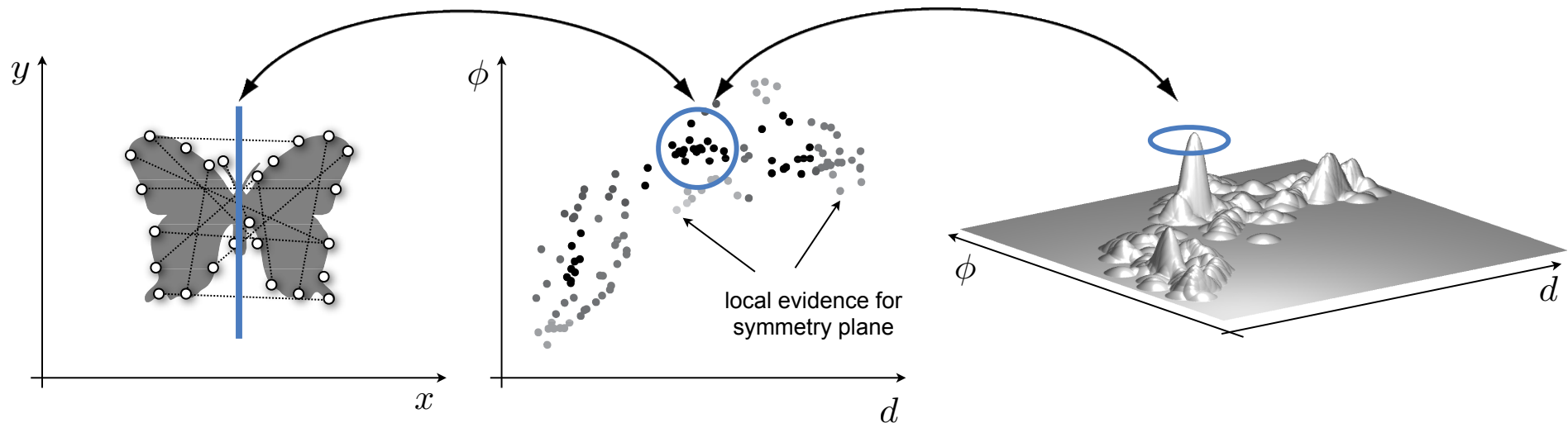
Symmetry

- Accumulation of local evidence
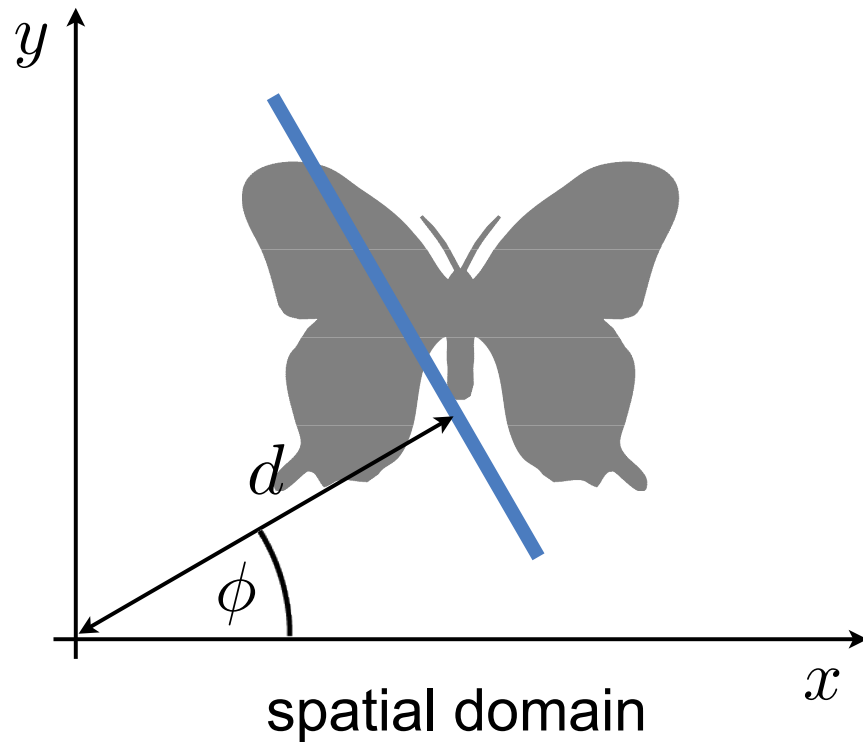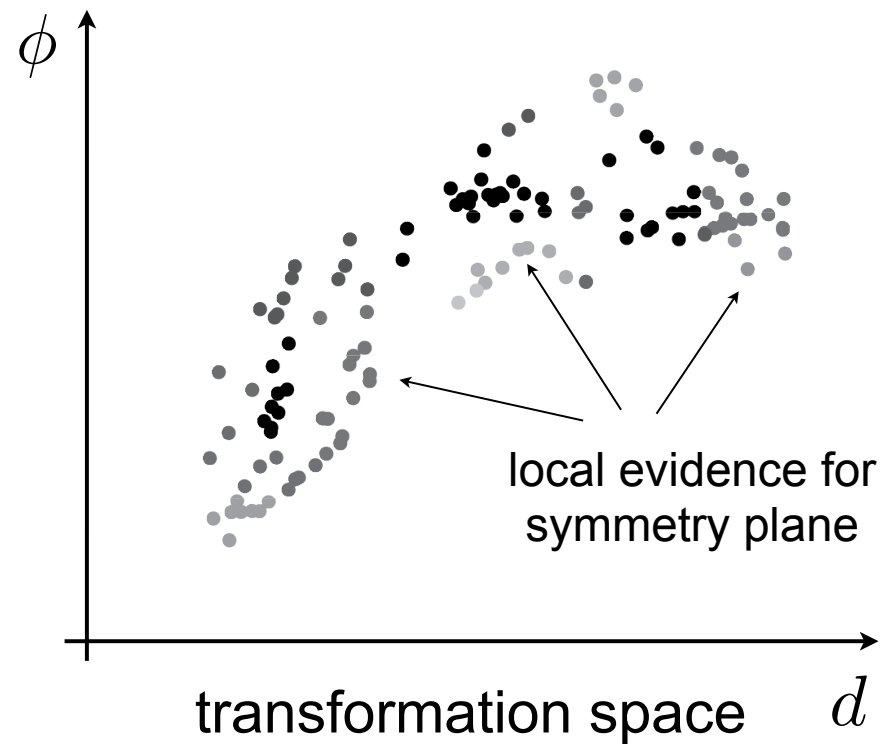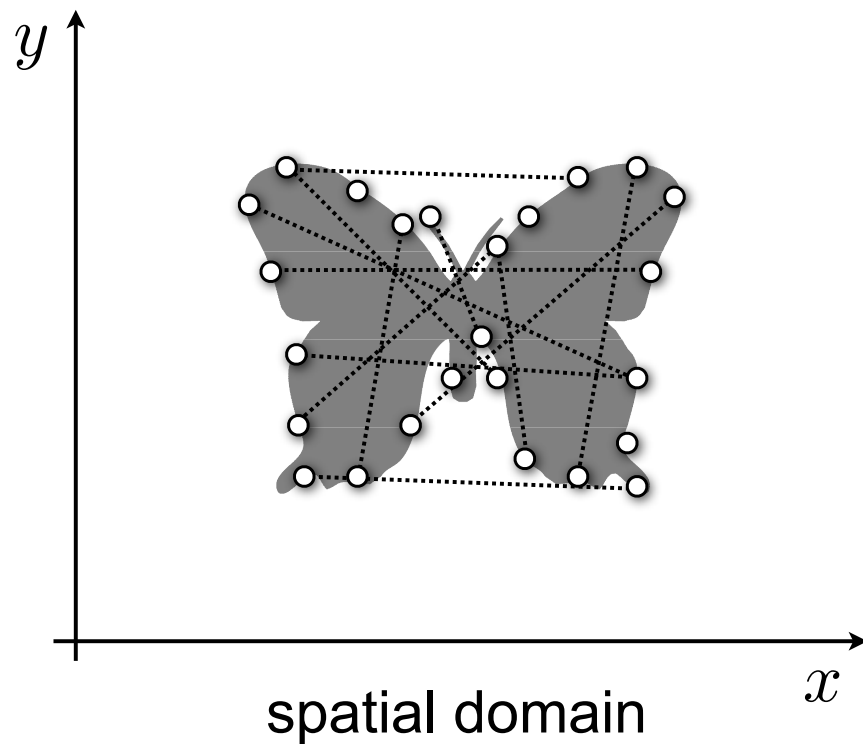
local evidence for
symmetry plane
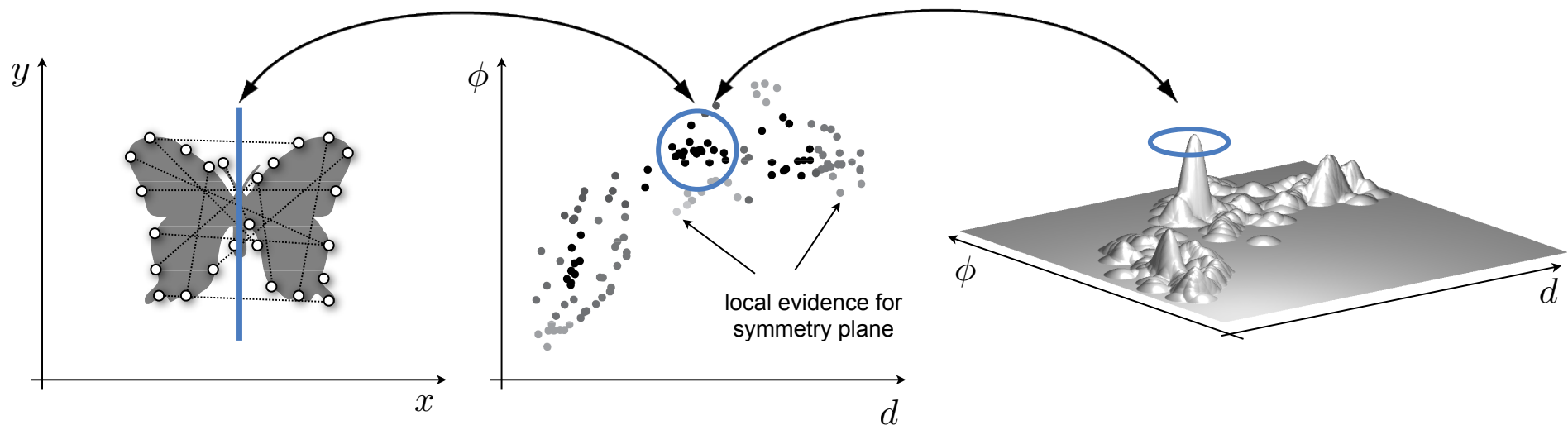
– clustering to extract symmetry transformation
– verification to extract symmetric patches
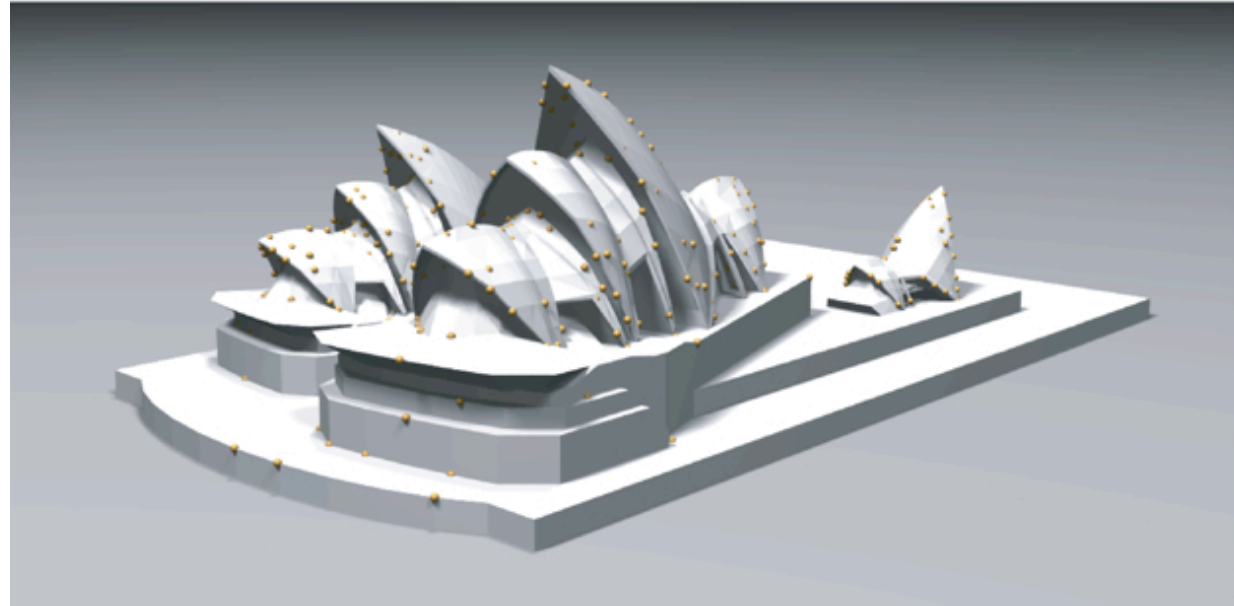
Symmetry

