Eurographics 2012
Cagliari, Italy
May 13-18
33rd ANNUAL CONFERENCE OF THE EUROPEAN ASSOCIATION FOR COMPUTER GRAPHICS

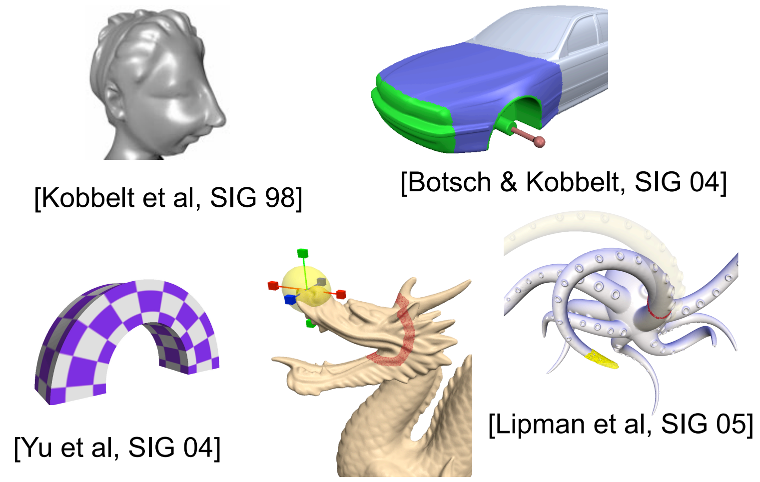
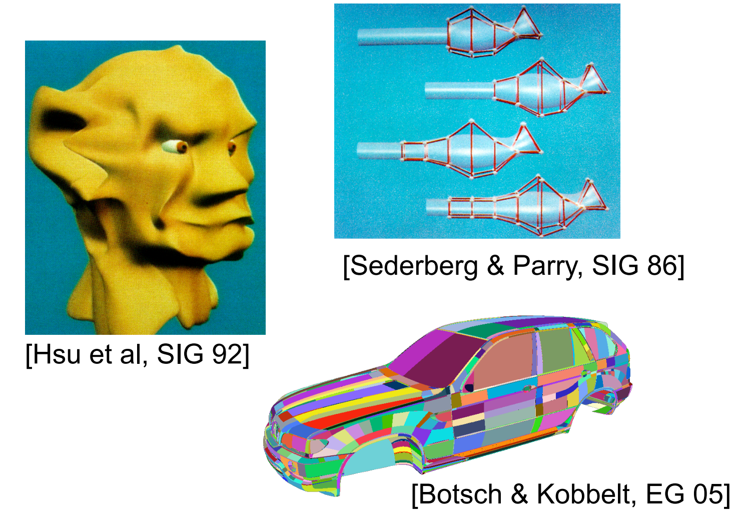
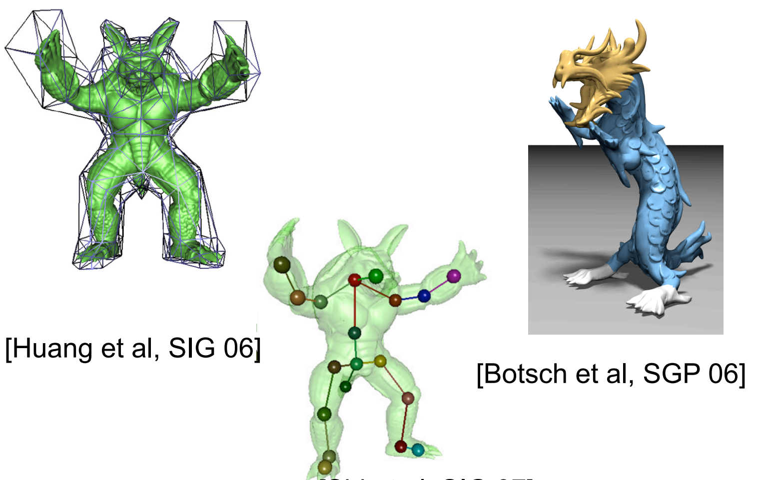
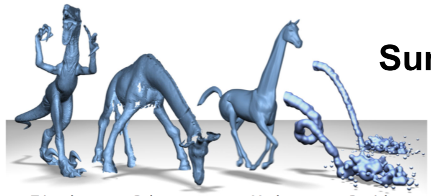
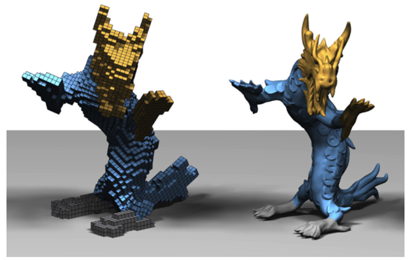
Deformation Graphs

