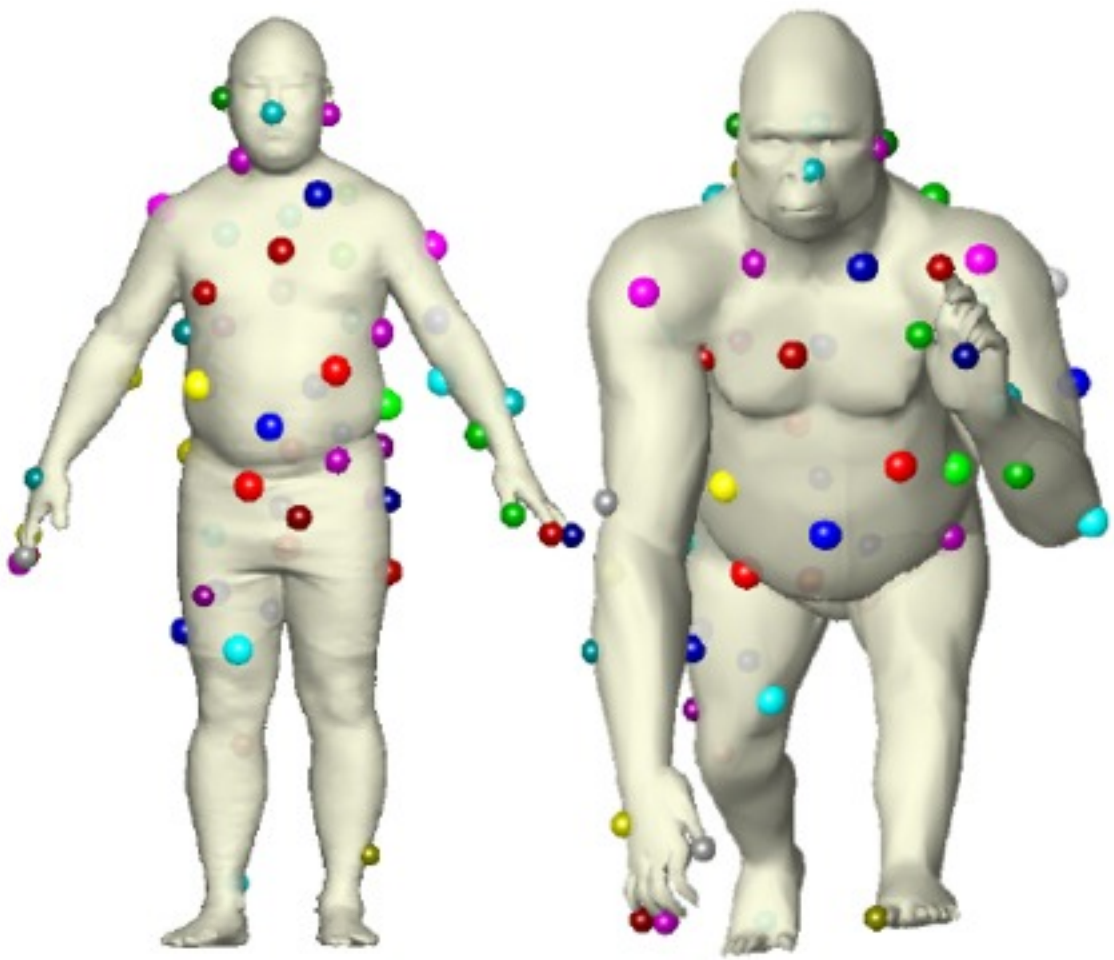
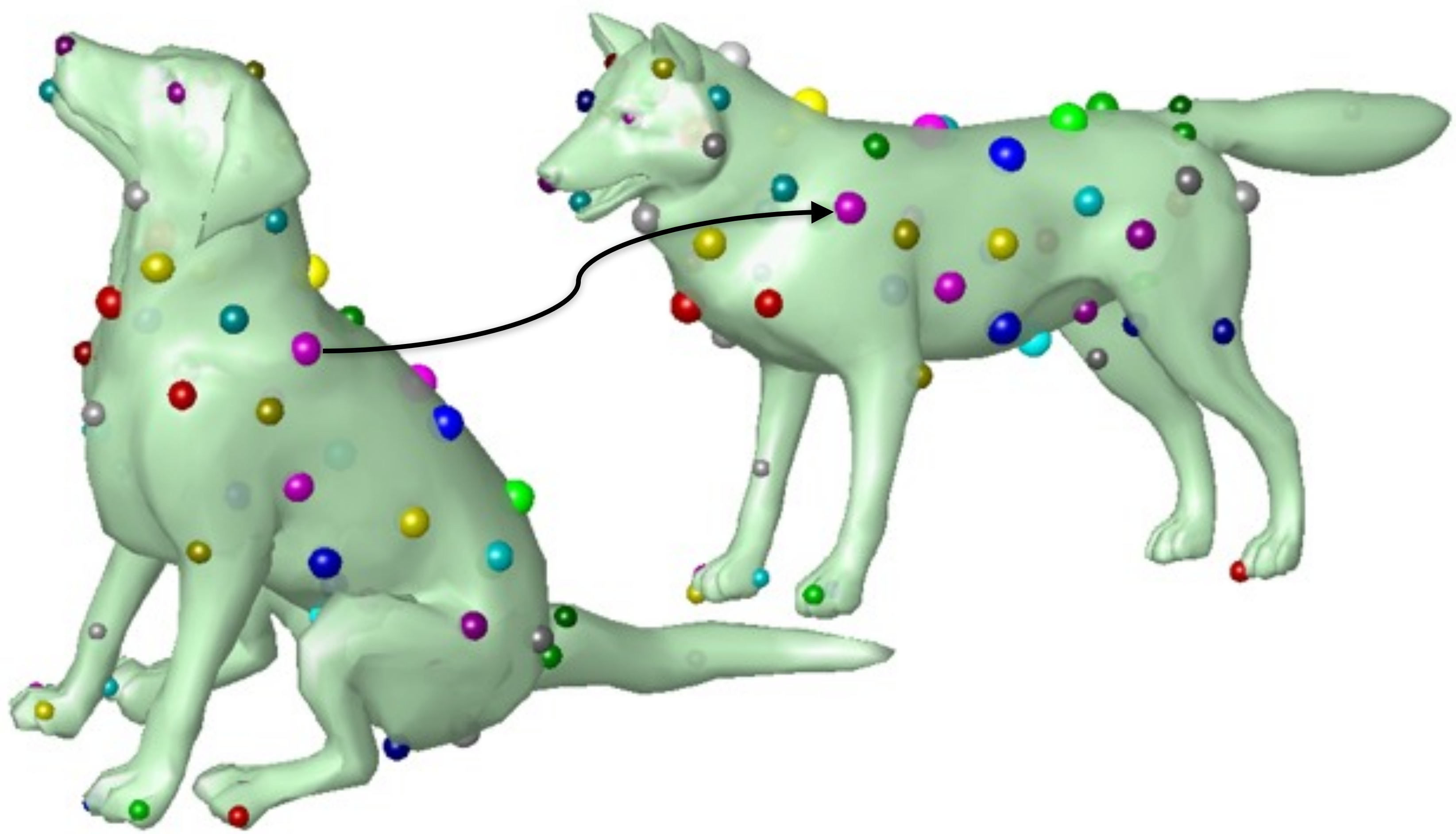
Markerless Correspondence

Symmetry Detection and Applications



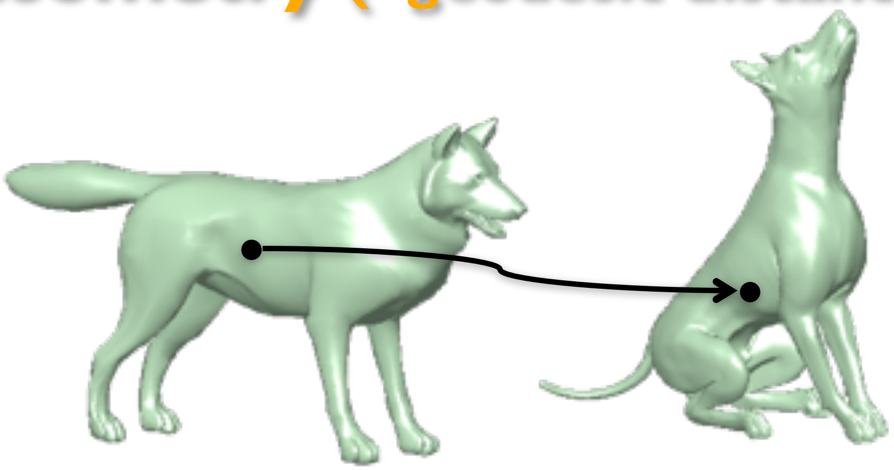
Correspondence Detection

Given two surfaces, find a set of corresponding points.



Mobius Voting

Goal: Find correspondences likely to participate in an **isometry** (=geodesic distance preserving)



Method: Use the **Möbius group as low DOF model** for non-rigid alignment.

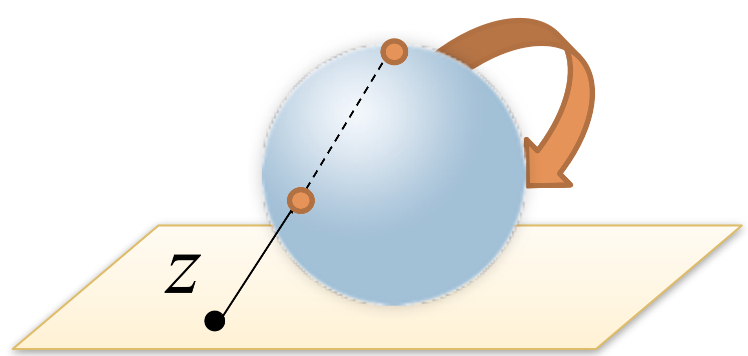
Rationale:

- **6 DOF** of the Möbius group
- **contains** perfect isometries

for devising randomized geometric algorithm.

Mobius Transformation

- All the global 1-1 and onto conformal map on the sphere.



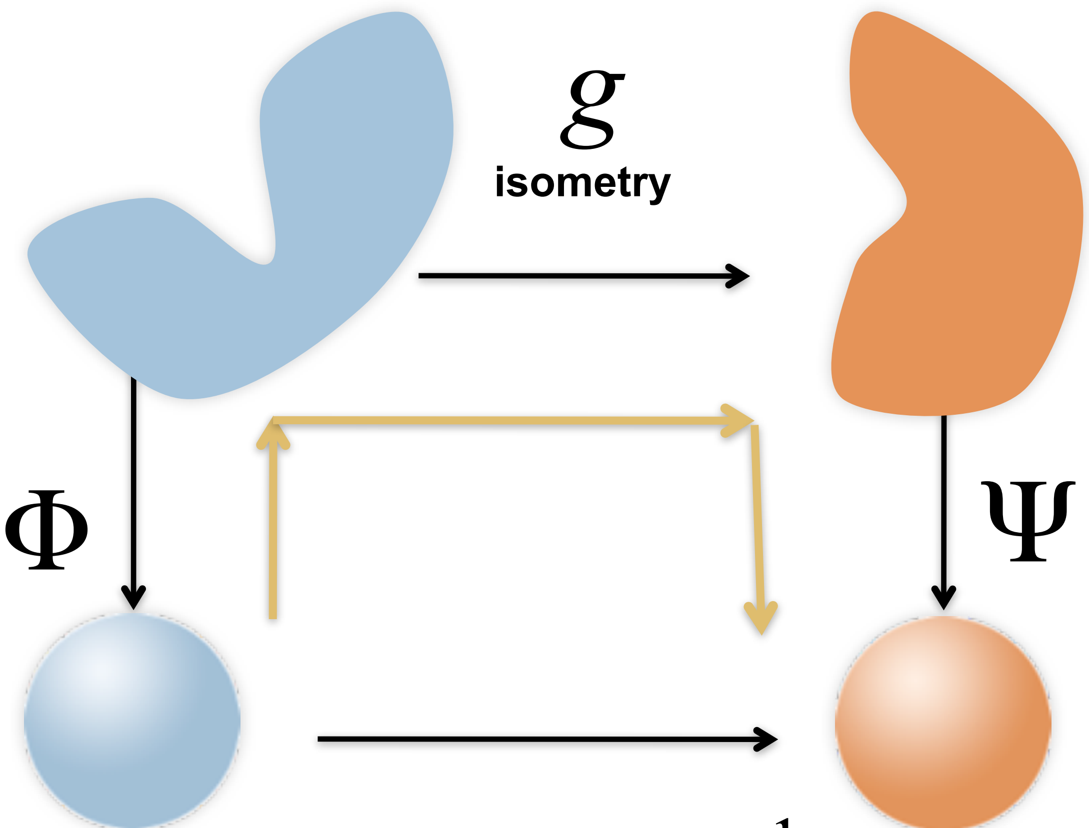
$$f(z) = \frac{az+b}{cz+d}, ad - bc \neq 0$$

$$a, b, c, d \in \mathbb{C}$$

- 6 DOF: prescribing three points uniquely defines a Möbius transformation.