- Accumulation of local evidence

local evidence for
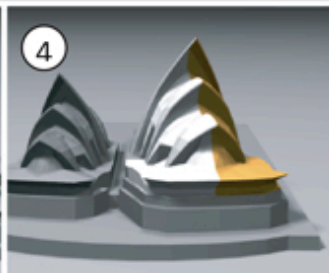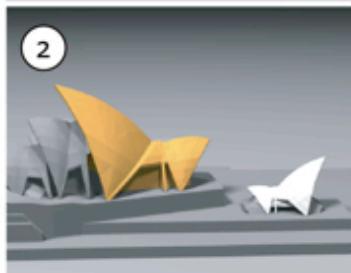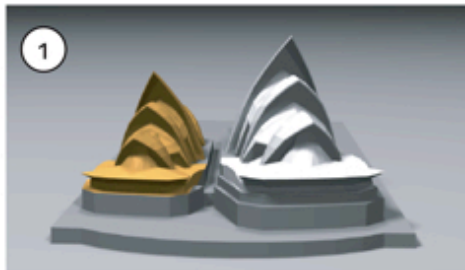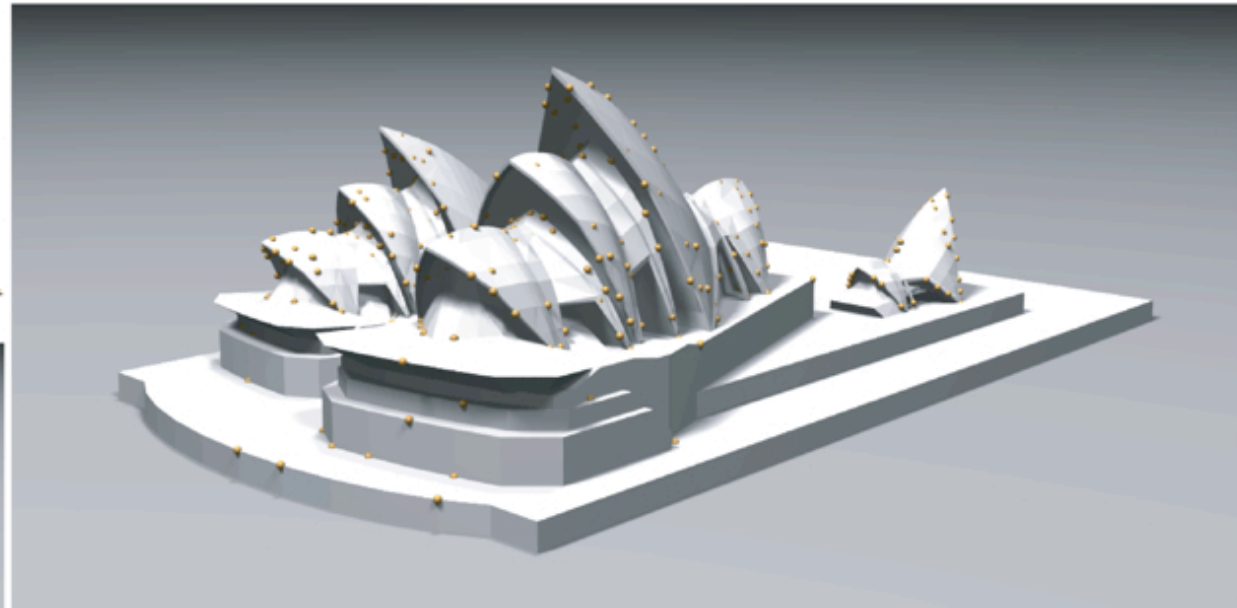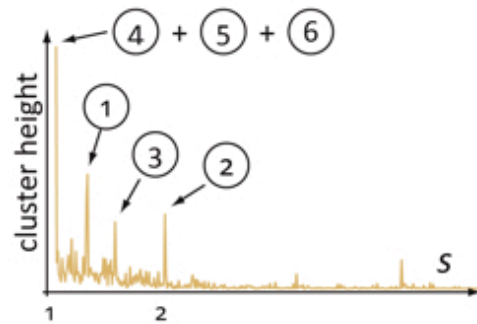symmetry plane