Mark Pauly
ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

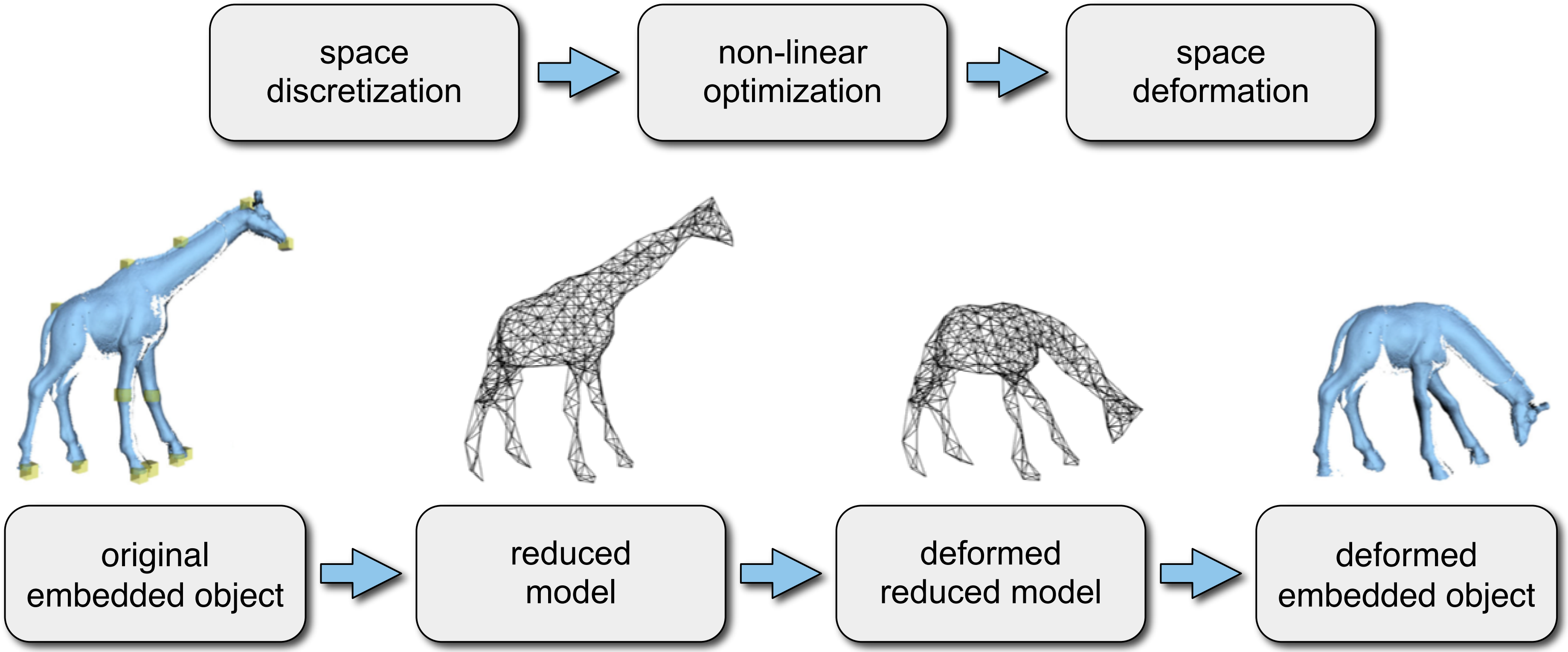
Deformation Models

Desirable Properties

- Generality
  - handle different geometry representations
  - predictable, physically plausible deformation behavior
- Efficiency, scalability
  - processing of large data sets, realtime feedback
- Robustness
  - stable even for bad input and drastic deformations
- Simplicity
  - ease of implementation
  - adaptability, extensibility, re-use

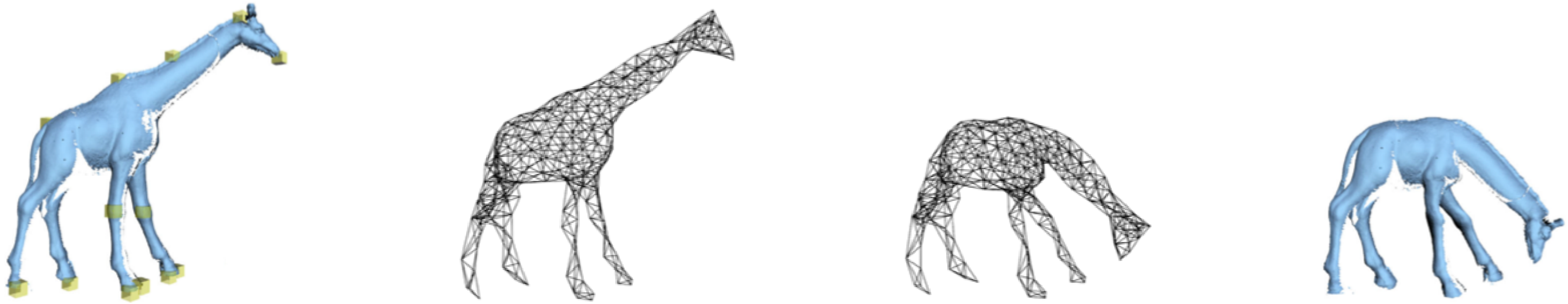
	Surface Deformation	Space Deformation
Linear	 [Kobbelt et al, SIG 98] [Botsch & Kobbelt, SIG 04] [Yu et al, SIG 04] [Sorkine et al, SGP 04] [Lipman et al, SIG 05]	 [Hsu et al, SIG 92] [Sederberg & Parry, SIG 86] [Botsch & Kobbelt, EG 05]
Nonlinear	 [Huang et al, SIG 06] [Shi et al, SIG 07] [Botsch et al, SGP 06]	 **Sumner et al, SIG 07**  Botsch et al, EG 07