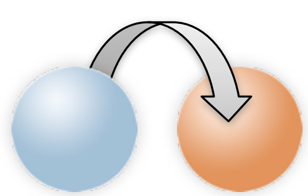
$$f(z_i) = y_i, i = 1, 2, 3 \quad \Rightarrow (a, b, c, d)$$

Algorithm for Perfect Isometries



$$\Psi \circ g \circ \Phi^{-1}$$

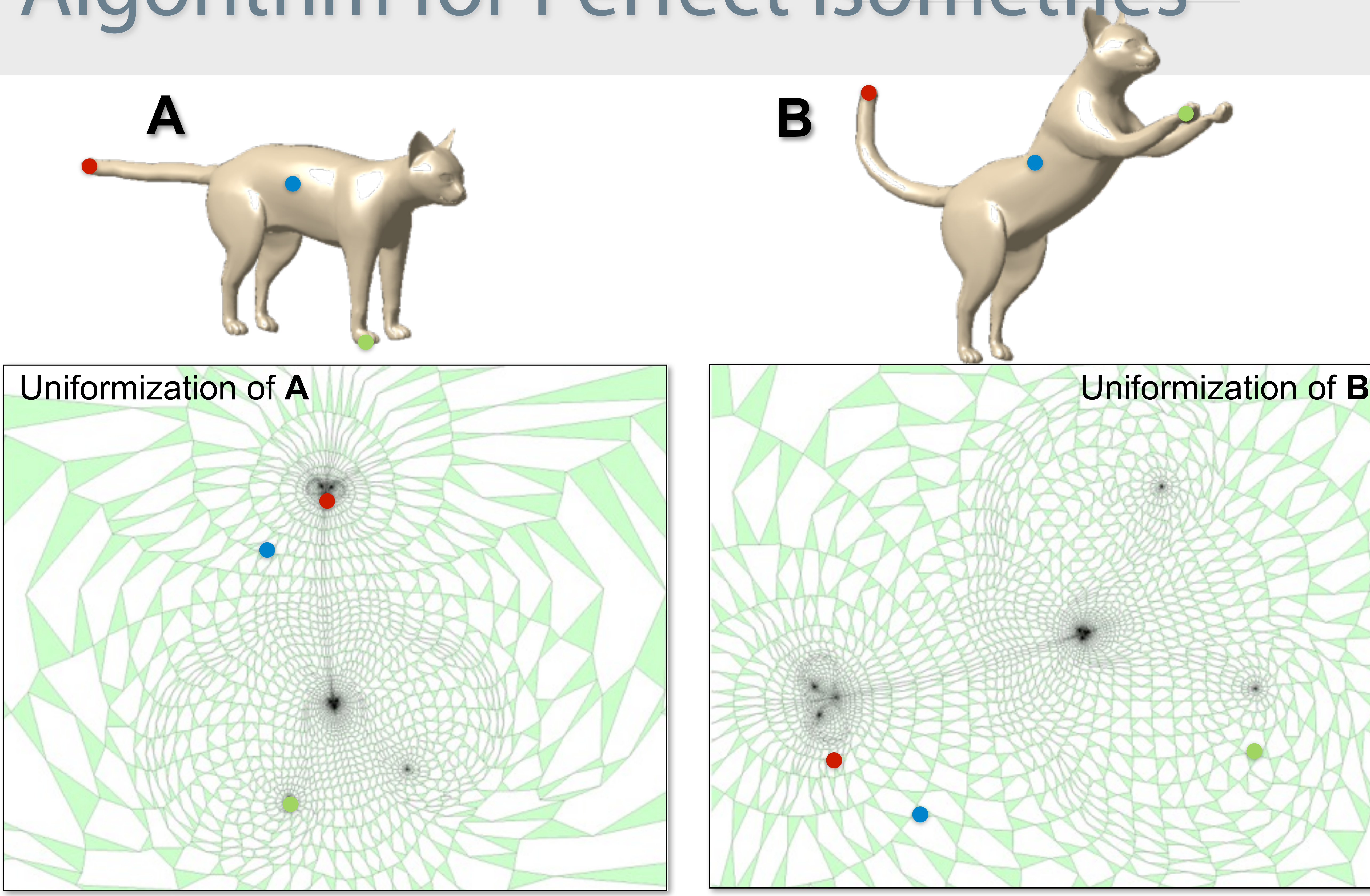
$$\Psi \circ g \circ \Phi^{-1} \in$$ Conformal



$$\Rightarrow \Psi \circ g \circ \Phi^{-1}(z) = \frac{az+b}{cz+d}$$

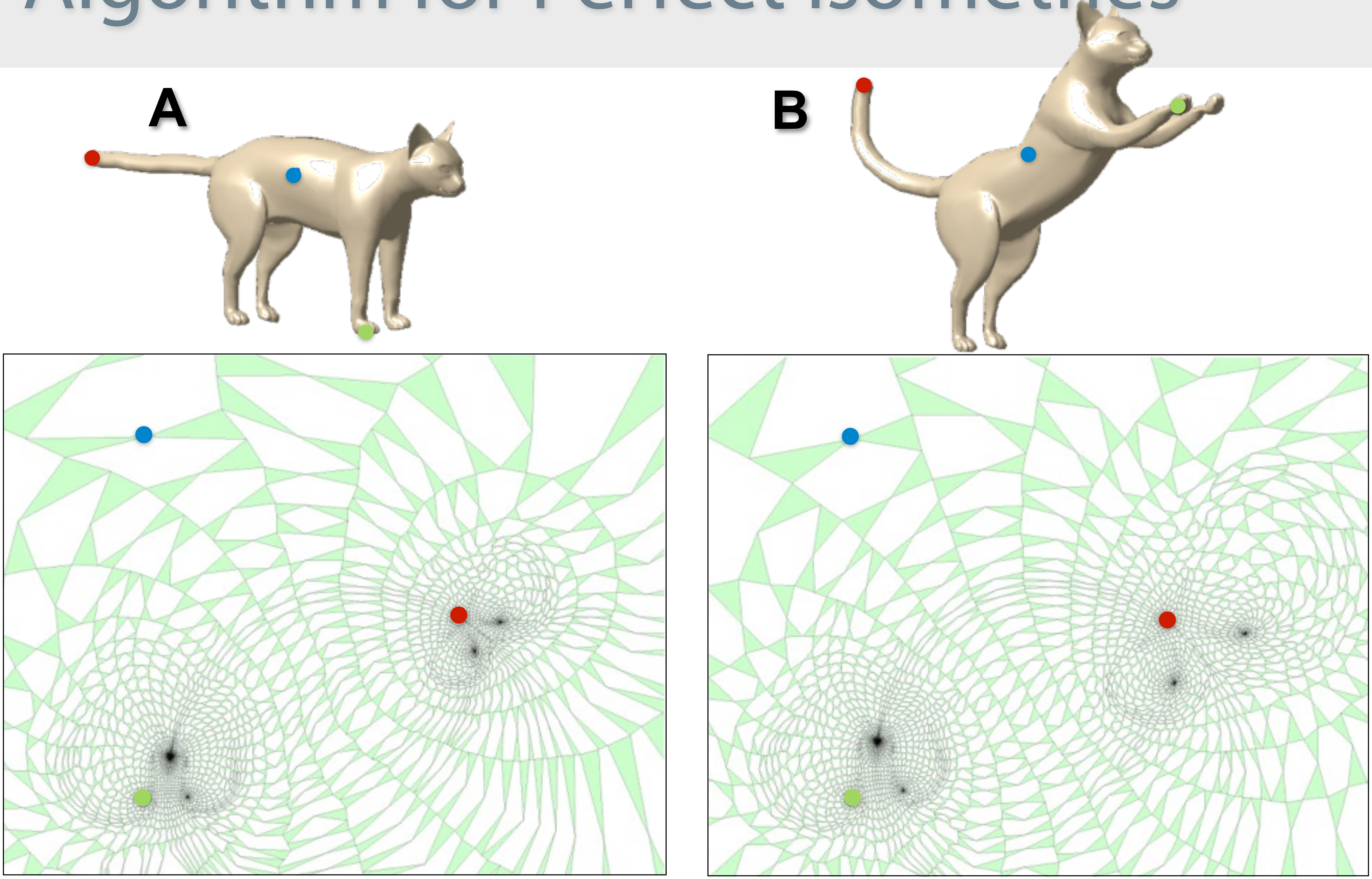
search the **Möbius** group (6 DOF) for your correspondence

Algorithm for Perfect Isometries



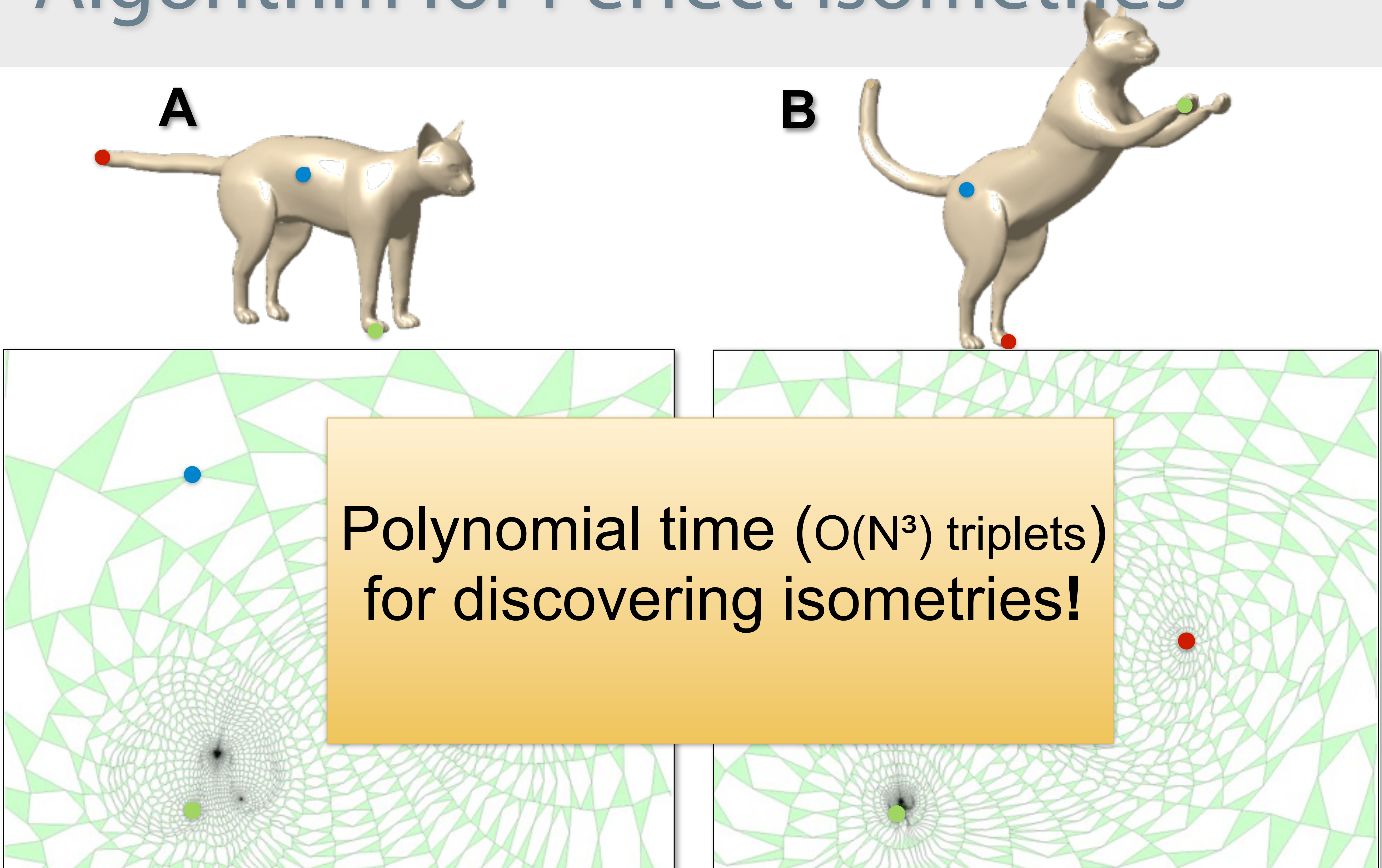
3 **Correct** Correspondences

Algorithm for Perfect Isometries



3 **Correct** Correspondences

Algorithm for Perfect Isometries

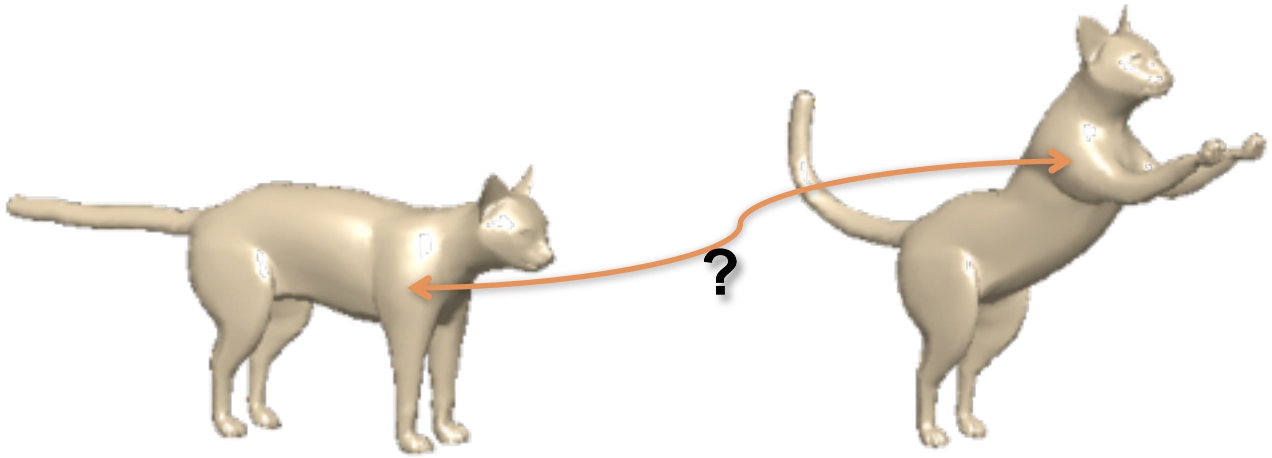


Polynomial time ($O(N^3)$ triplets) for discovering isometries!