– stochastic Algorithm with provable guarantees

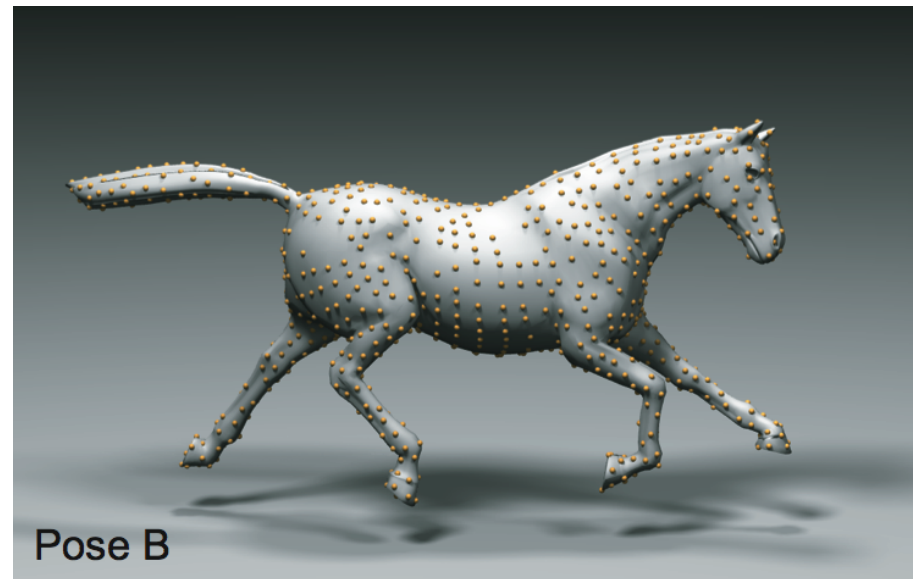$$E(n, \mu, \Delta\sigma) < \left(1 - \sqrt{-2\log\alpha/np}\right) np/2^d$$
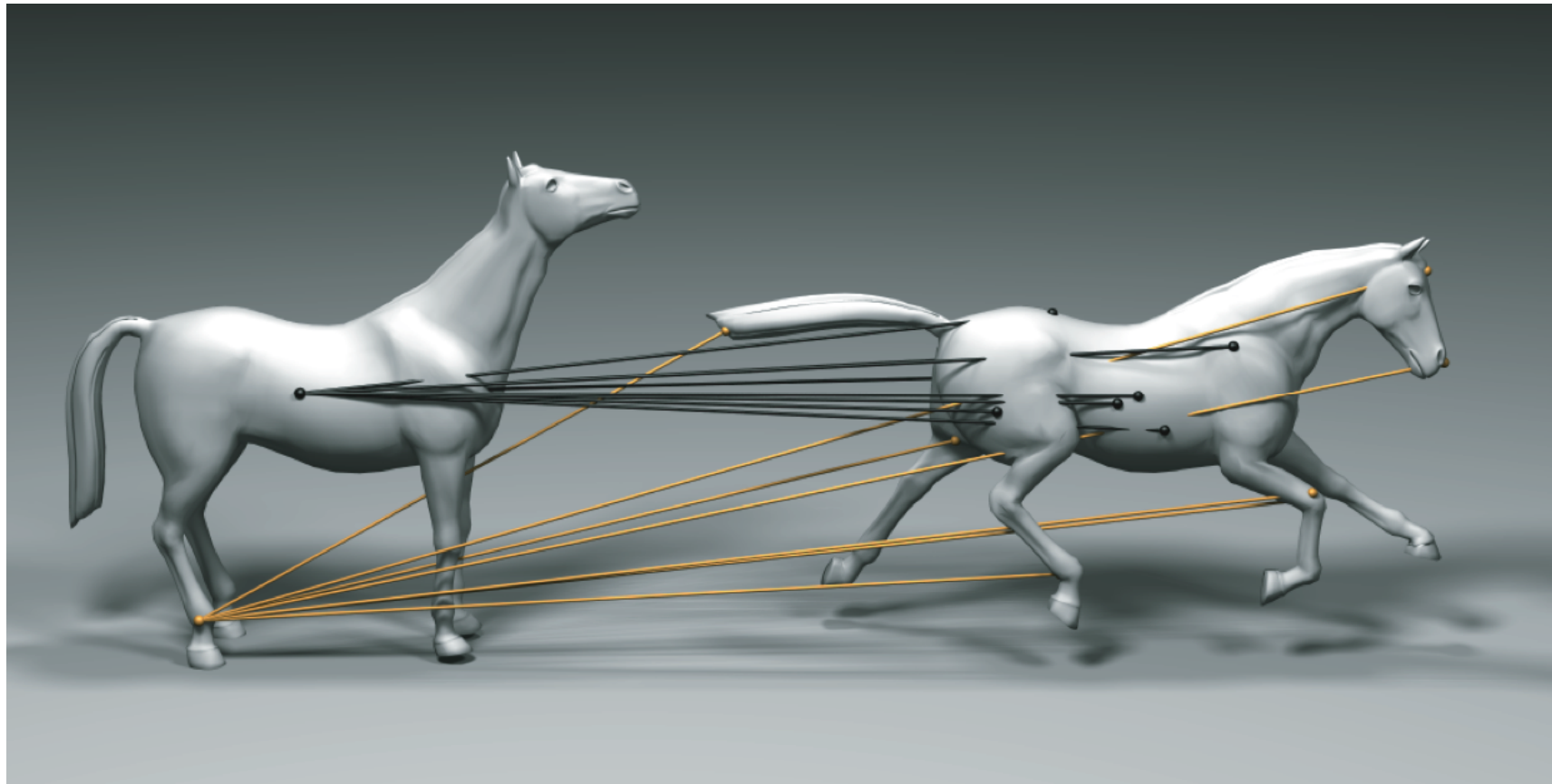
Sydney Opera House

Sydney Opera House
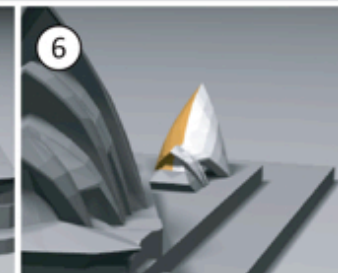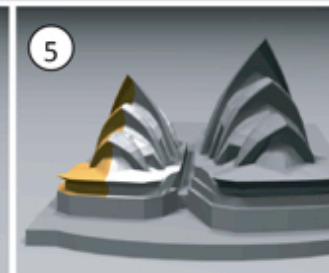
Articulated Shapes

- Random samples on two poses
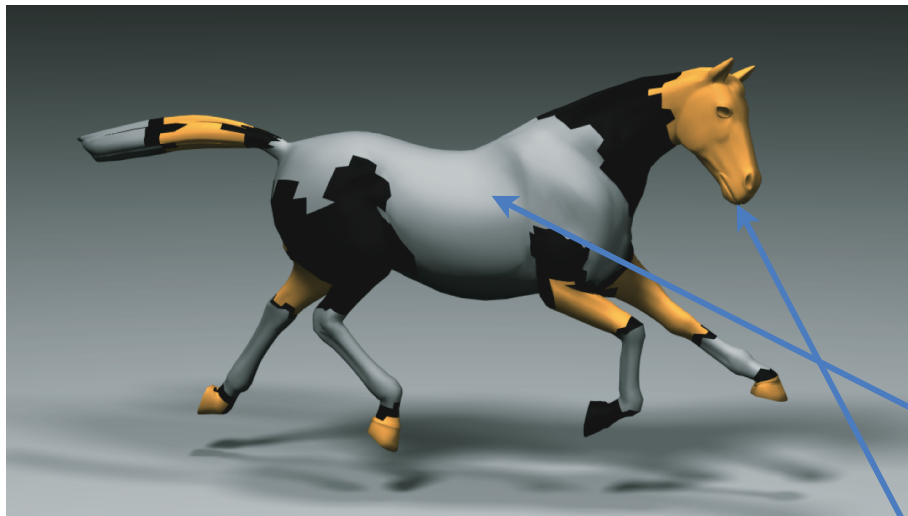 - Correspondences between points are not known