Deformation Pipeline



Deformation Pipeline

space discretization → non-linear optimization → space deformation



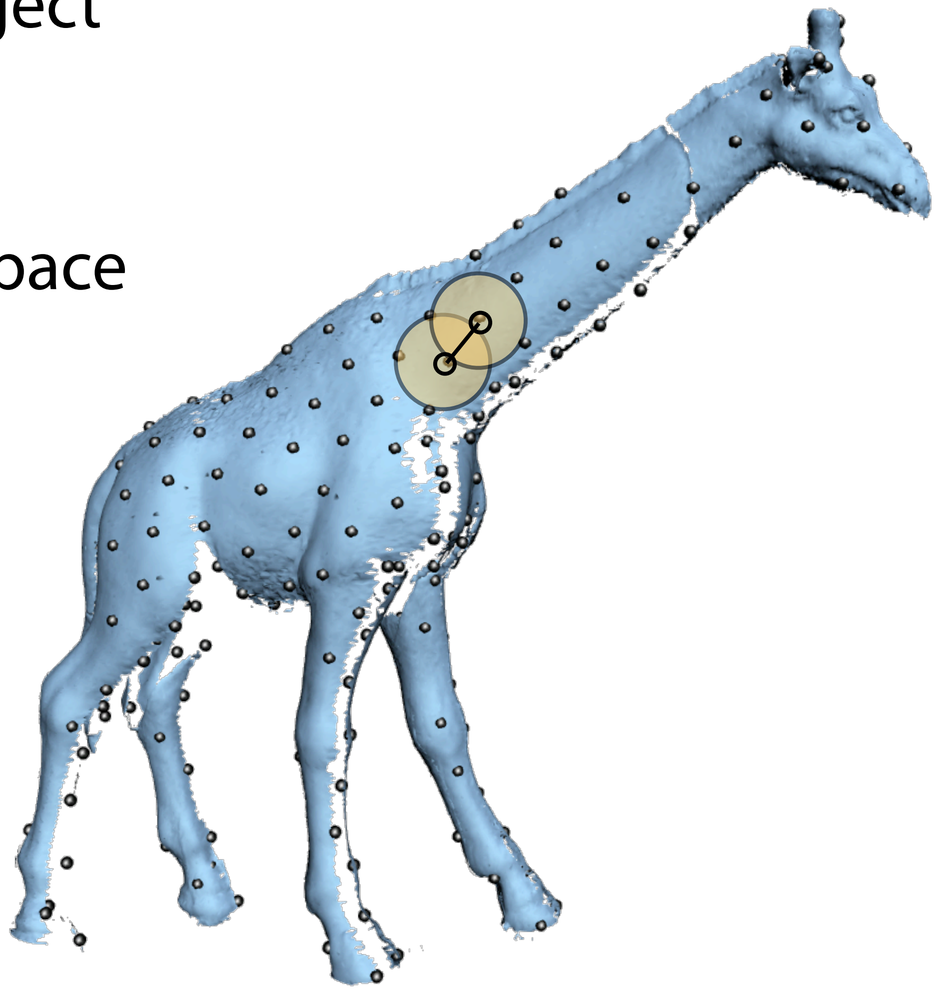
original embedded object → reduced model → deformed reduced model → deformed embedded object

