SIGGRAPH2005

Course 10
Realistic Materials in Computer Graphics

Scattering from fibers

Steve Marschner
Cornell University

Overview

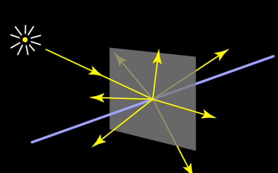
- Reflection from fibers
  - *reflection geometry*
  - *radiometry*
- Scattering from human hair
  - *the nature of hair*
  - *hair scattering phenomena*
  - *a model for scattering from hair*

Fiber reflection

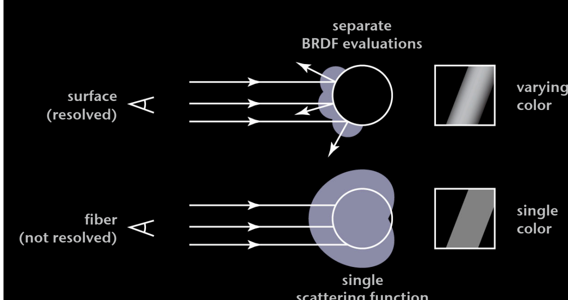
- Fibers are nearly 1D
- Choice: model as 1D or 3D
- 1D model leads to different radiometry from surfaces

1D fibers

- Hair is not resolved
- Fiber scattering function



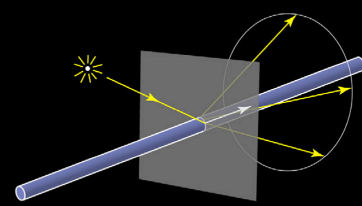
Surface vs. fiber



Scattering function

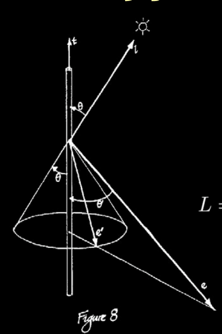
- BDRF analog for 1D fibers
- Job = summarize scattering behavior for the non-resolved case
  this means averaging reflected radiance across the width of the fiber

Specular fiber reflection



- Parallel incident rays
- All interfaces parallel to fiber axis
- Surface normals are coplanar (normal plane)
- Reflected vectors' directions lie in a cone

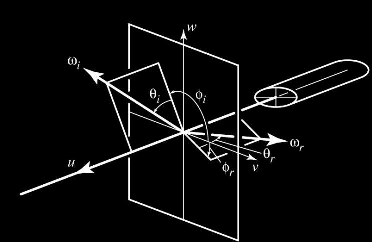
Kajiya–Kay model



- Based on conical reflection argument
- Diffuse and specular terms

$$L = (k_d \cos\theta + k_s \cos^p(\theta - \theta'))\, E$$

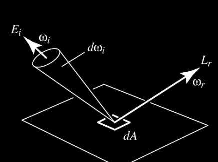
Scattering geometry



Surfaces to fibers

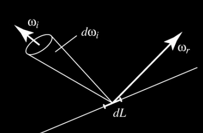
$$f_r(\omega_i, \omega_r) = \frac{dL_r(\omega_r)}{dE_i(\omega_i)}$$

$$L_r(\omega_r) = \int_{H^2} f_r(\omega_i, \omega_r) L_i(\omega_i) \cos\theta_i d\omega_i$$



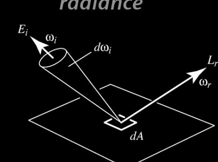
$$f_s(\omega_i, \omega_r) = \frac{d?}{d?}$$

$$\int_{S^2} f_s(\omega_i, \omega_r) L_i(\omega_i) \cos\theta_i d\omega_i$$

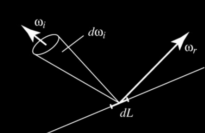


Surfaces to fibers

- in: power / area
  irradiance
- out: intensity / proj. area
  radiance



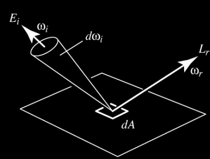
- in: power / length
  curve irradiance
- out: intensity / proj. length
  curve radiance



Surfaces to fibers

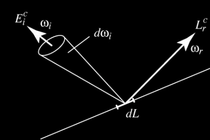
irradiance: $E = \frac{dP}{dA} \left(\frac{\mathrm{W}}{\mathrm{m}^2}\right)$

radiance: $L = \frac{d^2P}{dA^{\perp} d\omega} \left(\frac{\mathrm{W}}{\mathrm{m}^2\,\mathrm{sr}}\right)$



curve irradiance: $E^c = \frac{dP}{dL} \left(\frac{\mathrm{W}}{\mathrm{m}}\right)$

curve radiance: $L^c = \frac{d^2P}{dL^{\perp} d\omega} \left(\frac{\mathrm{W}}{\mathrm{m}\,\mathrm{sr}}\right)$