Pose A

Pose B

Articulated Shapes

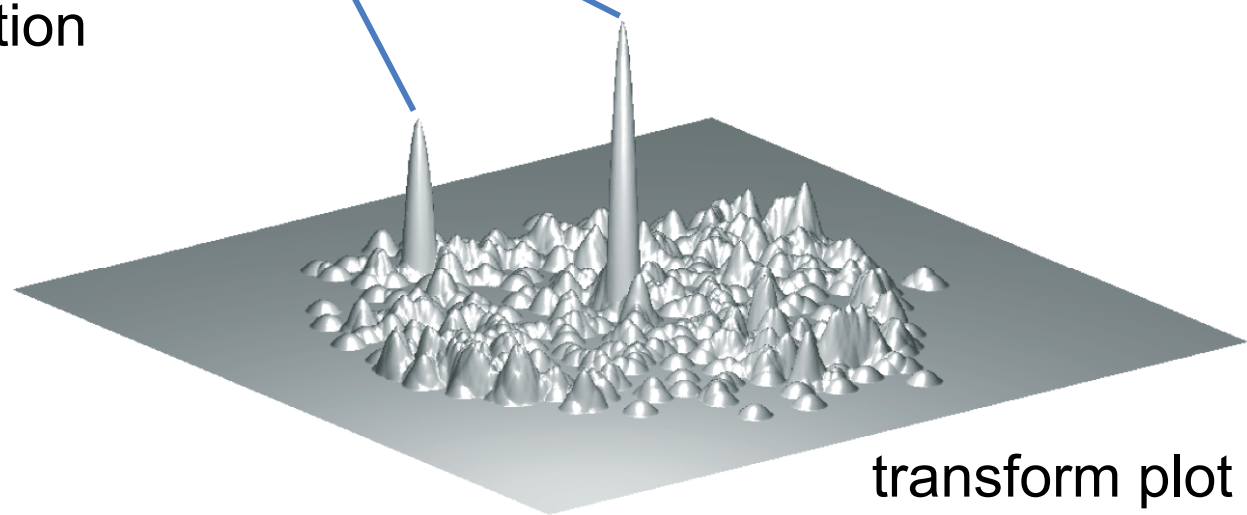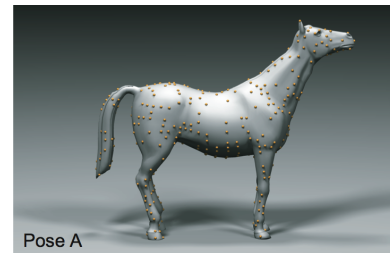- Correspondence candidates
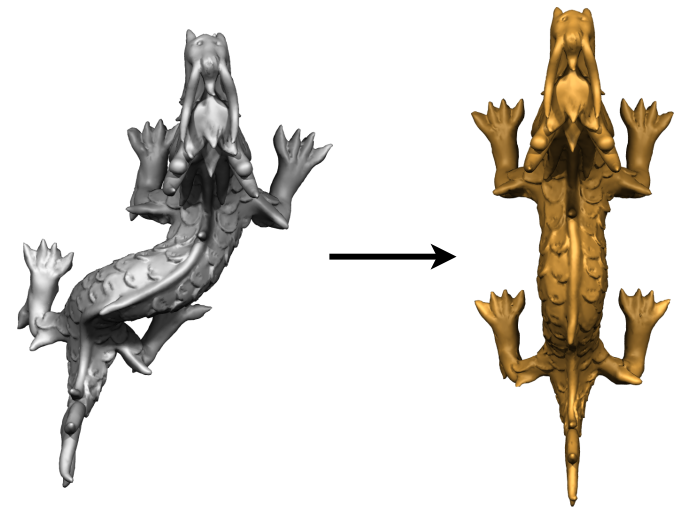
Articulated Shapes
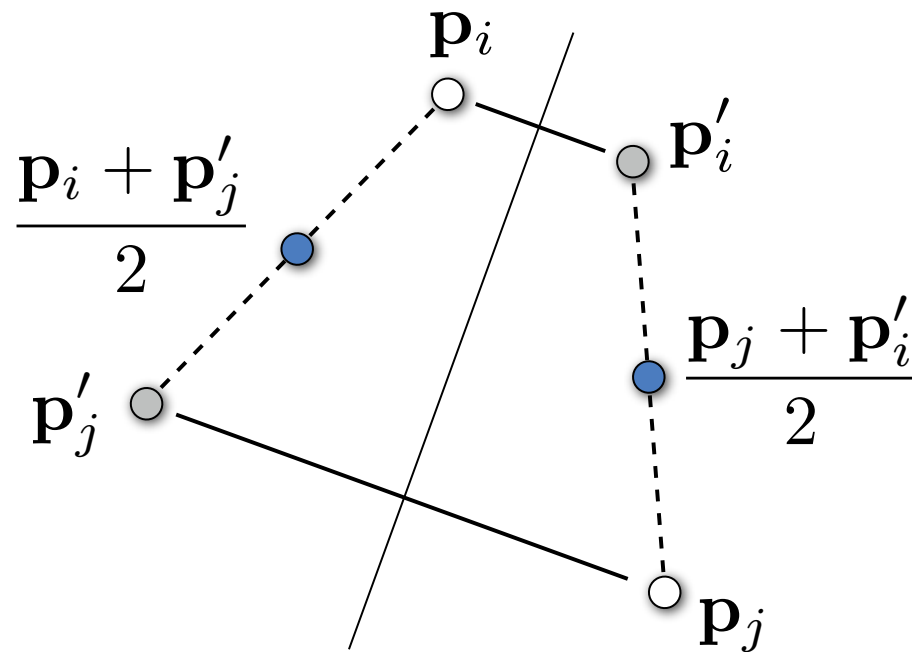

segmentation

Pose A Pose B

transform plot

Symmetrization

- Goal: Symmetrize 3D geometry

- Applications
  - reverse engineering
  - recognition, retrieval
  - compression
  - symmetric meshing, etc.

- Approach
  - Minimally deform the model by optimizing the distribution in transformation space
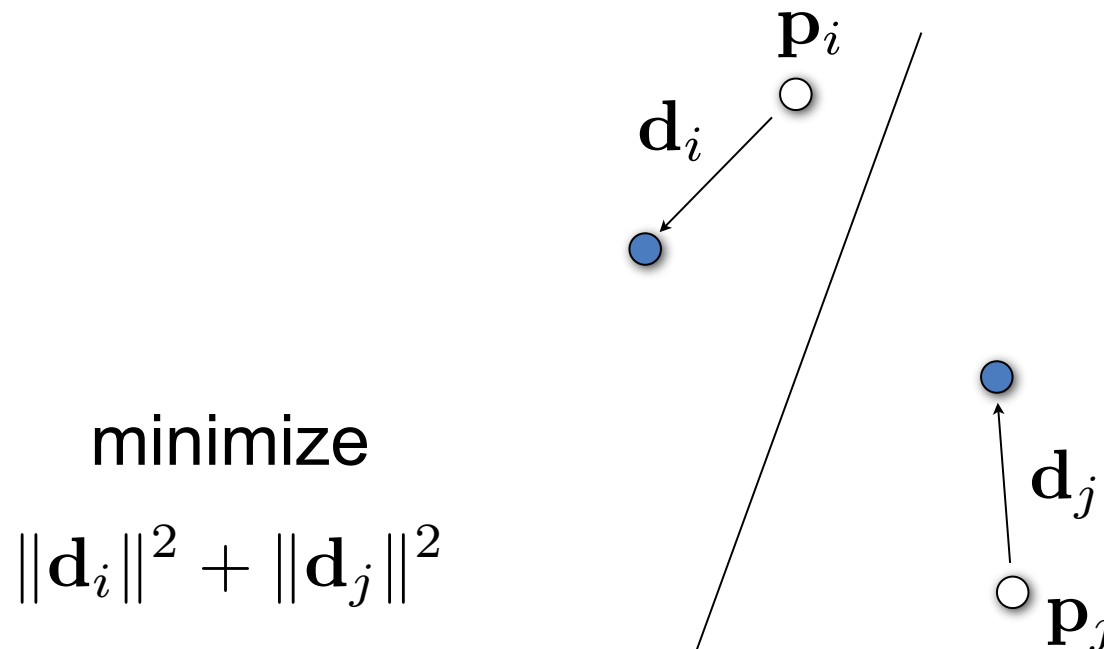
Optimal Displacements

- Find minimal displacements that make two points symmetric with respect to a given transformation
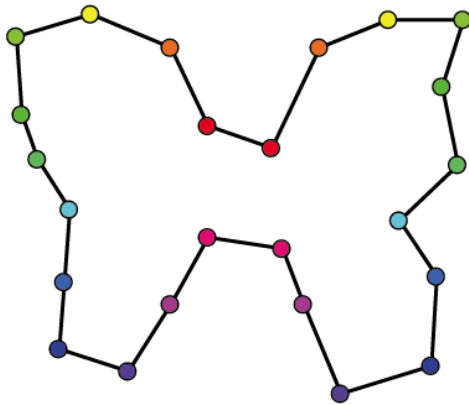
Optimal Displacements