Space Discretization

- Begin with an embedded object

Space Discretization

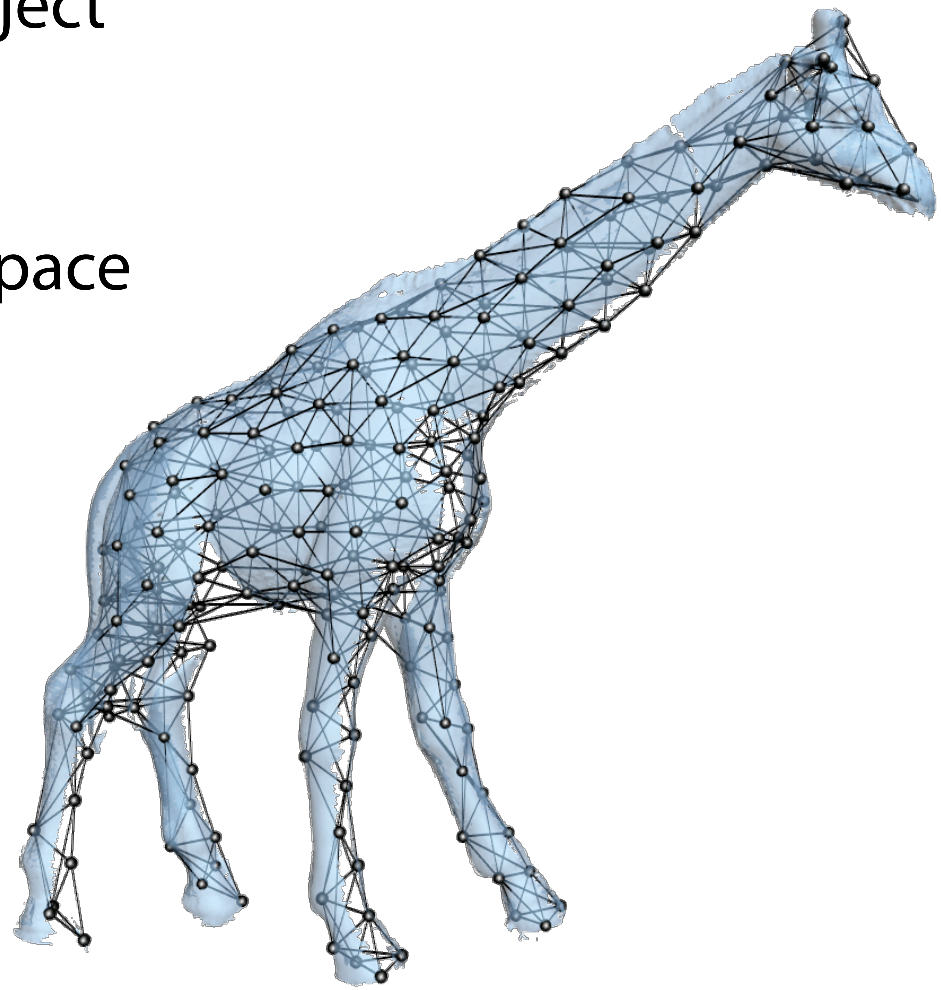
- Begin with an embedded object
- Sample the object
- Each node deforms nearby space
- Edges connect nodes of overlapping influence