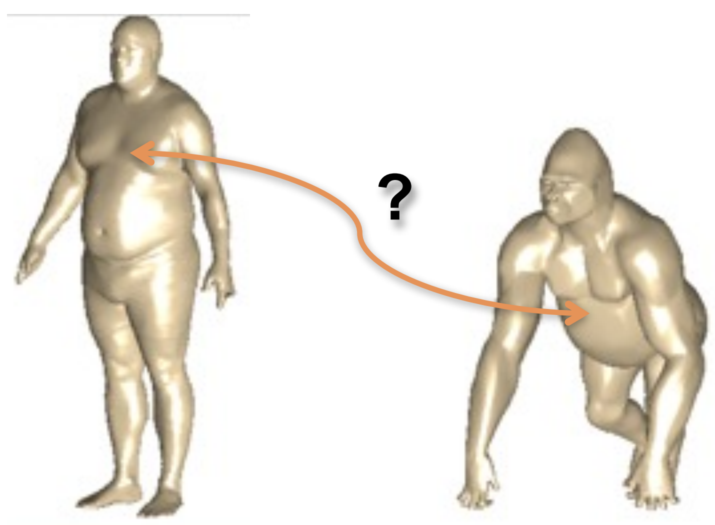
3 **Incorrect** Correspondences

Voting for Imperfect Isometries

Even the same shape in different pose is hardly exactly isometric so single global Möbius is not enough…