- Find minimal displacements that make two points symmetric with respect to a given transformation

$$\mathbf{p}_i$$

$$\mathbf{d}_i$$

$$\mathbf{d}_j$$

$$\mathbf{p}_j$$

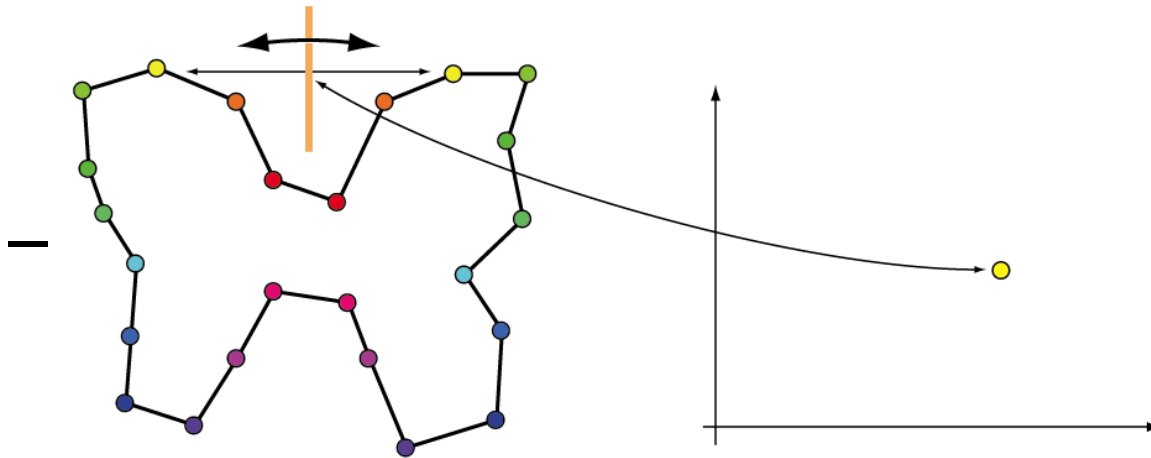minimize

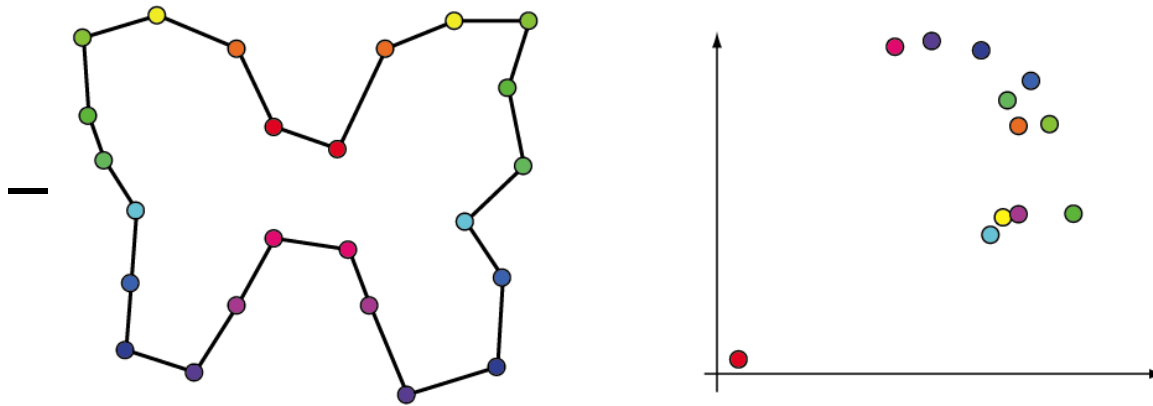$$\|\mathbf{d}_i\|^2 + \|\mathbf{d}_j\|^2$$

Optimal Displacements

- Find optimal transformation and minimal displacements for a set of corresponding points

Optimal Displacements

- Find optimal transformation and minimal displacements for a set of corresponding points
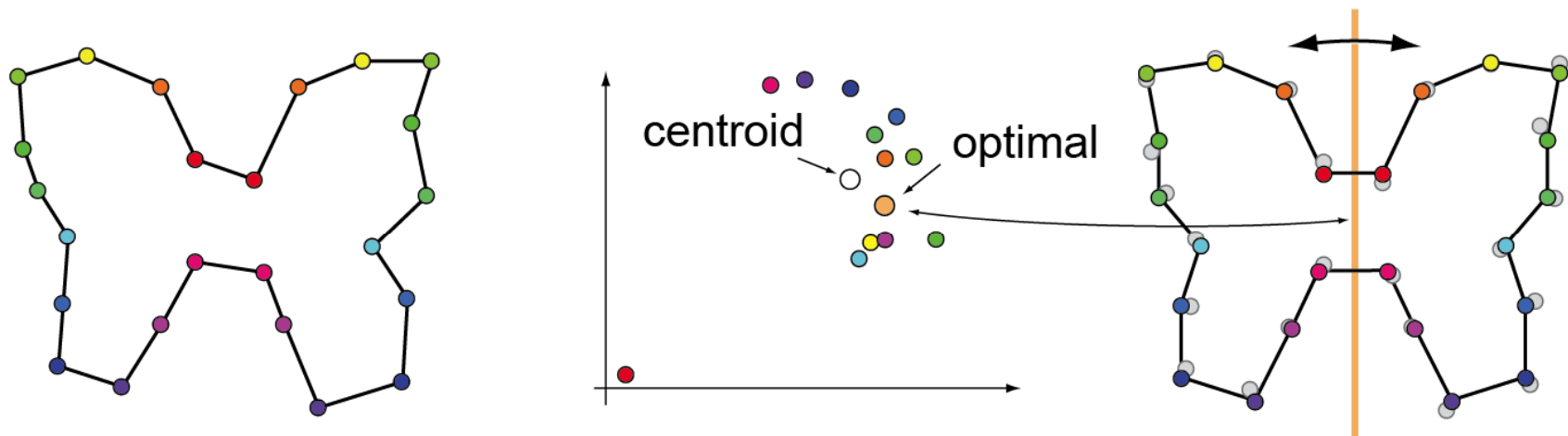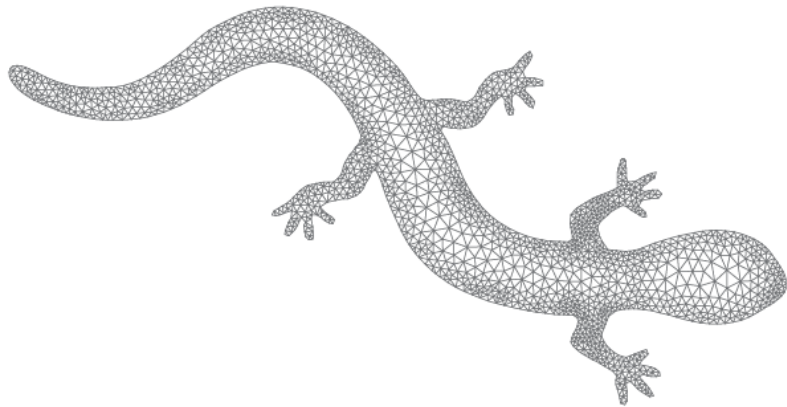
Optimal Displacements

- Find optimal transformation and minimal displacements for a set of corresponding points