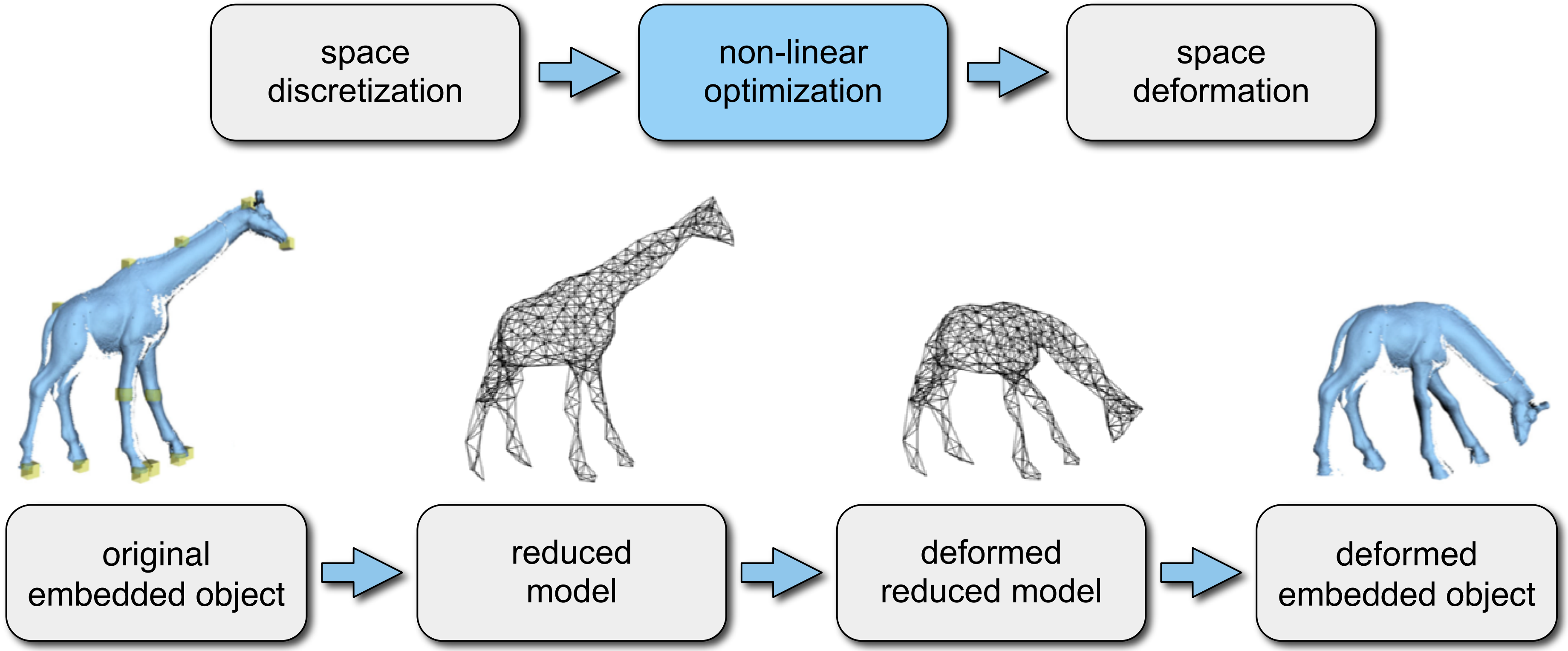
Space Discretization

- Begin with an embedded object
- Sample the object
- Each node deforms nearby space
- Edges connect nodes of overlapping influence