Curve radiometry

- Midway between 2D surface and 0D particle
  particle scatters power to intensity
- Same units as BRDF (1 / sr)
- Related to surface units via fiber diameter
  diameter might depend on direction (non-circular fibers)

Scattering integral

- Integrate curve radiance over sphere weighted by scattering function

$$L_r^c(\omega_r) = \int_{S^2} f_s(\omega_i, \omega_r) L_i^c(\omega_i) \cos\theta_i d\omega_i$$

If renderer has radiance, substitute (diam x radiance):

$$L_r^c(\omega_r) = \int_{S^2} f_s(\omega_i, \omega_r)(D_i L_i(\omega_i)) \cos\theta_i d\omega_i$$

If renderer wants radiance out, divide by diameter:

$$\langle L_r(\omega_r)\rangle = \frac{1}{D_r}\int_{S^2} f_s(\omega_i, \omega_r)(D_i L_i(\omega_i)) \cos\theta_i d\omega_i$$

For circular fibers the diameters cancel.

Example: ideal diffuse

- As with BRDF, corresponds to constant scattering function f_s

$$\begin{aligned} L_r^c &= \int_{S^2} f_s L_i^c(\omega_i) \cos\theta_i \, d\omega_i \\ &= f_s E_i^c \end{aligned}$$

to find reflectance feed in constant L^c_i:

$$\begin{aligned} L_r^c &= \int_{S^2} f_s L_i^c \cos\theta_i \, d\omega_i \\ &= f_s L_i^c \int_{S^2} \cos\theta_i \, d\omega_i \\ &= (\pi^2 f_s) L_i^c \end{aligned}$$

Kajiya–Kay as scat. fn.

- Their model gives radiance directly

$$L = (k_d \cos\theta + k_s \cos^p(\theta - \theta'))\, E$$

to put in terms of scattering function, divide by incident cosine:

$$f_s(\theta_i, \phi_i, \theta_r, \phi_r) = k_d + k_s \frac{\cos^p(\theta_i + \theta_r)}{\cos(\theta_i)}$$

Example: ideal specular

- Reflection confined to specular cone

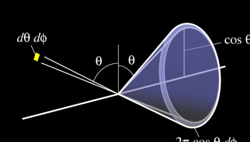
$$f_s(\theta_i, \phi_i, \theta_r, \phi_r) \approx \delta(\theta_i, \theta_r)$$

but light that comes in at different angles reflects into different sized cones

$$f_s(\theta_i, \phi_i, \theta_r, \phi_r) = \frac{\delta(\theta_i, \theta_r)}{2\pi \cos\theta_i \cos\theta_r}$$

to cancel cos θ_i;

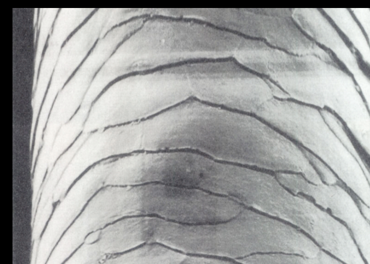
for cone s.a.



Scattering from hair

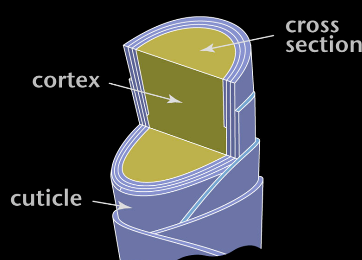
- Kajiya–Kay
- Modifications
  Goldman (fake fur rendering)
  Kim (thesis)
- Conical highlight, simple (or no) azimuthal dependence

Real Hair



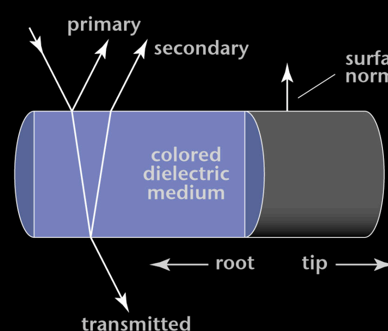
[Robbins 1994]

Hair Structure

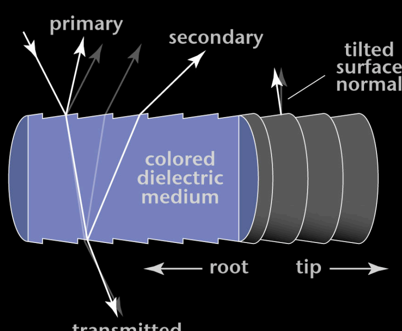


(after [Robbins 1994])