Optimal Displacements

- Find optimal transformation and minimal displacements for a set of corresponding points
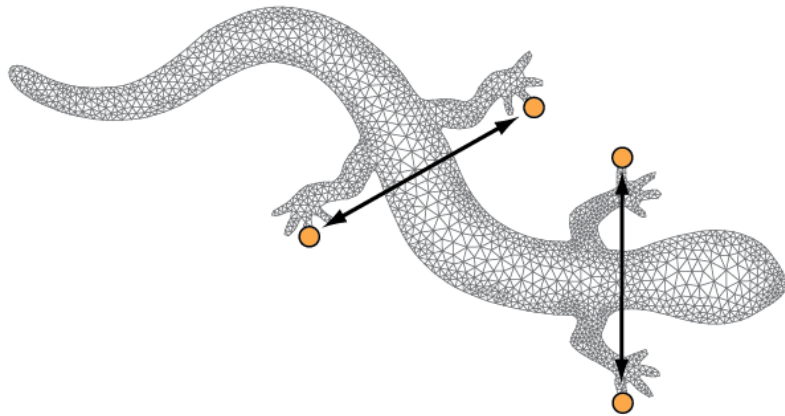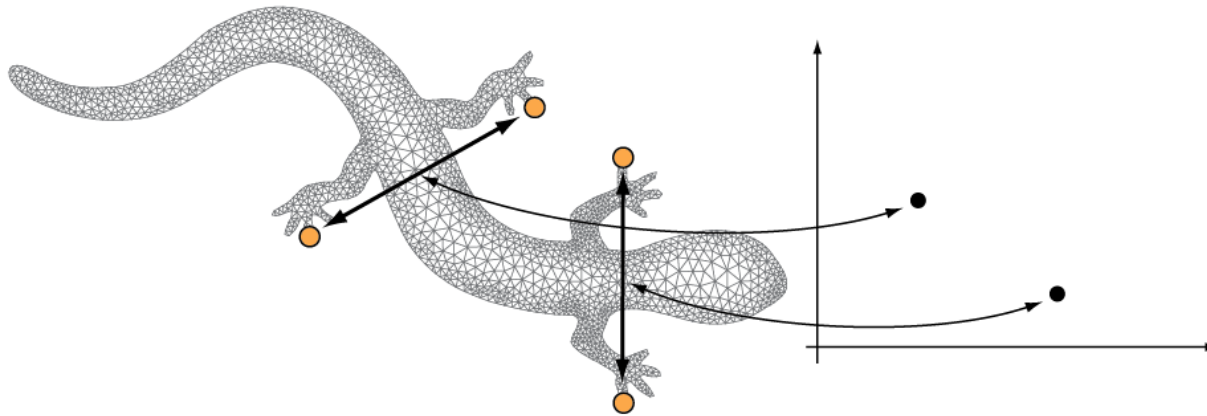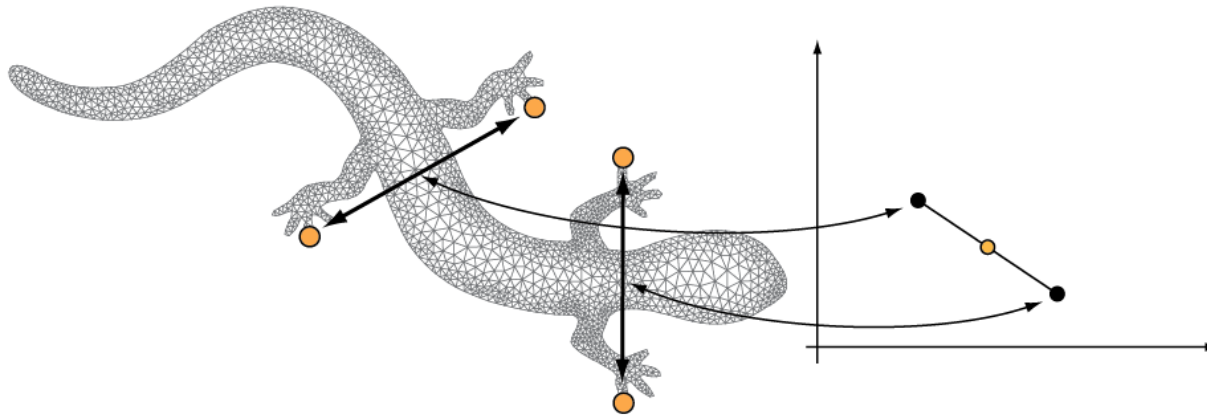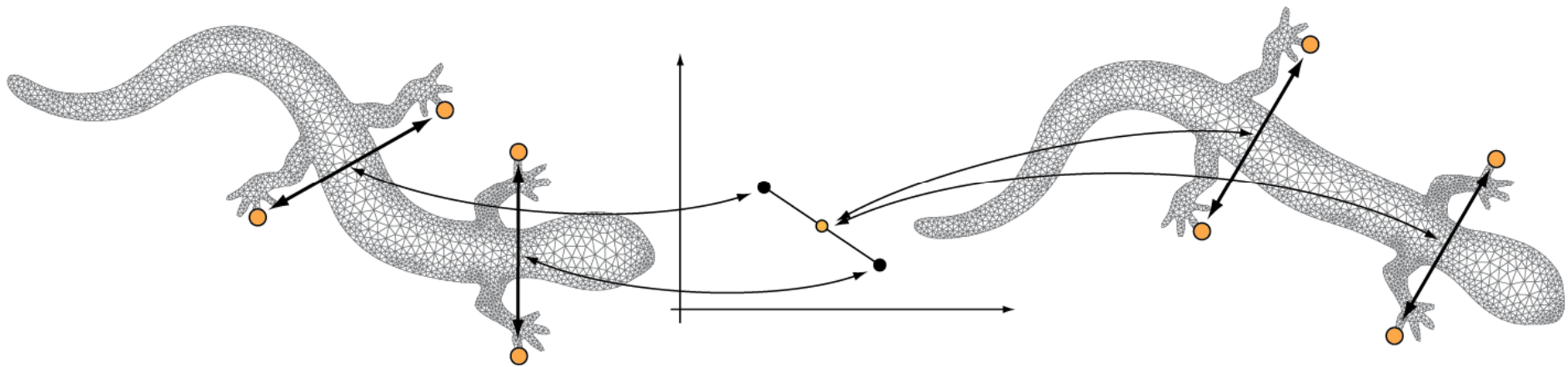

- – closed form solution exists!

Optimization

- Embedded deformation

Optimization

- Embedded deformation

Optimization

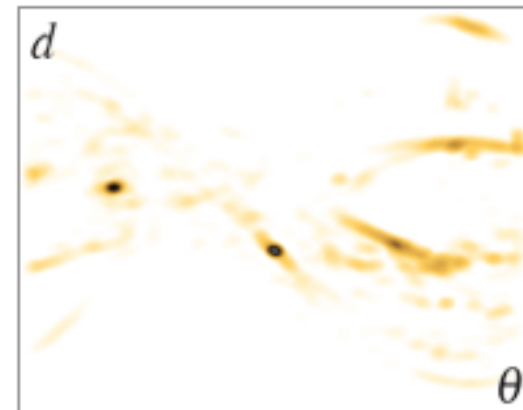- Embedded deformation

Optimization

- Embedded deformation

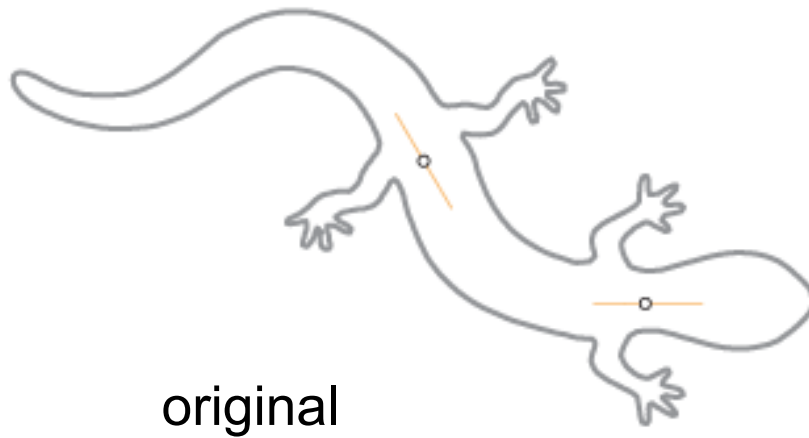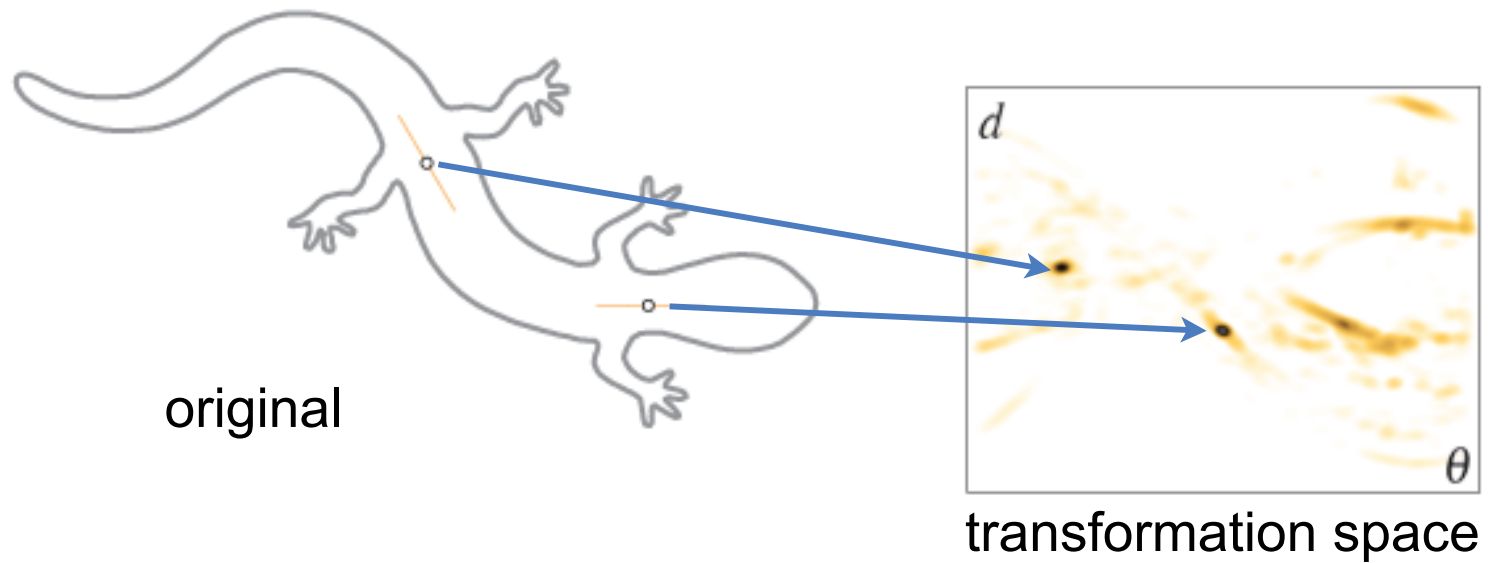# Optimization