Deformation Graph

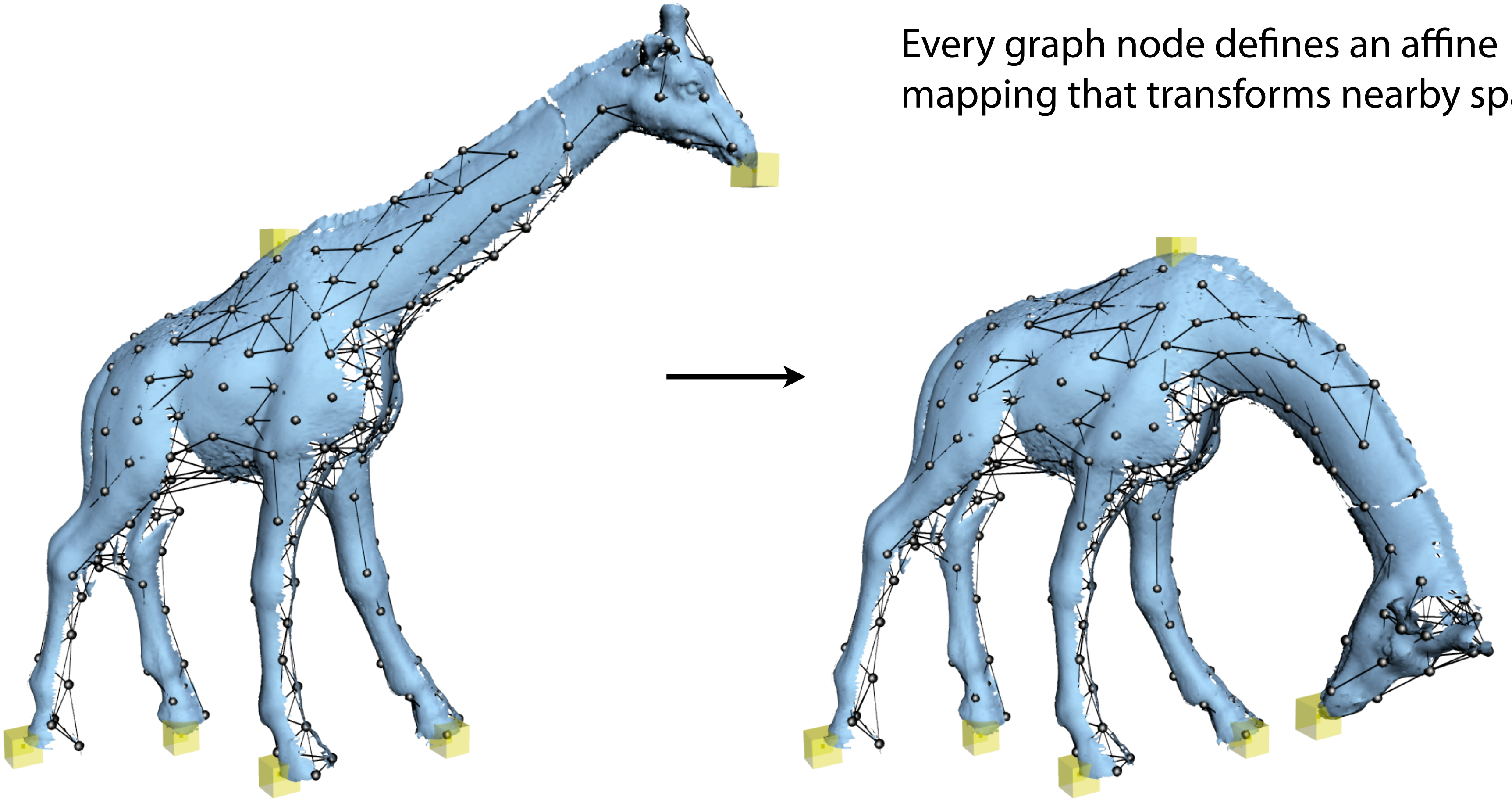


Deformation Pipeline



Optimization

Every graph node defines an affine mapping that transforms nearby space



Optimization

Energy formulation

$$\min_{\mathbf{R}_1,\mathbf{t}_1\ldots\mathbf{R}_m,\mathbf{t}_m} w_{\mathrm{rot}}\mathrm{E}_{\mathrm{rot}} + w_{\mathrm{reg}}\mathrm{E}_{\mathrm{reg}} + w_{\mathrm{con}}\mathrm{E}_{\mathrm{con}}$$

Graph parameters

Rotation term

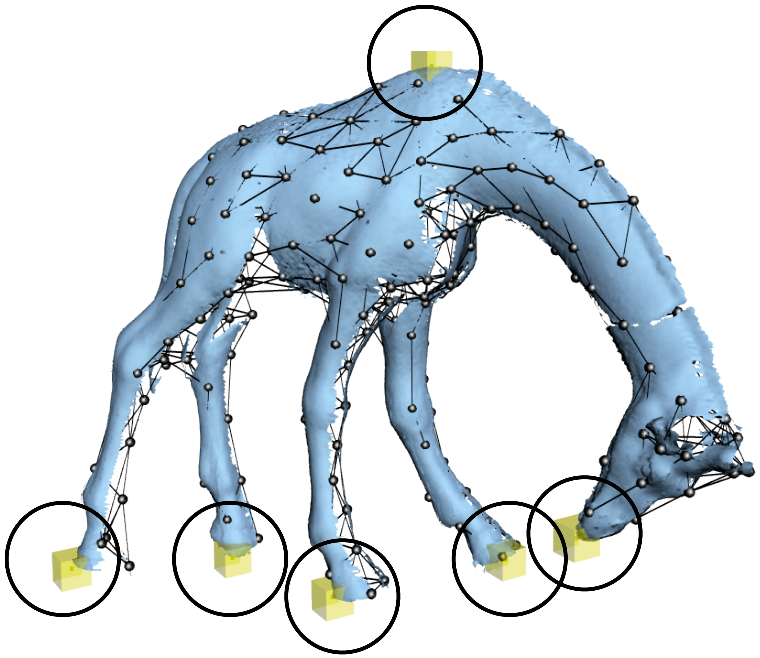
Regularization term

Constraint term

Optimization

Constraint term

$$\mathrm{E_{con}} = \sum_{l=1}^{p} \left\| \tilde{\mathbf{v}}_{\mathrm{index}(l)} - \mathbf{q}_l \right\|_2^2$$

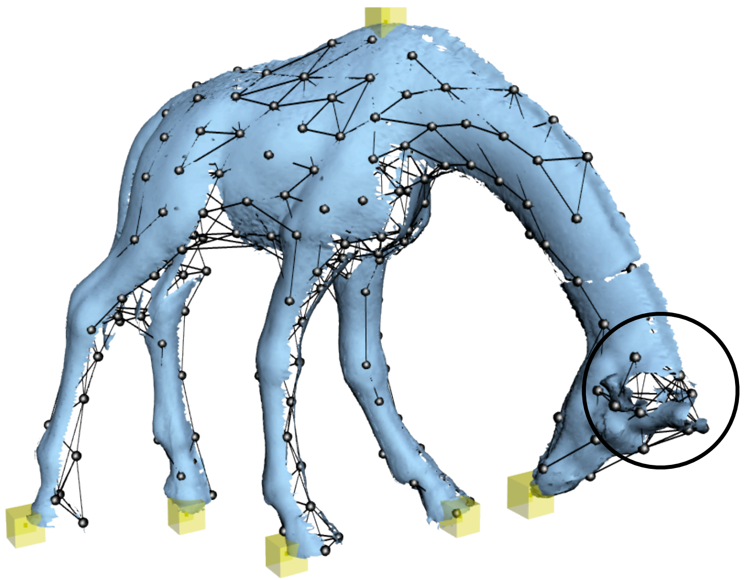


Constrained vertices should move according to correspondences

Optimization

Rotation term

$$\mathrm{Rot}(\mathbf{R}) = (\mathbf{c}_1 \cdot \mathbf{c}_2)^2 + (\mathbf{c}_1 \cdot \mathbf{c}_3)^2 + (\mathbf{c}_2 \cdot \mathbf{c}_3)^2 + (\mathbf{c}_1 \cdot \mathbf{c}_1 - 1)^2 + (\mathbf{c}_2 \cdot \mathbf{c}_2 - 1)^2 + (\mathbf{c}_3 \cdot \mathbf{c}_3 - 1)^2$$