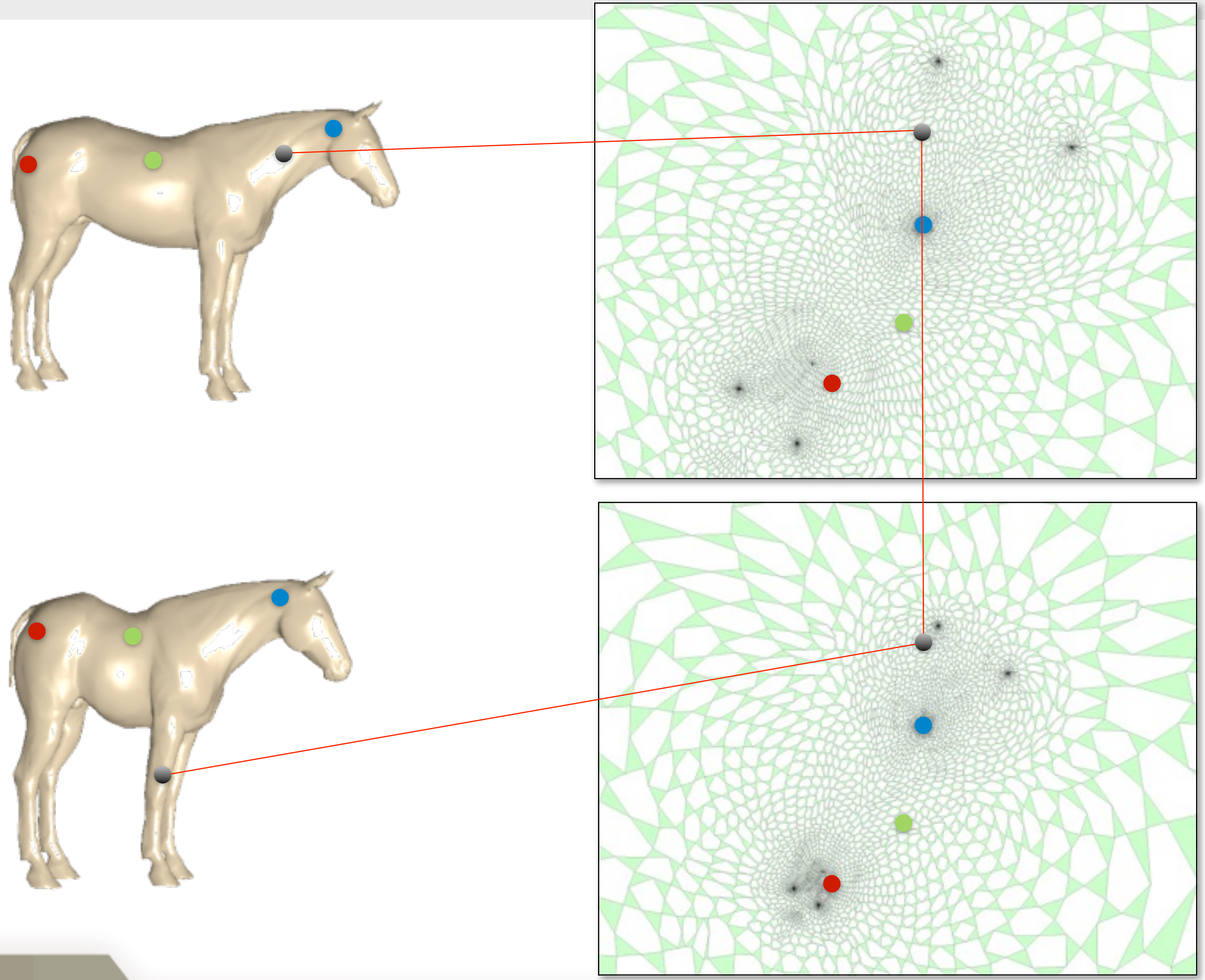


Furthermore, we want to compare different (non-isometric) surfaces…

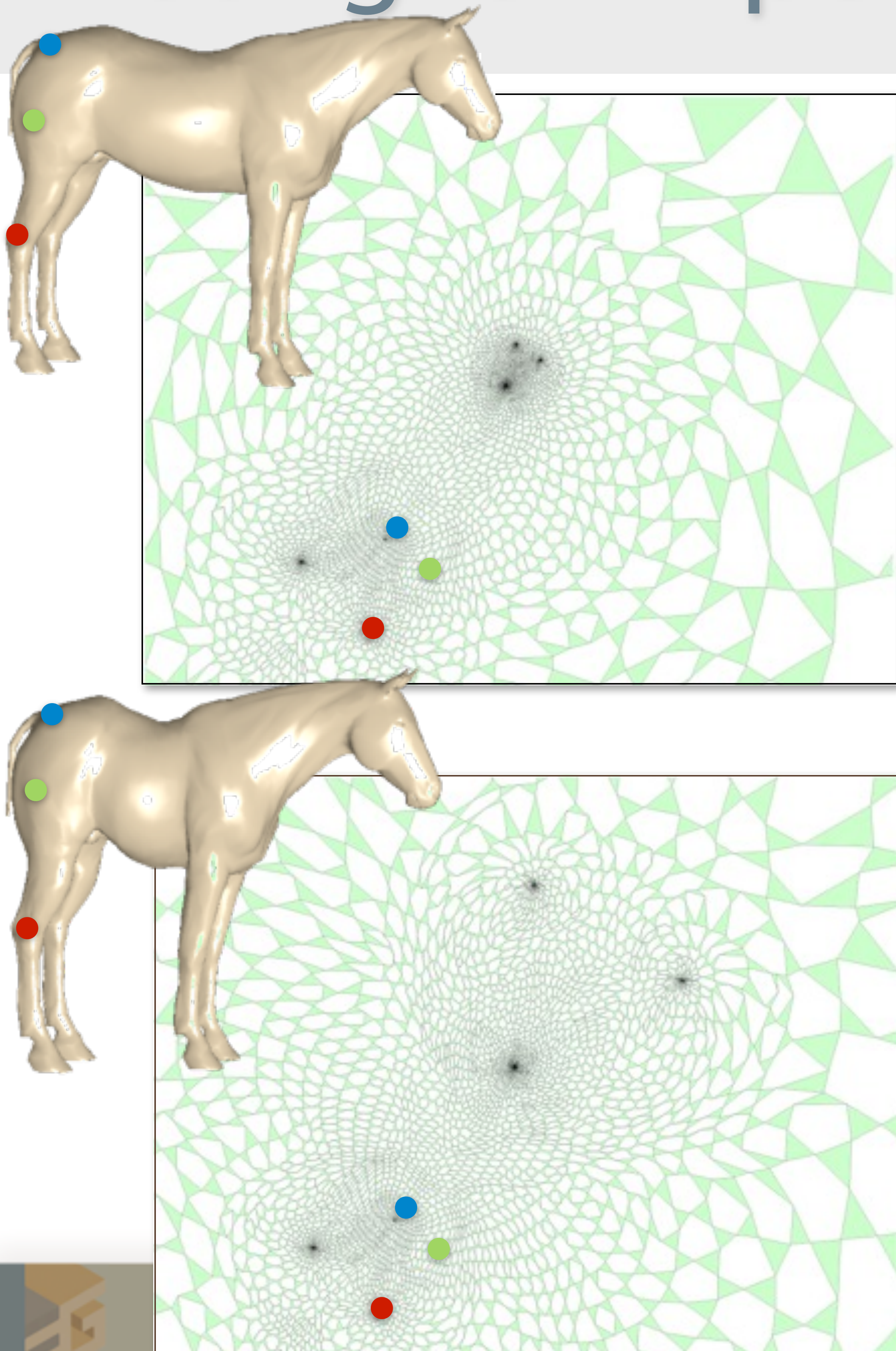


How do we extend to "near isometries?" – with Voting, locality

Voting for Imperfect Isometries

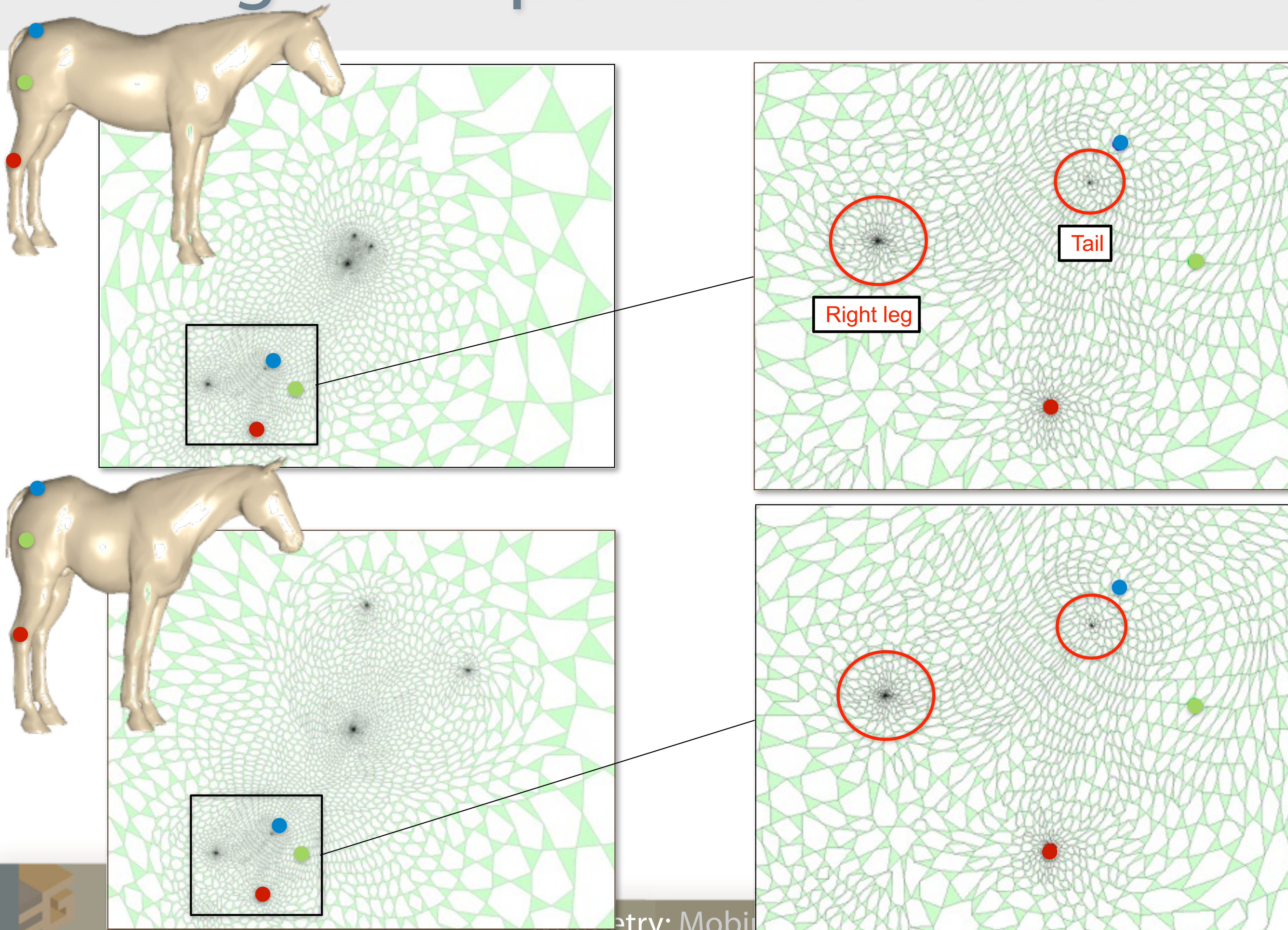


Voting for Imperfect Isometries

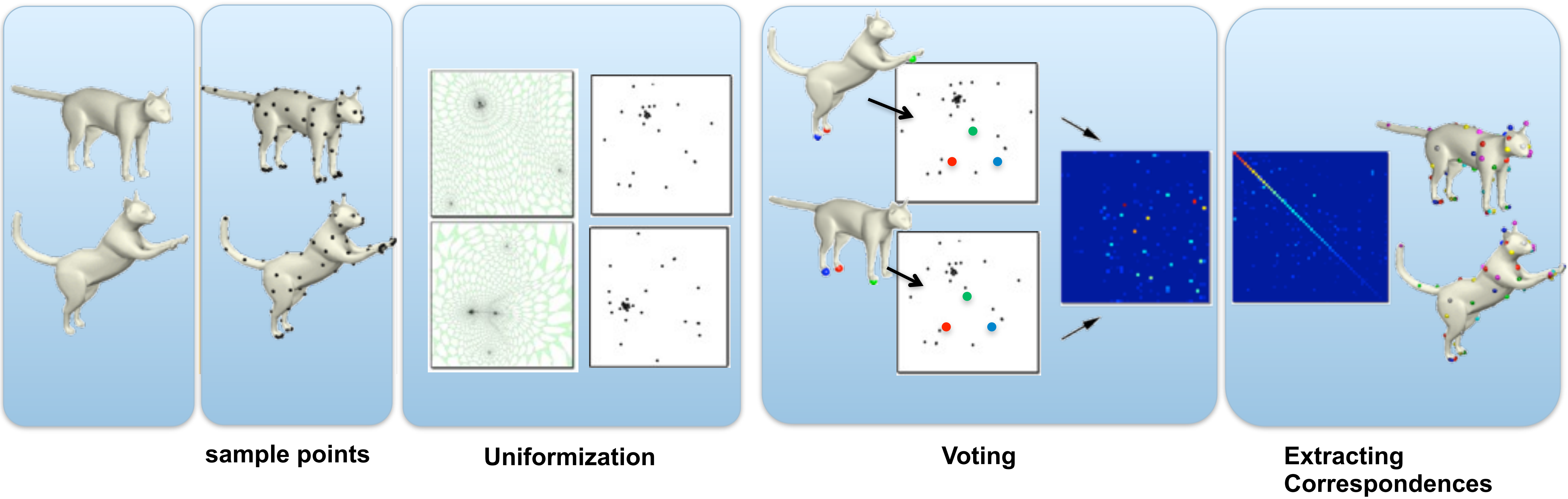


Key: **Uniformization is local**

Voting for Imperfect Isometries



Algorithm Overview



Algorithm Stages

Sampling points

Uniformization

Scoring Votes

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Sampling points

Uniformization

Scoring Votes

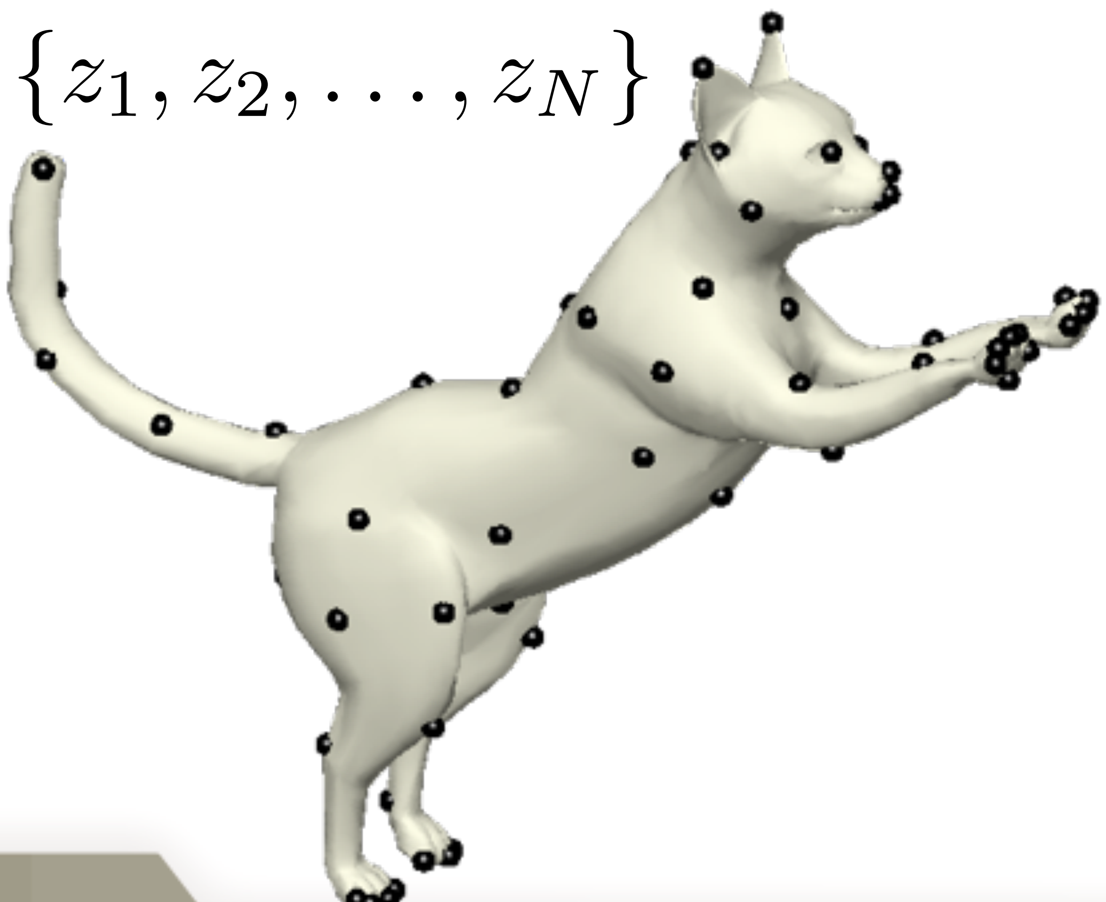
Sampling points

Sample by:

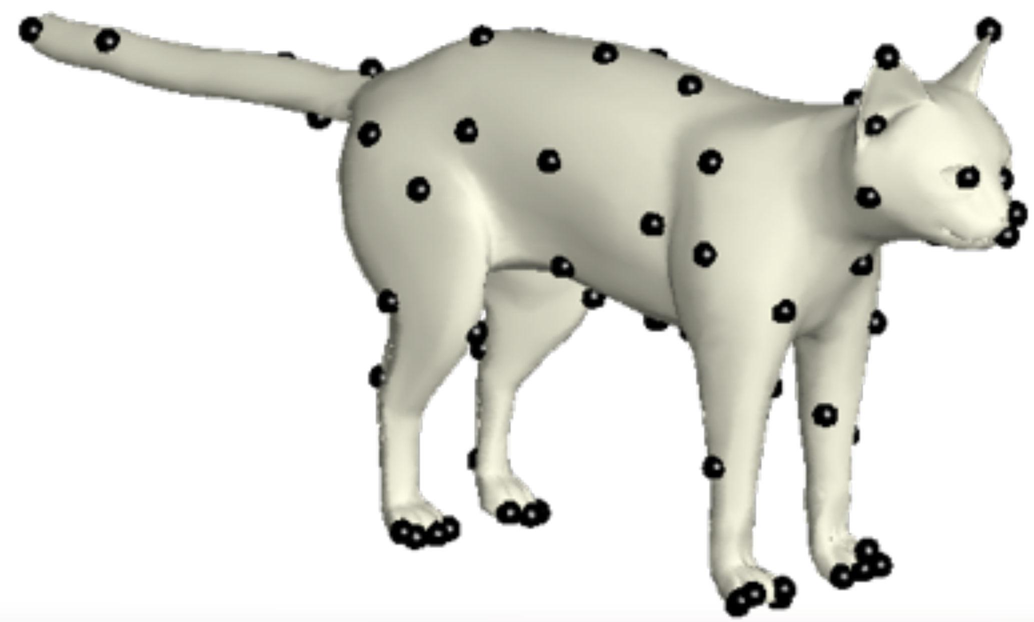
1) Extrema of Gauss curvature (isometry invariant)
2) Uniform samples

Each point represent a surface patch of “equal importance”

$$\{z_1, z_2, \ldots, z_N\}$$



$$\{w_1, w_2, \ldots, w_N\}$$



Algorithm Stages

Sampling points

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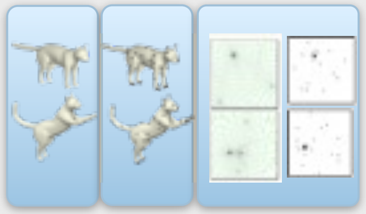
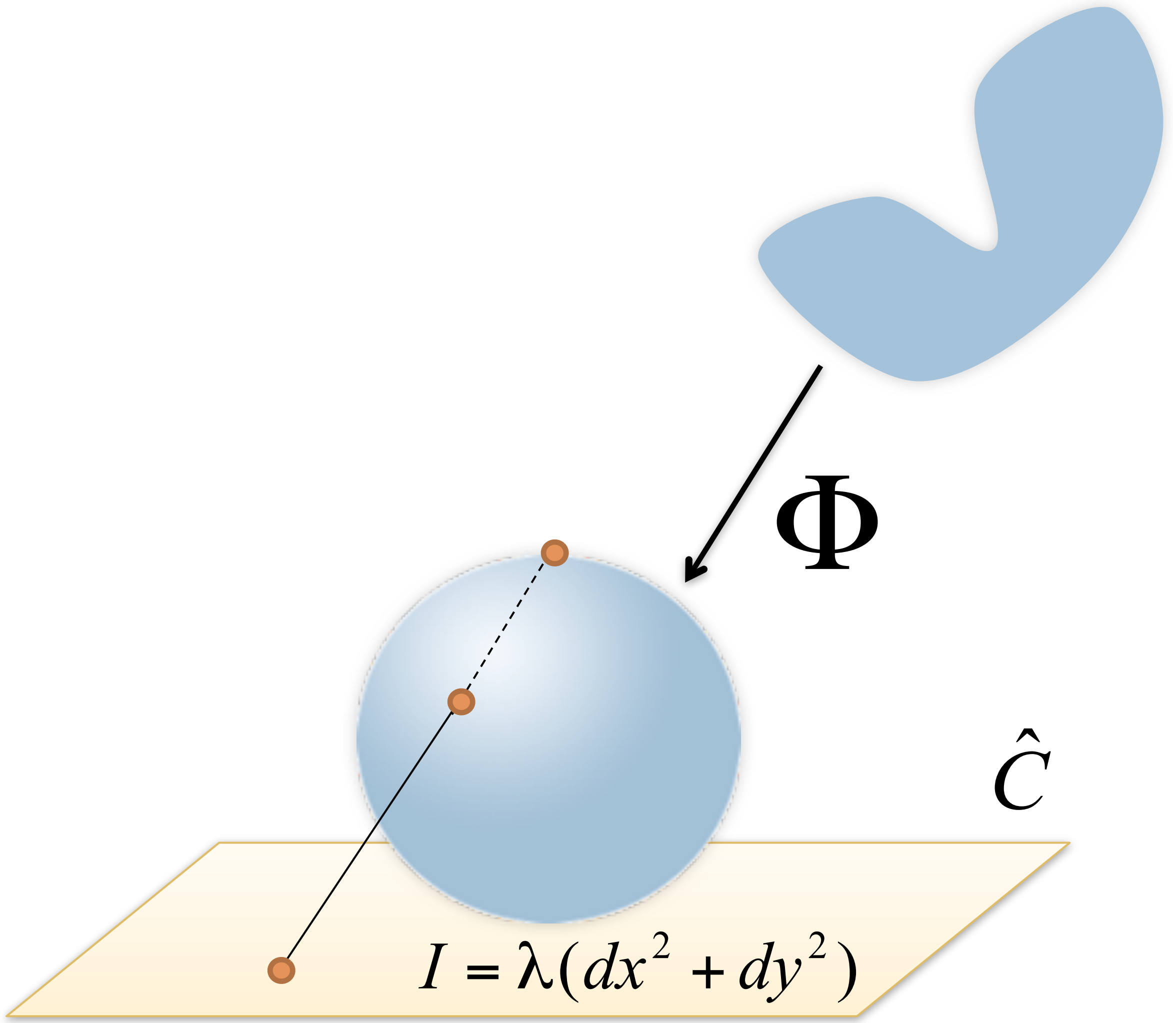
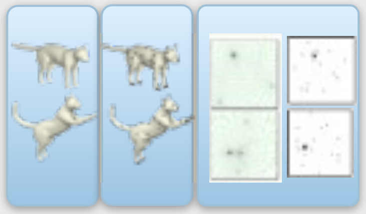
Uniformization