- Embedded deformation

Symmetrization

- 2D Example

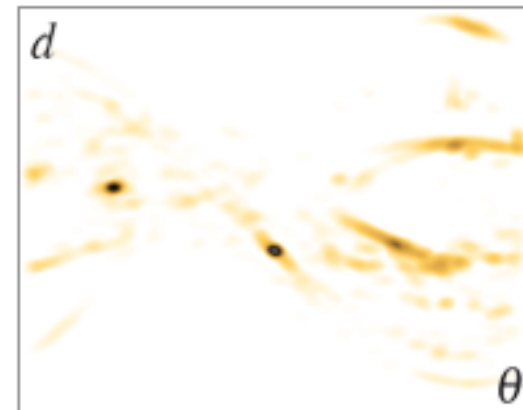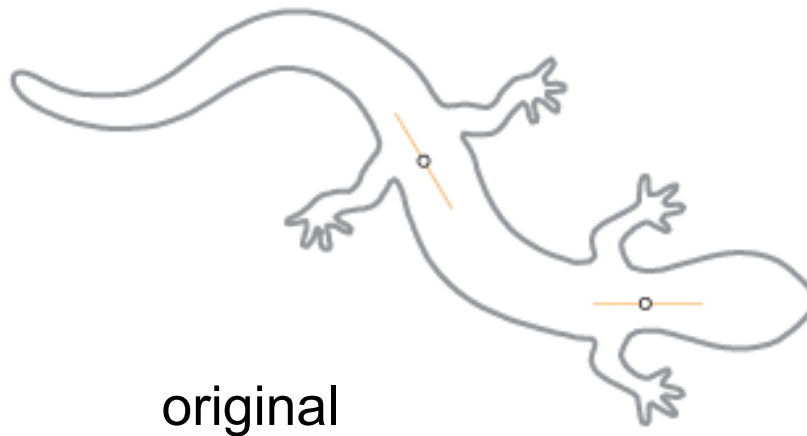
original

transformation space

Symmetrization

- 2D Example

original

transformation space

Symmetrization

- 2D Example

original

transformation space

Symmetrization

- 2D Example

transformation space
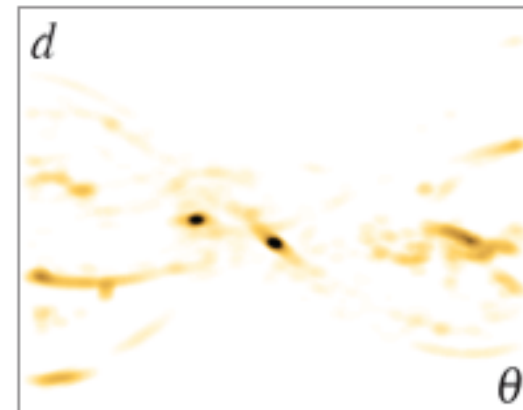
Symmetrization

- 2D Example

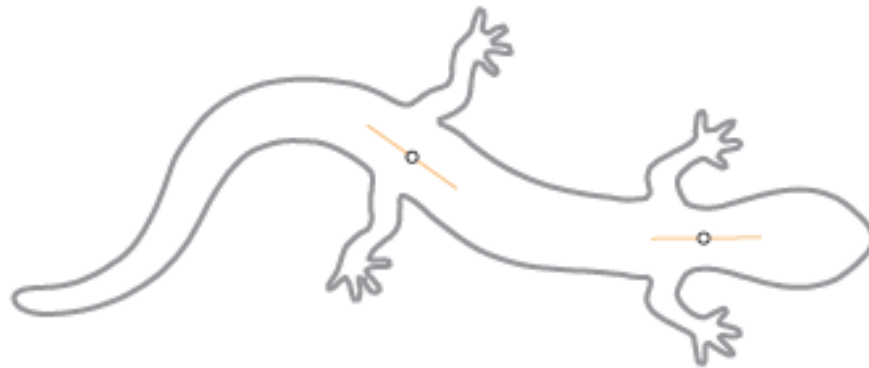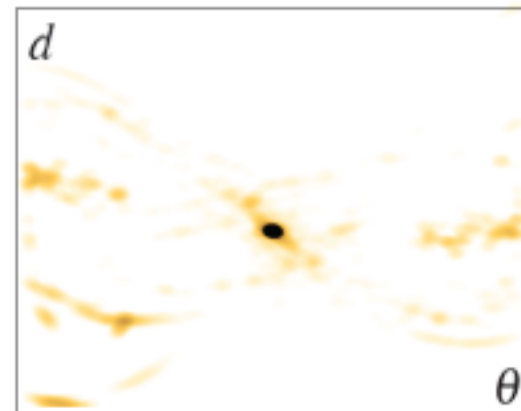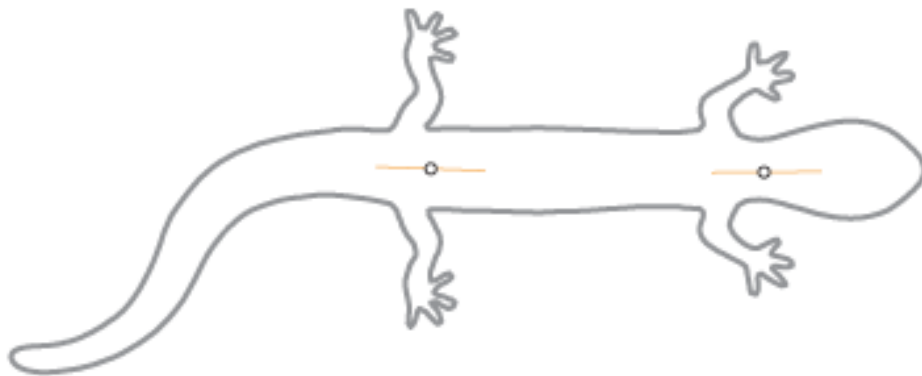transformation space

Symmetrization

- 2D Example

transformation space
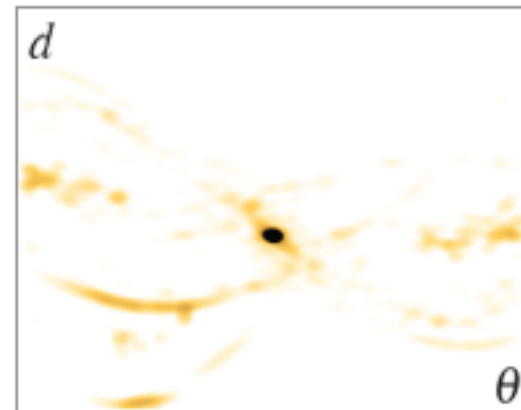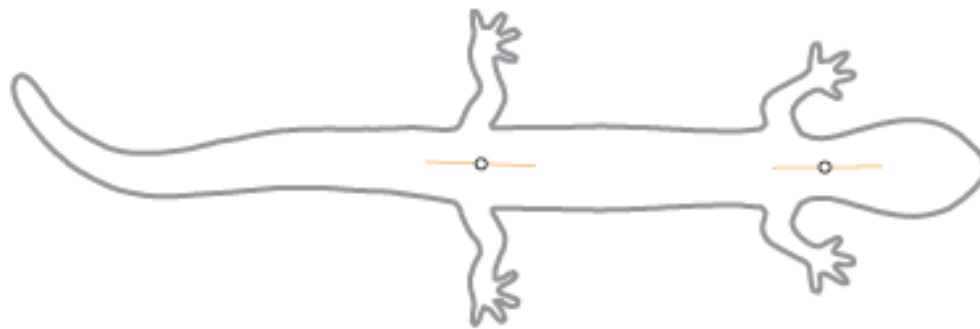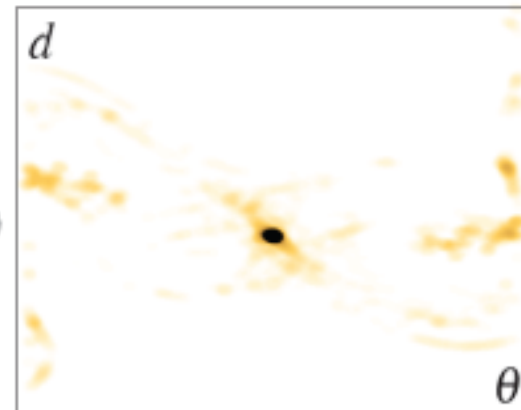
Symmetrization