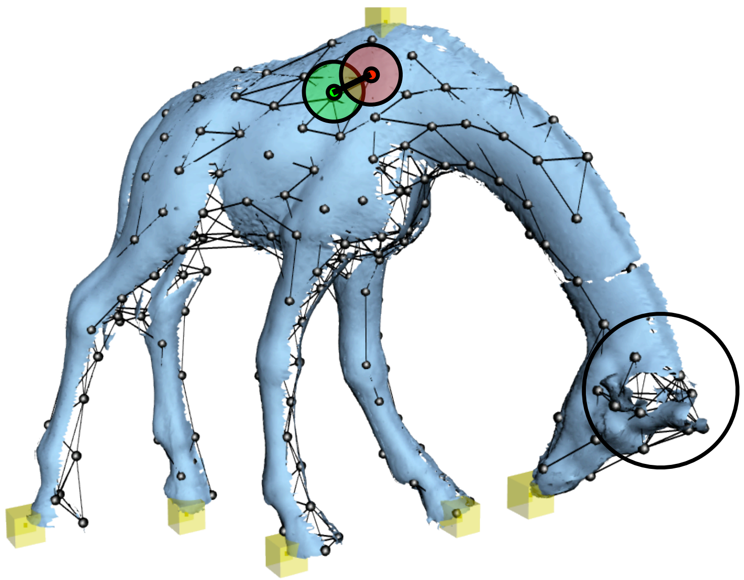
$$\mathrm{E}_{\mathrm{rot}} = \sum_{j=1}^{m} \mathrm{Rot}(\mathbf{R}_j)$$

For detail preservation,
features should rotate and
not scale or skew.

Optimization

Regularization term

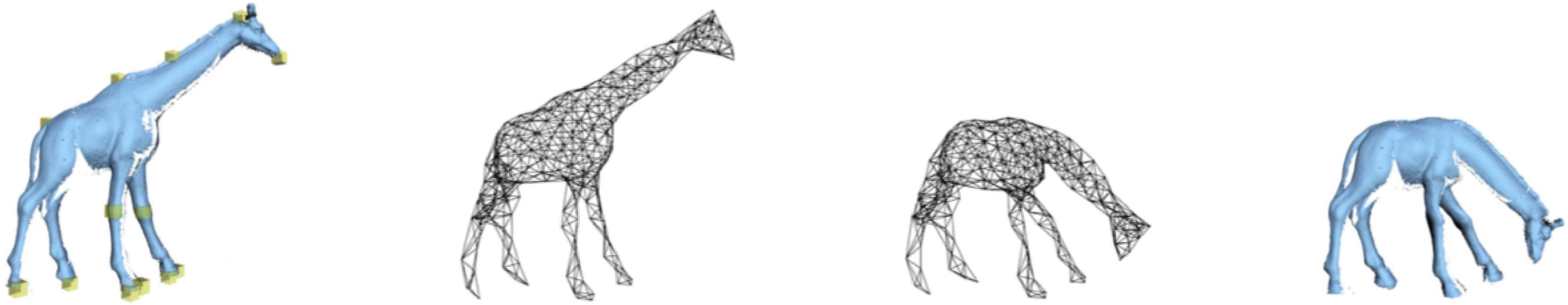
$$E_{\text{reg}} = \sum_{j=1}^{m} \sum_{k \in \mathcal{N}(j)} \alpha_{jk} \left\| \underbrace{\mathbf{R}_j(\mathbf{g}_k - \mathbf{g}_j) + \mathbf{g}_j + \mathbf{t}_j}_{\text{where node } j \text{ thinks node } k \text{ should go}} - \underbrace{(\mathbf{g}_k + \mathbf{t}_k)}_{\text{where node } k \text{ actually goes}} \right\|_2^2$$



Neighboring nodes should agree on where they transform each other.

Deformation Pipeline

space discretization → non-linear optimization → space deformation



original embedded object → reduced model → deformed reduced model → deformed embedded object