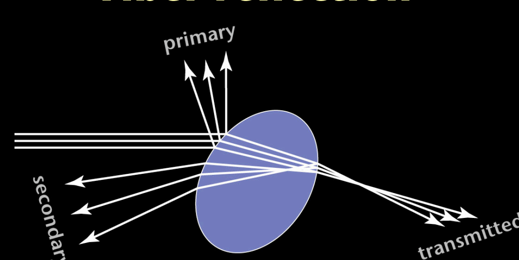
Fiber reflection



Fiber reflection

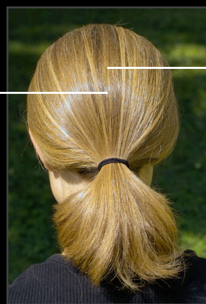


Fiber reflection



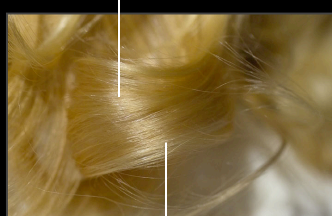
nonuniform scattered distribution

Hair appearance



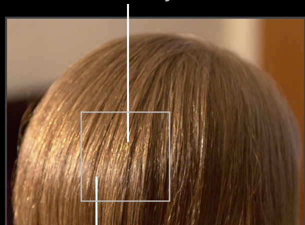
Hair appearance

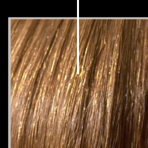
Hair appearance



Hair appearance

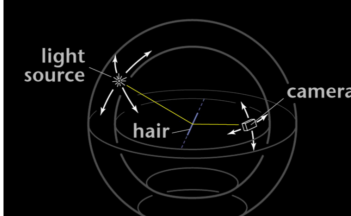
Hair appearance

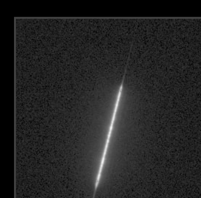




Measurement

- 4-axis scattering measurements

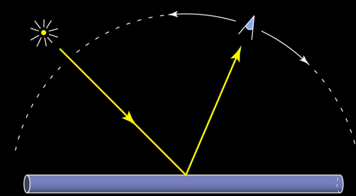




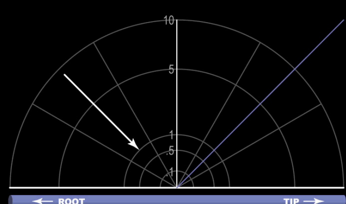
Measurement

- Previous work for cosmetics
  Stamm et al. 77; Bustard & Smith 91
- We cover full distribution