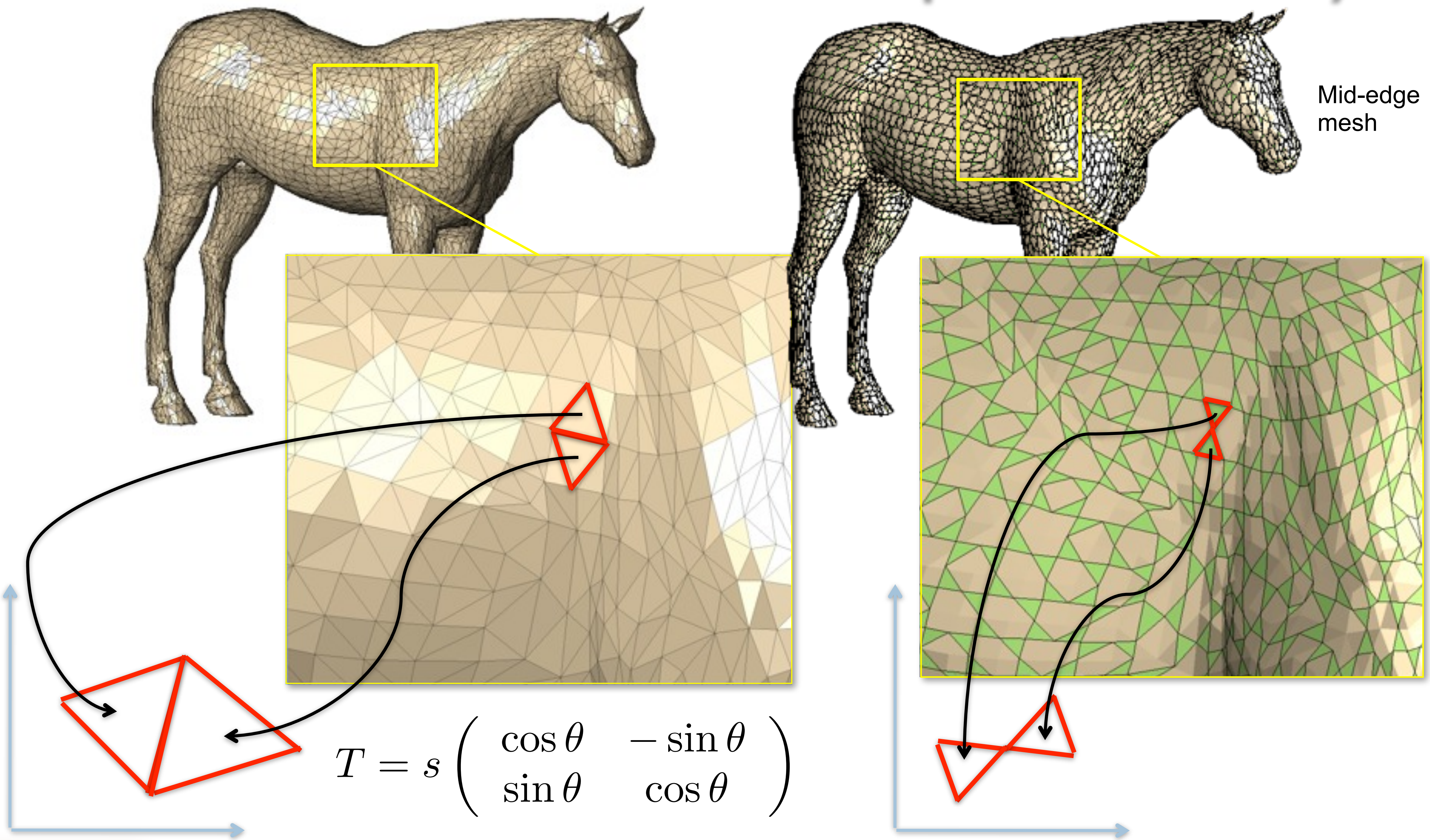
- Map the surface to space where **Möbius is easy** to apply and the metric represented by **density**.
- Every genus-0 surface can be **mapped globally** to a sphere conformally (angle preserving).



Uniformization



Natural definition of discrete conformal: piecewise similarity

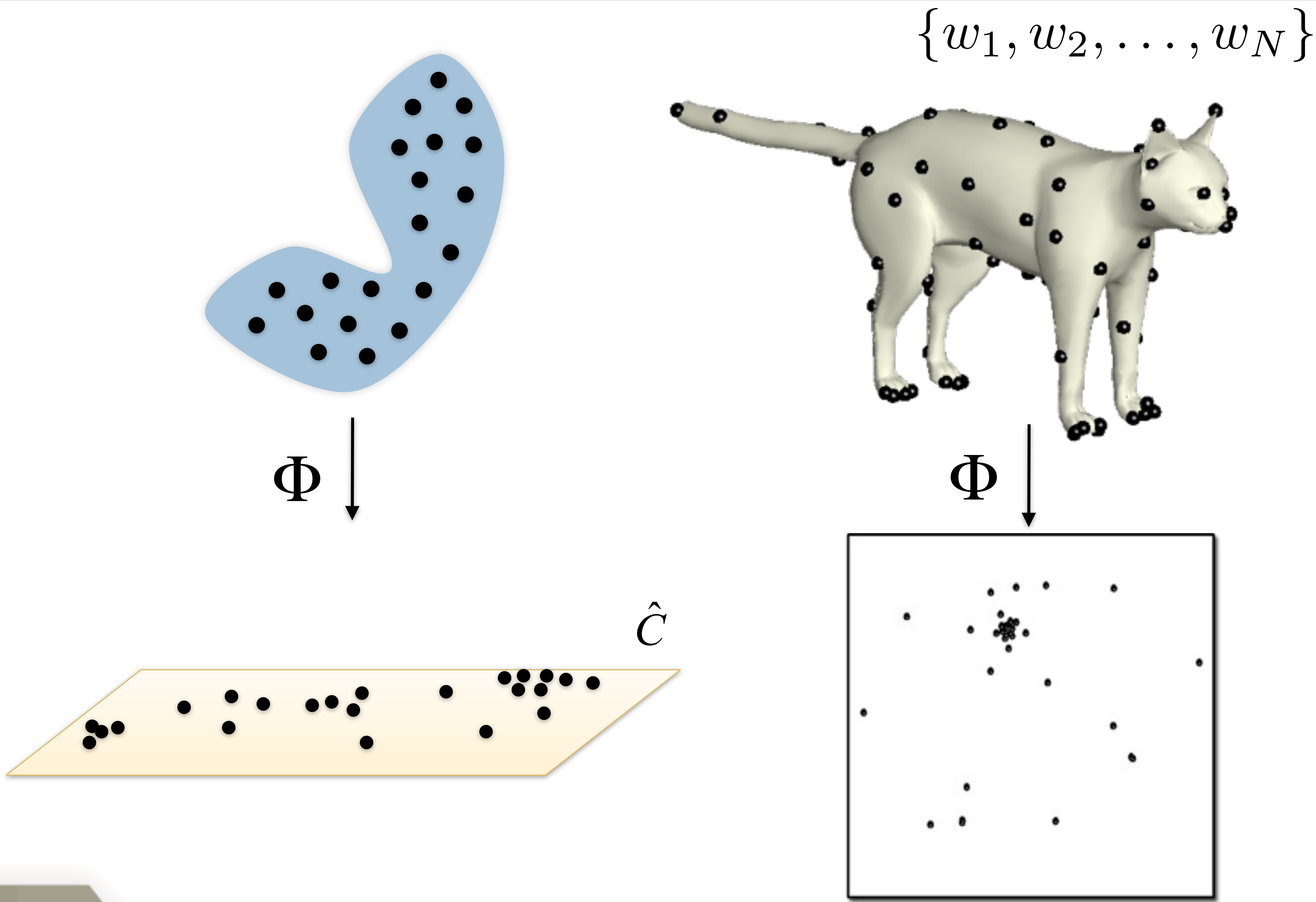


$$T = s \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

Too many constraints: generally **not possible**.

possible: using Pinkall & Polthier [93] conjugate discrete harmonics.