Space Deformation: Approach 1

Simple averaging of transformations

For each point $\mathbf{p}$

- Pick k closest graph nodes $\{\mathbf{T}_1, \mathbf{T}_2, \ldots, \mathbf{T}_k\}$ (k=4)
- Build weighted average of transformations

$$\mathbf{p} \mapsto \sum_{i=1}^{k} w_i \mathbf{T}_i(\mathbf{p})$$

Space Deformation: Approach 2

Each graph node yields displacement

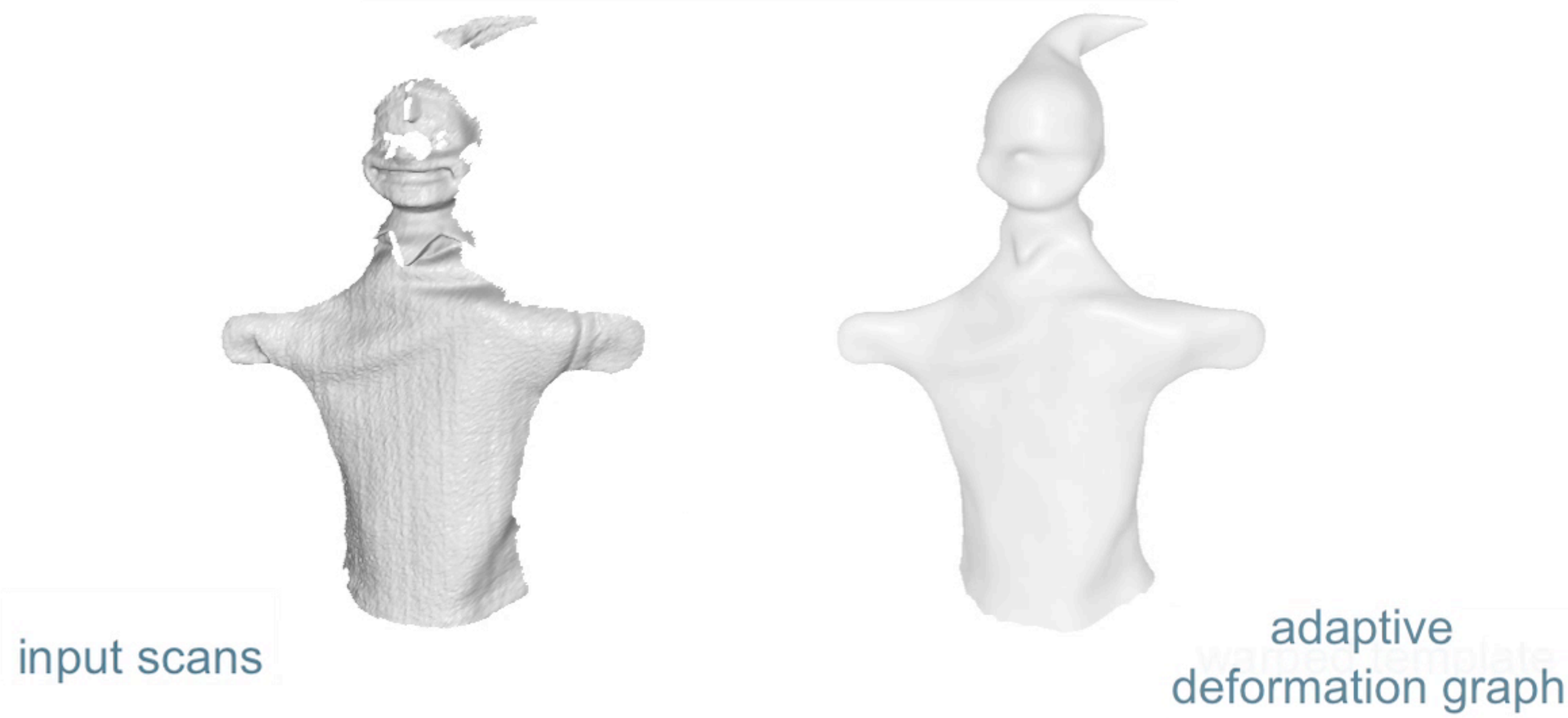
$$\mathbf{d}_i \;=\; \mathbf{T}_i(\mathbf{c}_i) - \mathbf{c}_i$$

Interpolate by triharmonic RBFs

$$\mathbf{d}(\mathbf{x}) \;=\; \sum_{i=1}^{n} \mathbf{w}_i \cdot \|\mathbf{c}_i - \mathbf{x}\|^3 \;+\; \mathbf{p}(\mathbf{x})$$

- Guarantees smooth & fair deformation
- Solve dense linear system for RBF coefficients

Adaptive Deformation Graphs



Deformation Models

Desirable Properties

- Generality
  - handle different geometry representations
  - predictable, physically plausible deformation behavior ✓
- Efficiency, scalability
  - processing of large data sets, realtime feedback ✓
- Robustness
  - stable even for bad input and drastic deformations (✓)
- Simplicity
  - ease of implementation (✓)
  - adaptability, extensibility, re-use

Sumner, Schmid, Pauly: Embedded Deformation for Shape Manipulation, SIGGRAPH 2007

Botsch, Pauly, Wicke, Gross: Adaptive Space Deformation based on Rigid Cells, Eurographics 2007

Li, Adams, Guibas, Pauly: Single-View Geometry and Motion Reconstruction, SIGGRAPH ASIA 2009