- 2D Example

transformation space
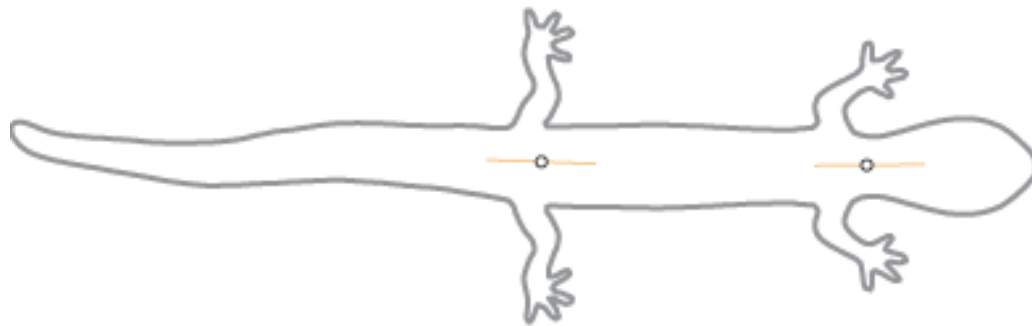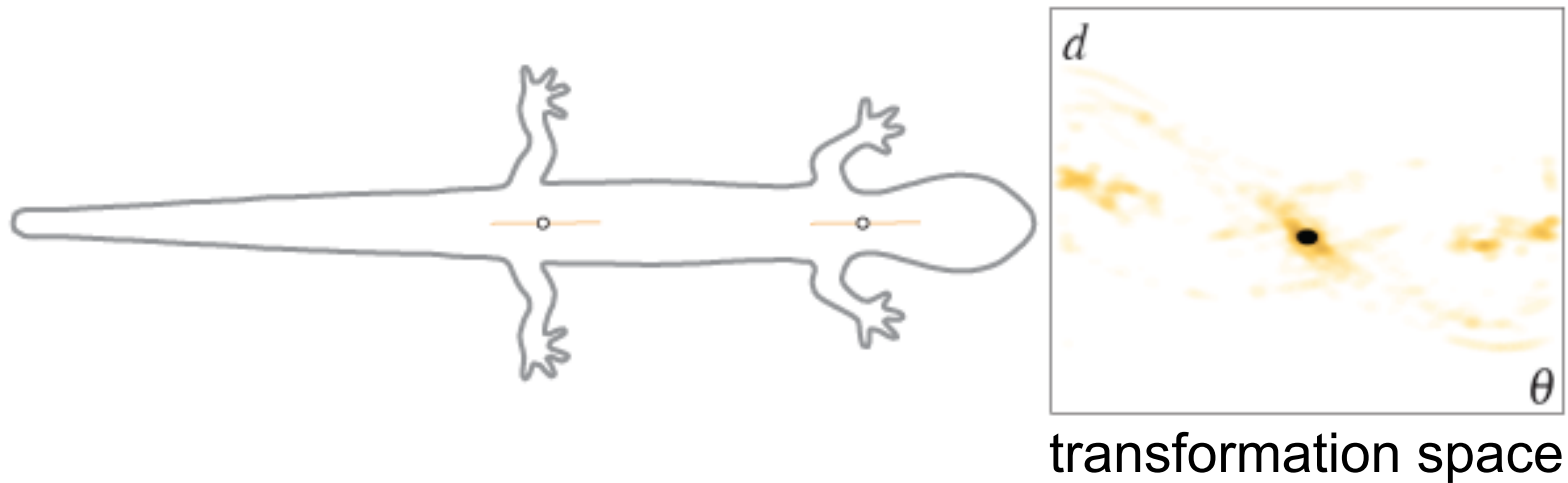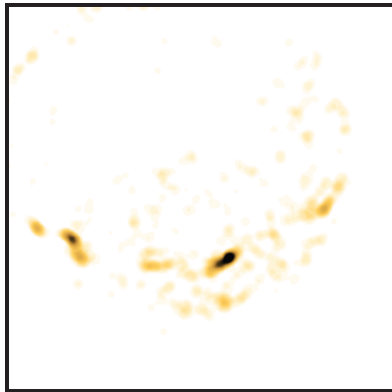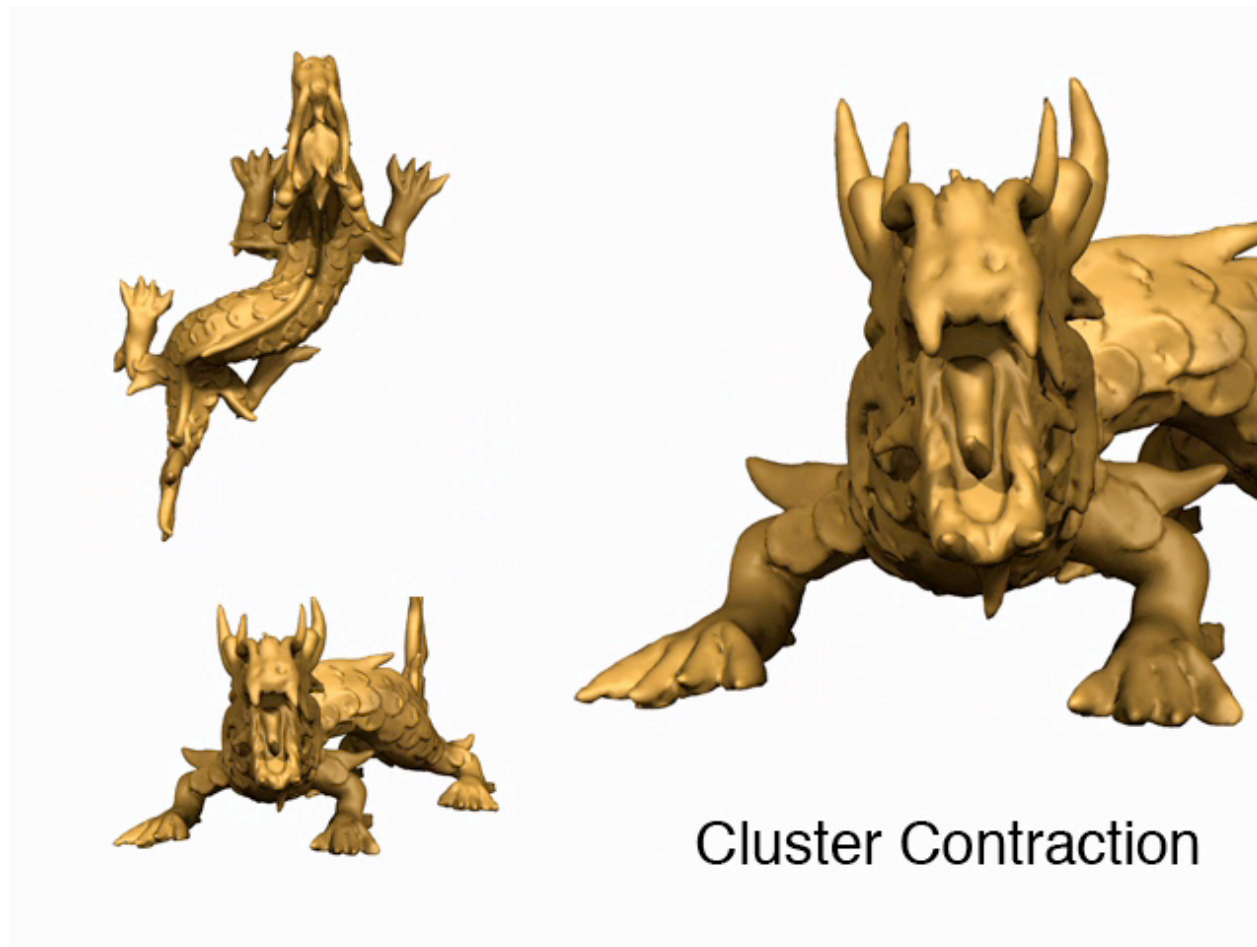
Symmetrization

- 2D Example

transformation space

Dragon

Dragon

Dragon

Symmetrizing the Dragon

Cluster Contraction

Shape Matching

Cluster Merging

References

- Mitra, Guibas, Pauly: ***Partial and Approximate Symmetry Detection for 3D Geometry***, ACM Transactions on Graphics (Proceedings of SIGGRAPH), 2006

- Mitra, Guibas, Pauly: ***Symmetrization***, ACM Transactions on Graphics (Proceedings of SIGGRAPH), 2007