Uniformization



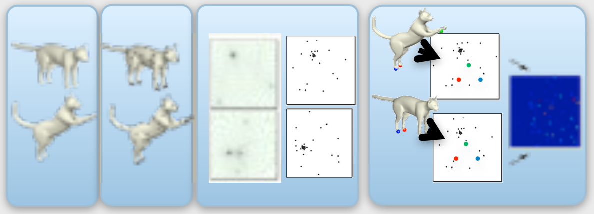
Algorithm Stages

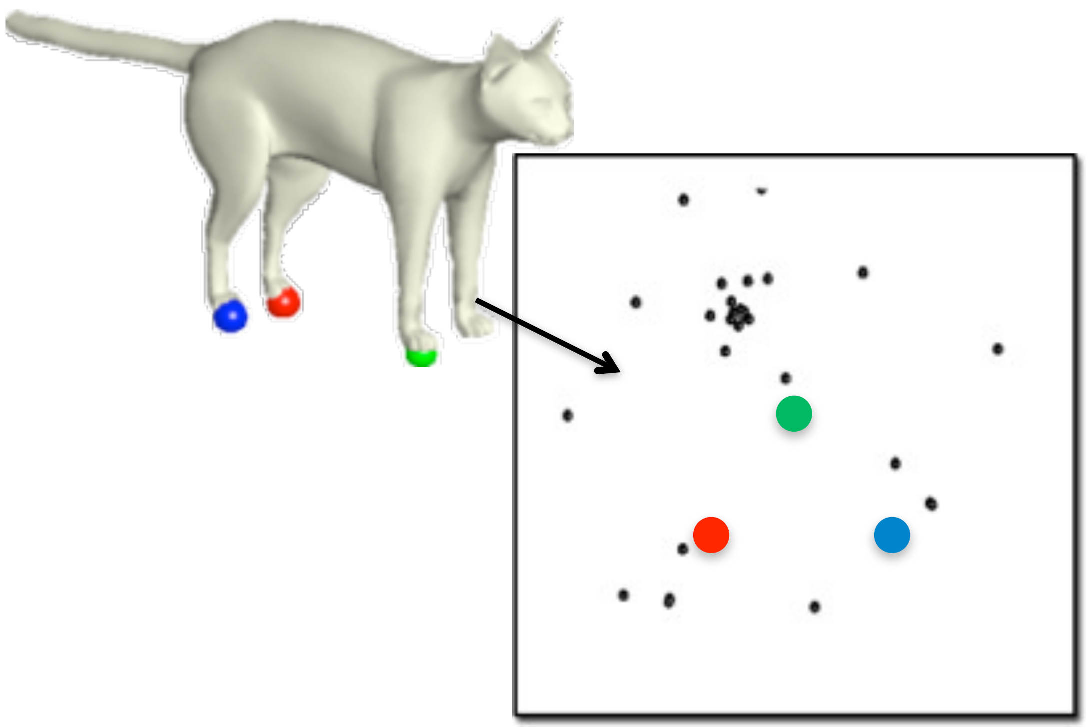
Sampling points

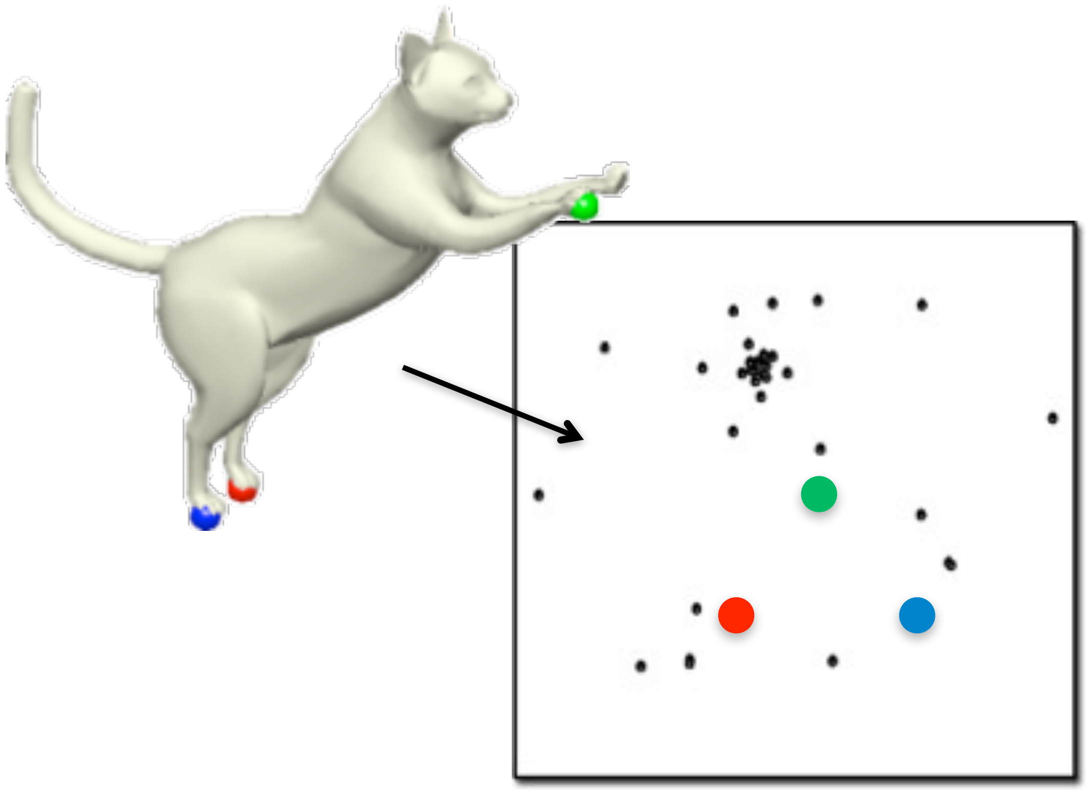
Uniformization

Scoring Votes

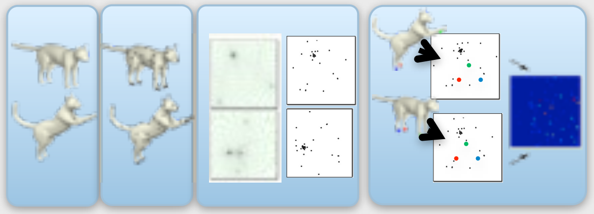
Scoring votes



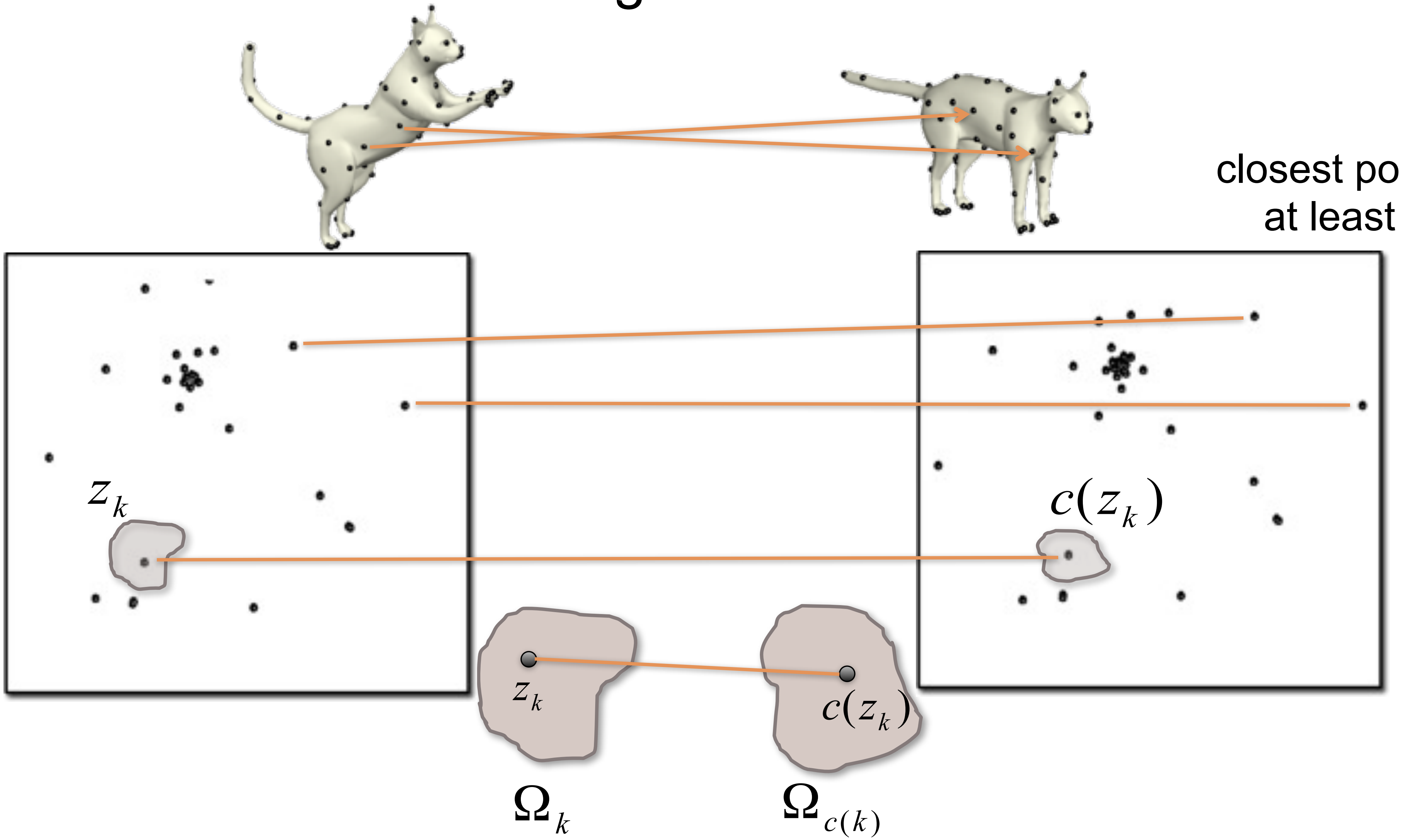




Scoring Votes



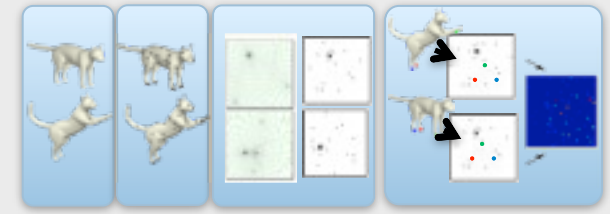
measuring deformation error

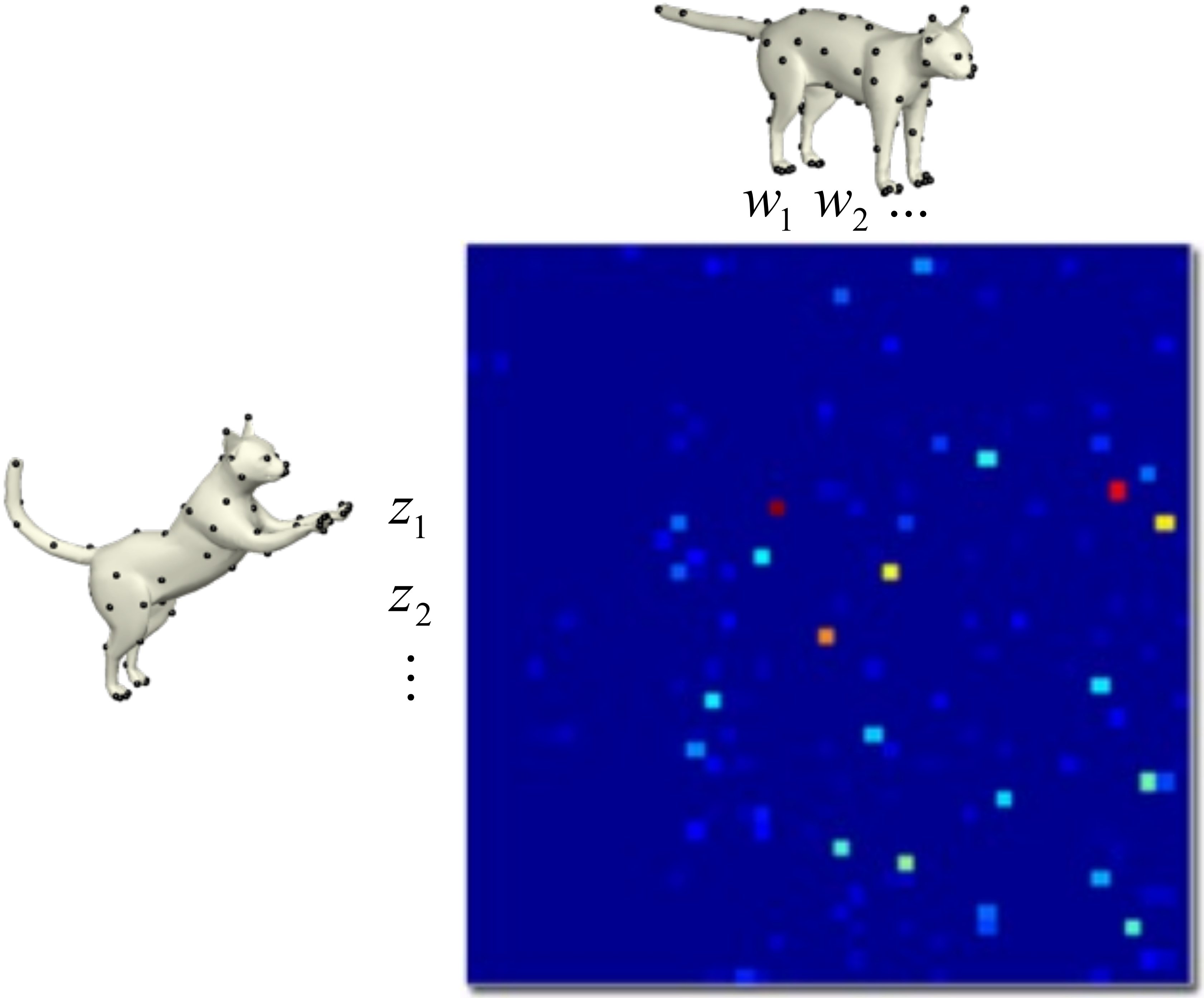


The vote value is the transportation "effort":

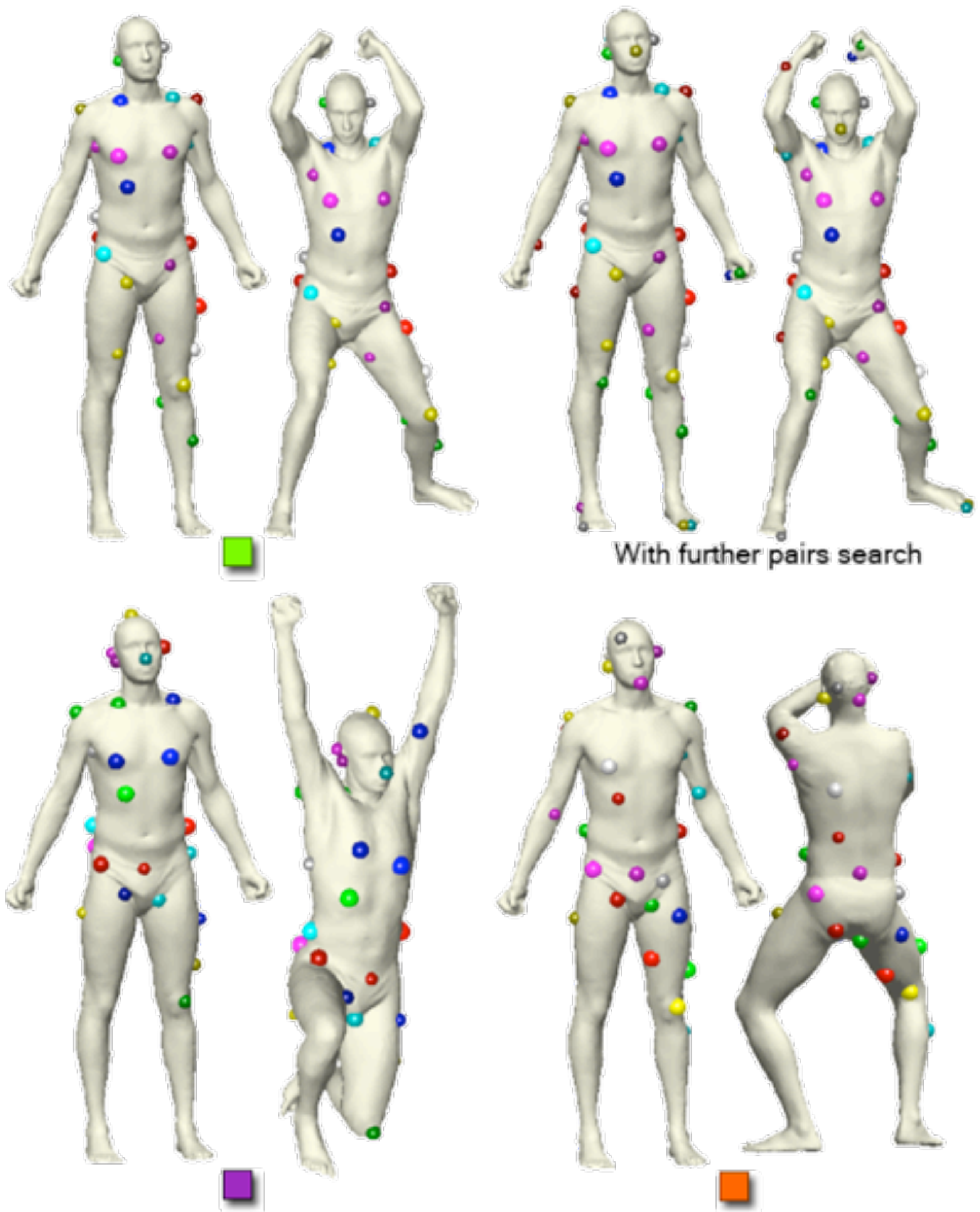
$$E(c) = \int_{\mathbb{C}} d(z, c(z))d\lambda \approx \sum_k d(z_k, c(z_k))\text{area}(\Omega_k) \qquad d(z,w) = \frac{|z-w|}{|1+\bar{z}w|}$$