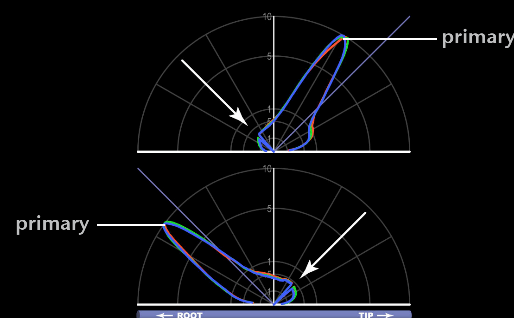
Incidence plane



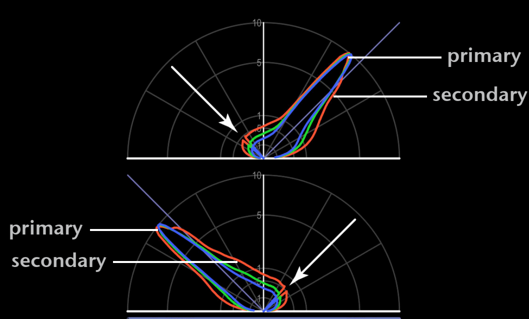
Incidence plane



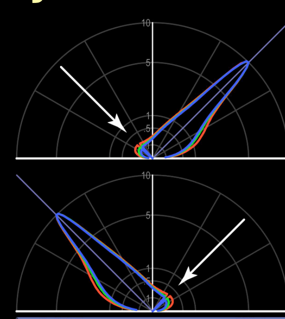
Black hair



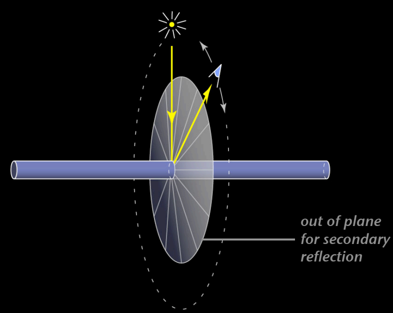
Blond hair



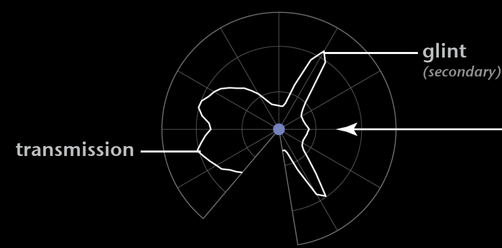
Synthetic hair

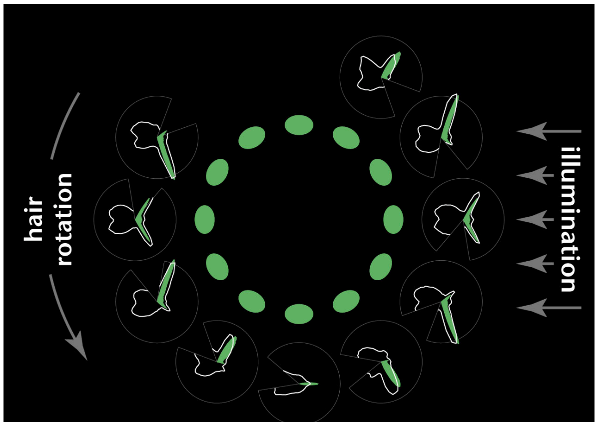


Transverse secondary



Blond hair





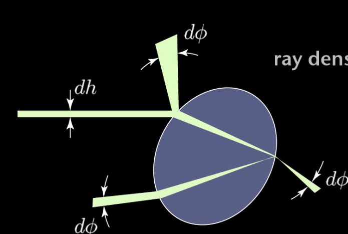
Optical model

- Based on structure of hair
- Explains:
  - double highlight
  - off-axis glints
  - effect of eccentricity
- Core is cross-section scattering

Optical model

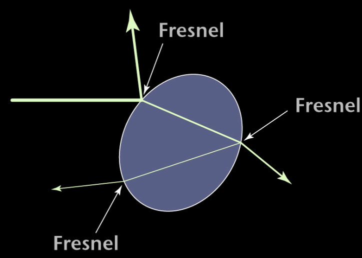
- Key analytical tools:
  - conical specular locus
    reduces to azimuthal dependence
  - Bravais' law
    reduces to 2D case (with different index)

Cross-section scattering

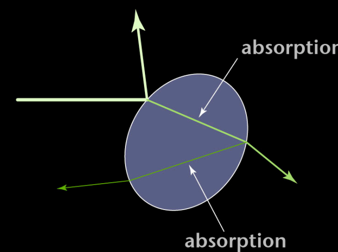


ray density $= \left|\dfrac{d\phi}{dh}\right|^{-1}$

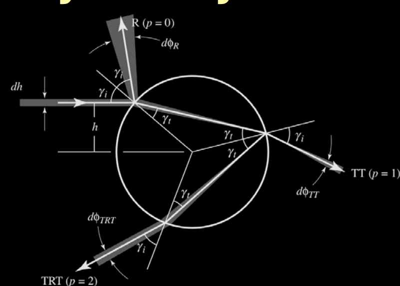
Cross-section scattering



Cross-section scattering



Ray density: circle



$$\phi(p, h) = 2p\gamma_t - 2\gamma_i + p\pi$$

Rendering

- Approximate scattering model
  based on circular fiber + 1st order eccentricity correction
- Ray trace fiber assemblies
- Compare to photograph
- Evaluate against Kajiya-Kay

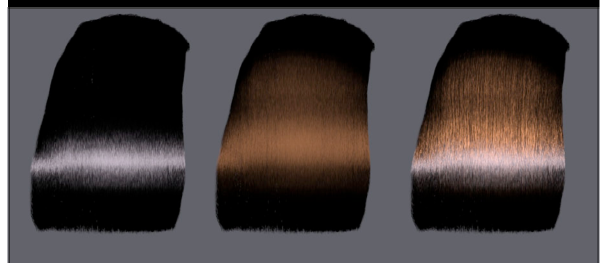
Photograph

[Gray 1997]

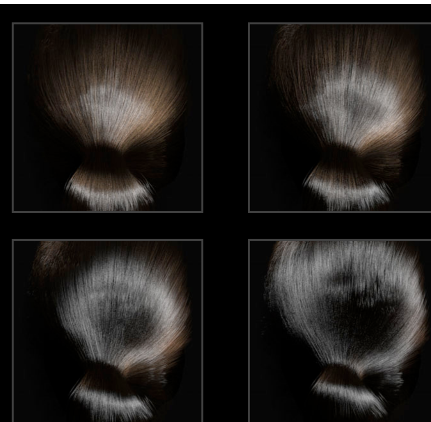
Photo vs. Kajiya-Kay

Photo vs. new model

Components of model



Hair texture



Conclusion

- Fibers are different from surfaces
- Similar types of behavior
  diffuse: bright at equator, dim at poles
  specular: bright on specular cone
- Hair has interesting structure
  causes deviations from the "usual" behavior

Other kinds of fibers

- Fabric is made of fibers
- Some minerals form fibers
- Plants contain fibers
- Wood is made of fibers

plug: papers, Tues. 10:30 am (Plants)