Scoring Votes

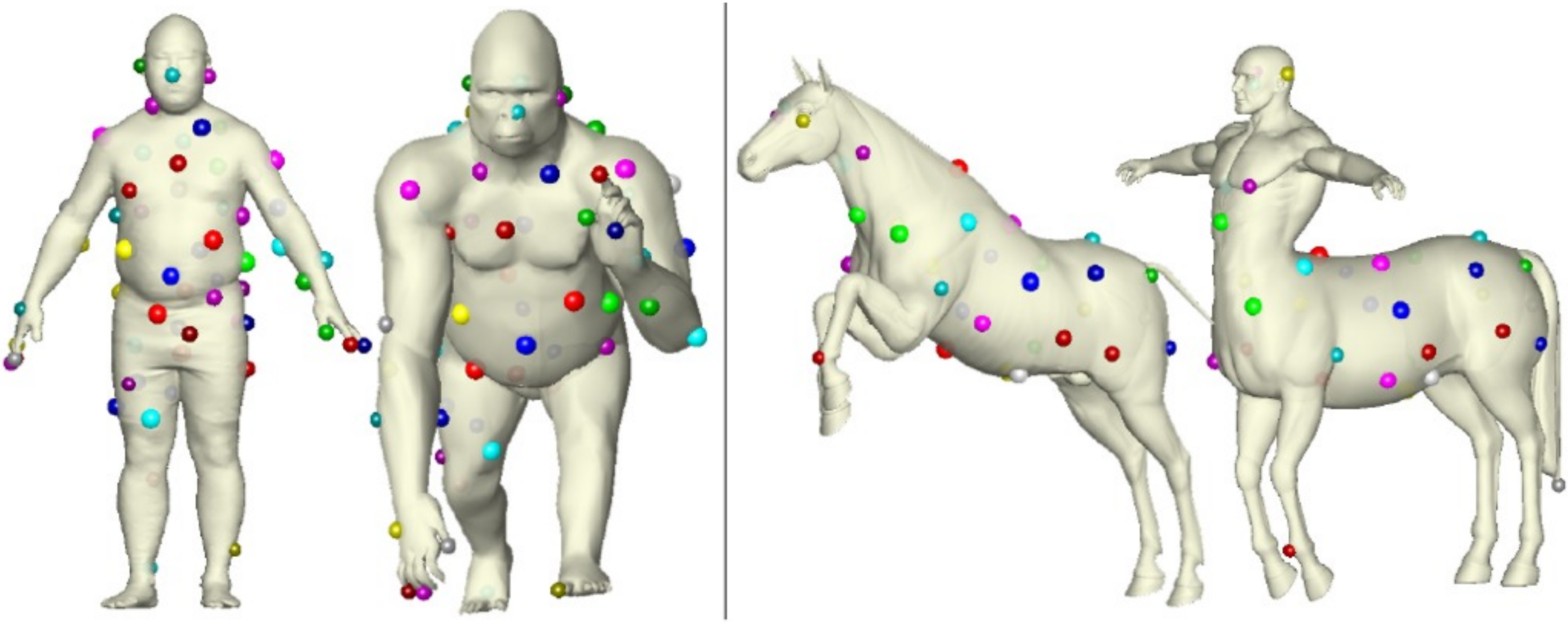




Results



Cross Correspondence

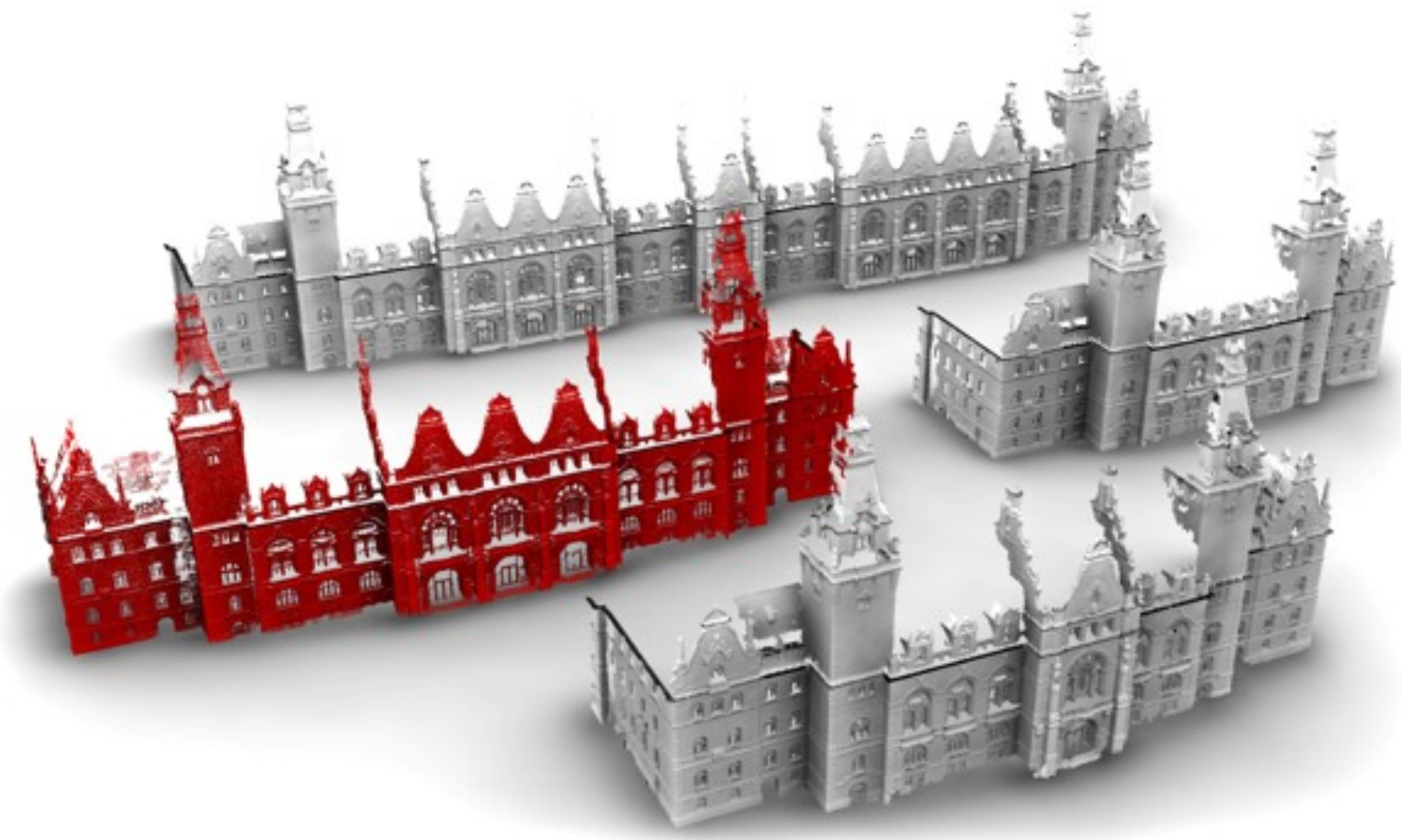


Reference

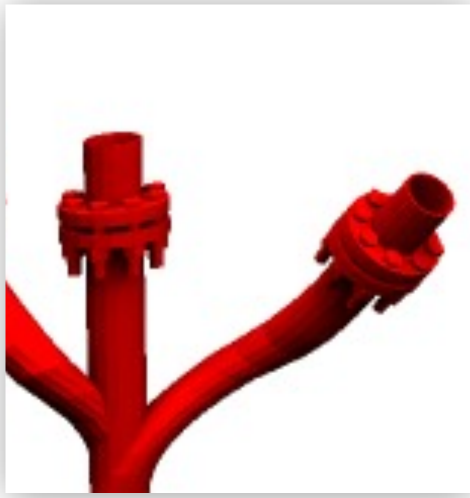
Mobius Voting for Surface Correspondence, Yaron Lipman, Thomas Funkhouser, SIGGRAPH 2009.

Applications

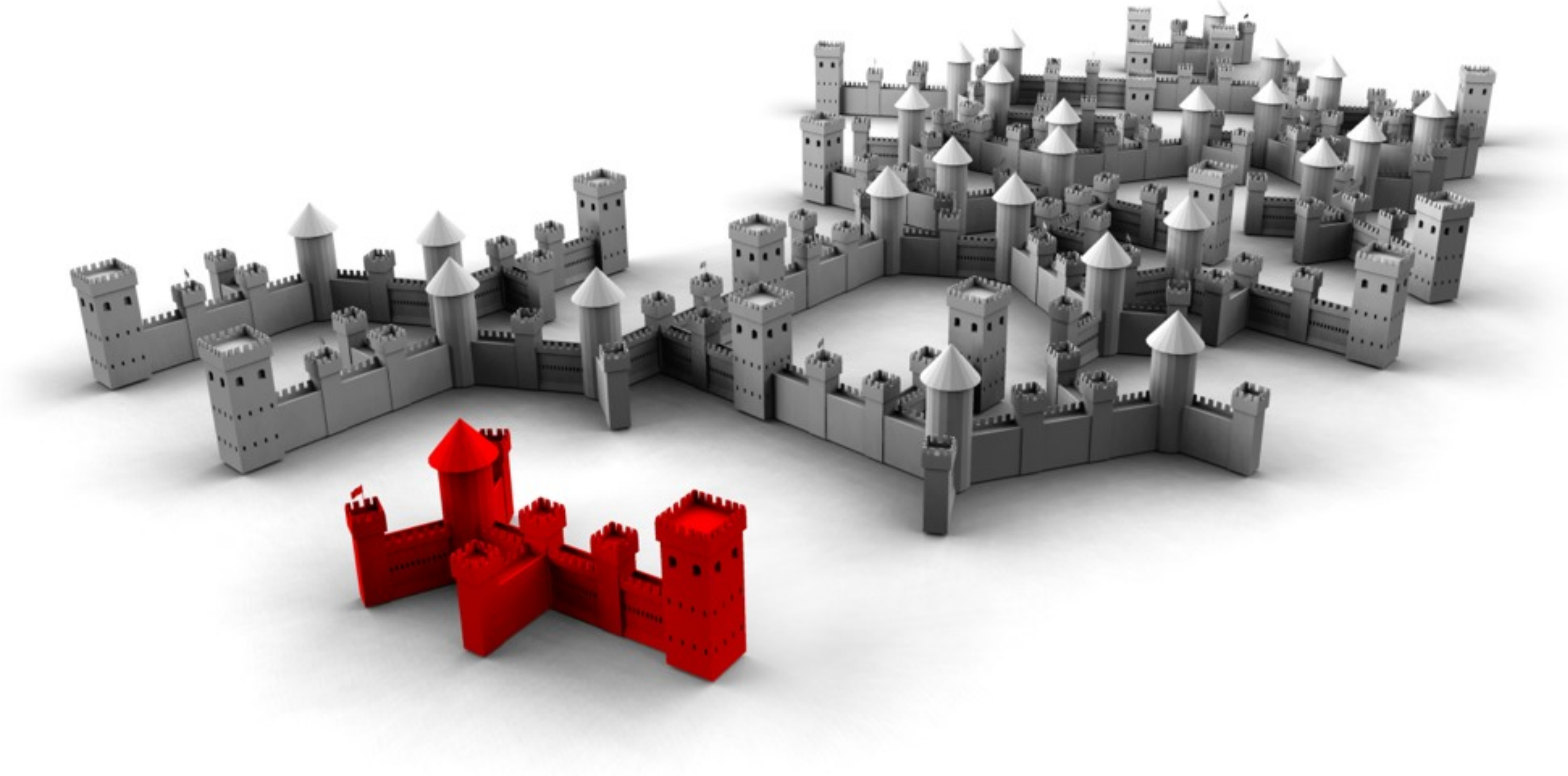
Symmetry Detection and Applications



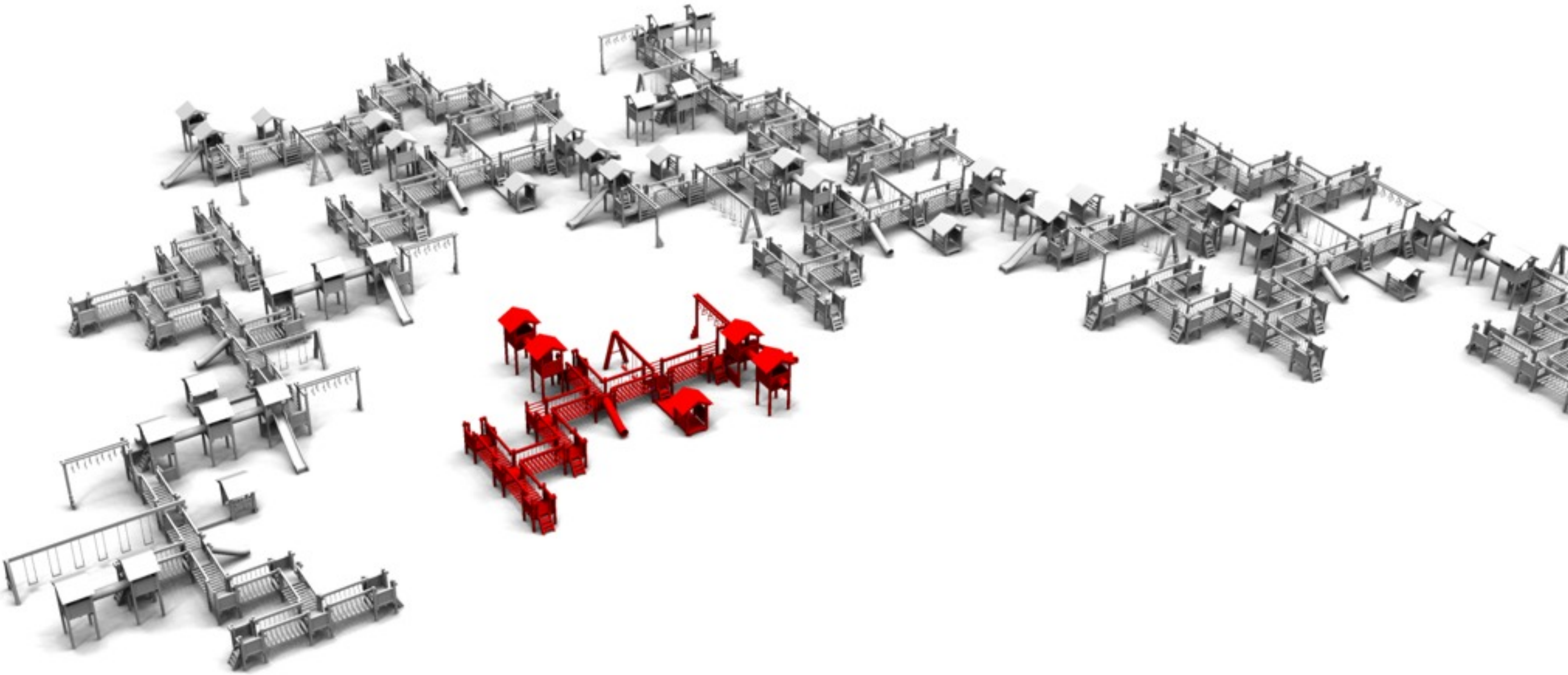
Pipe Tree



Random (Castle) Variations

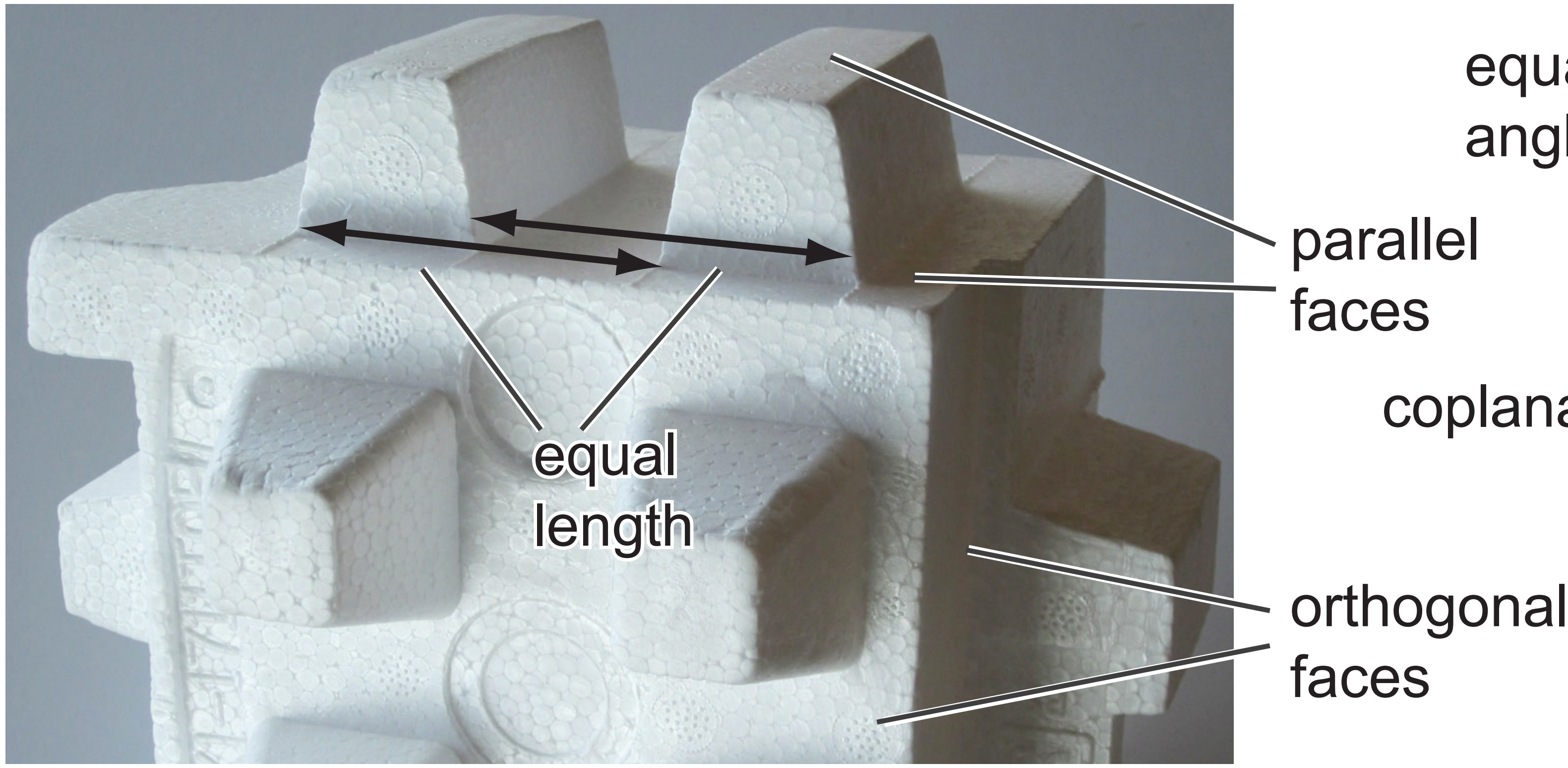


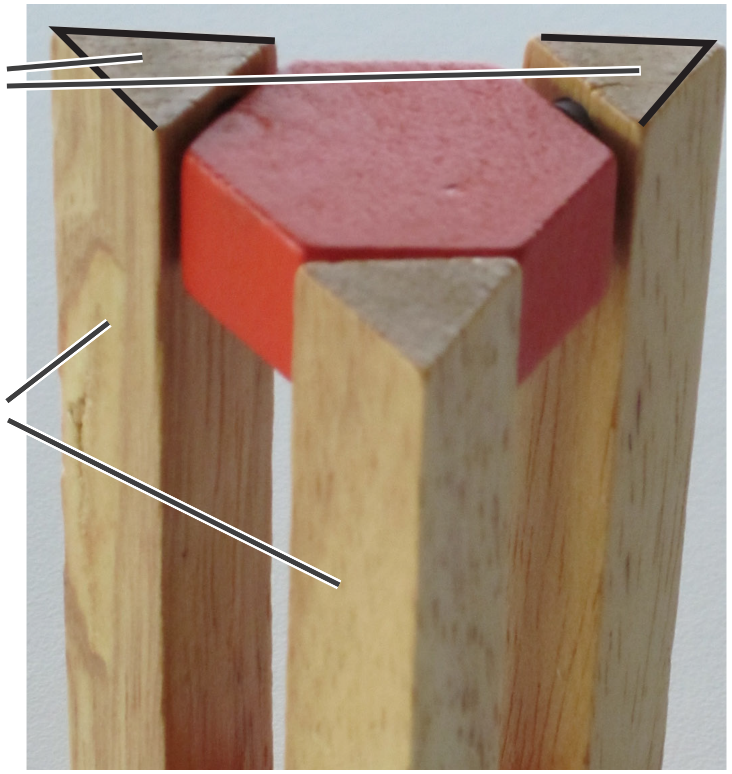
Random (Playground) Variations



Bus Stop Variations

Relations in Man-made Objects



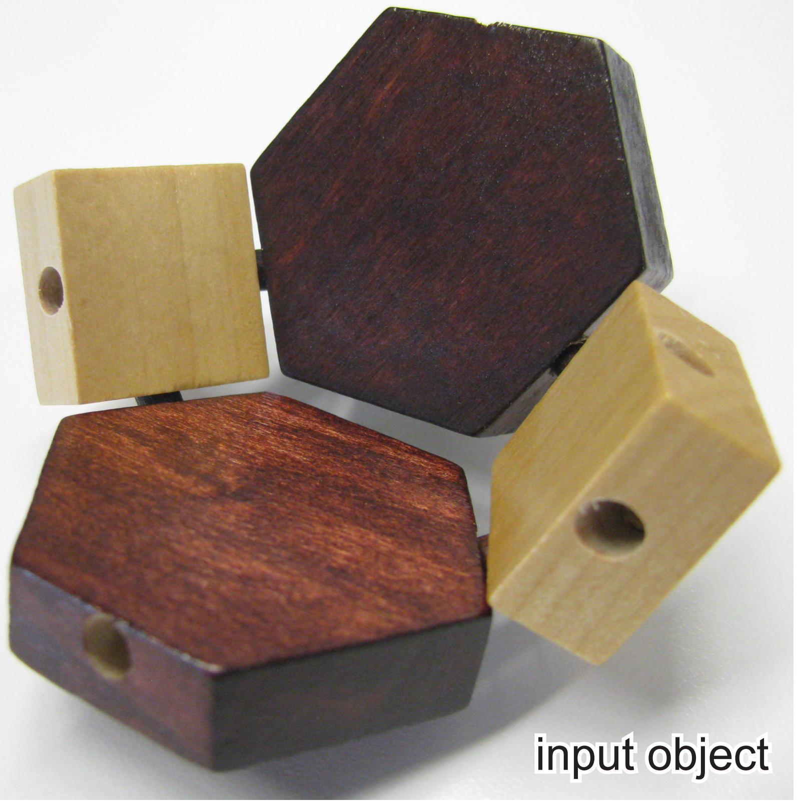


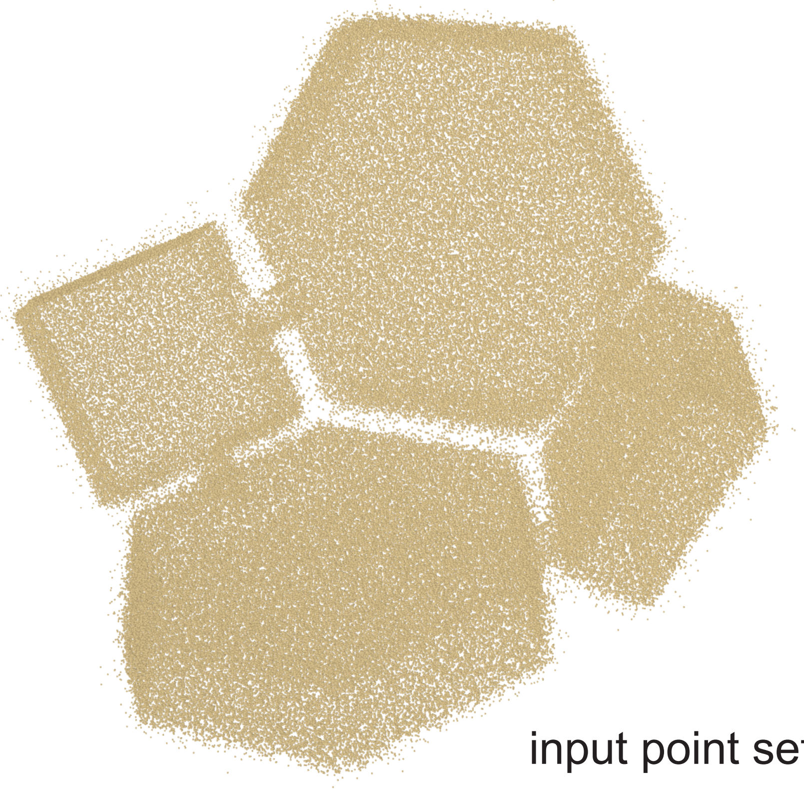
i) orthogonal/parallel relations; equal angle

ii) placement relation, e.g., coplanar, coaxial

iii) equal length/radii relations

Parallel/Orthogonal Relations



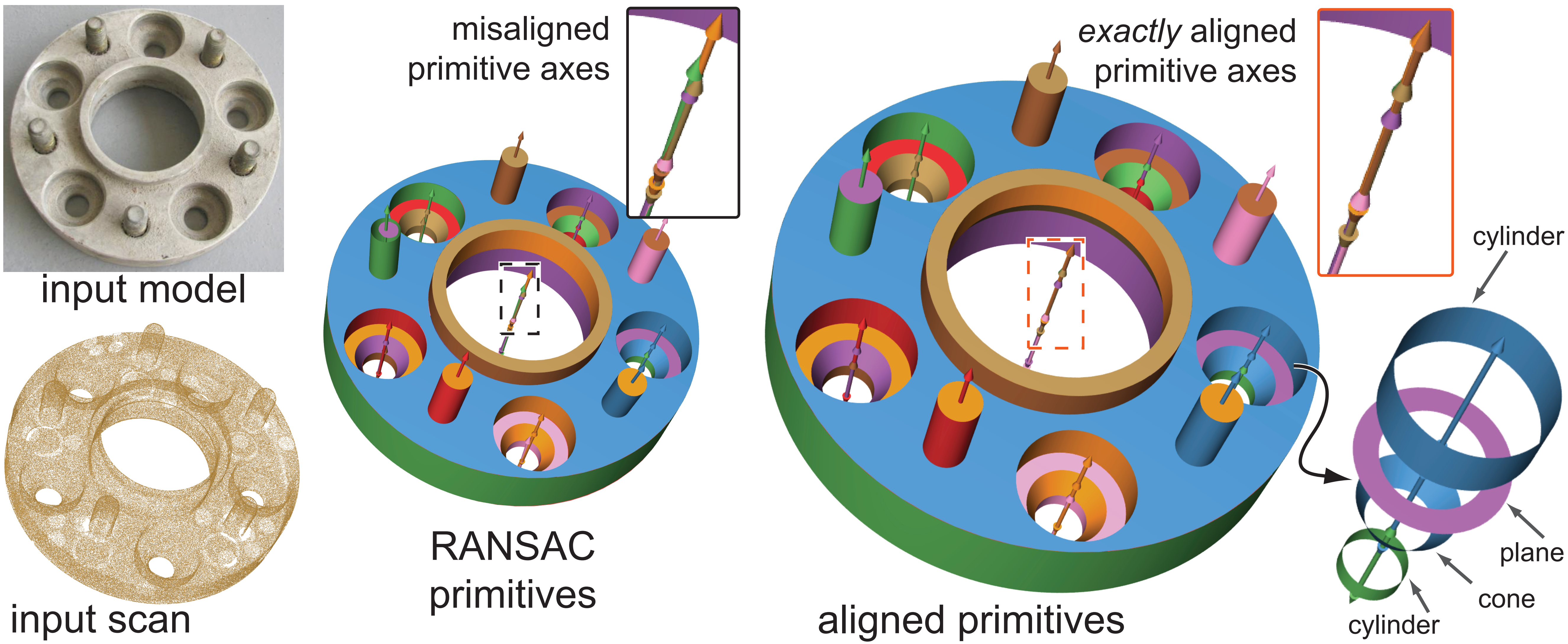


$$C_o = \{c_1, c_2, \ldots\}$$

$$C_o^* \subset C_o$$

Equal Angle Relations

Wheel Dataset



References

A Connection between Partial Symmetry and Inverse Procedural Modeling,
Martin Bokeloh, Michael Wand, Hans-Peter Seidel,
SIGGRAPH 2010.

GlobFit: Consistently Fitting Primitives by Discovering Global Relations,
Yangyan Li, Xiaokun Wu, Yiorgos Chrysanthou, Andrei Sharf, Daniel Cohen-Or, Niloy J. Mitra,
SIGGRAPH 2011 (conditional accept).