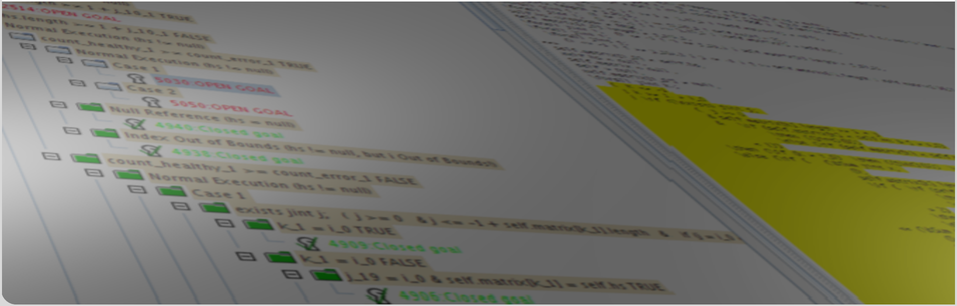
Karlsruhe Institute of Technology

Deductive Verification of Object-Oriented Software

Part B

Bernhard Beckert | VTSA, 24.–28.08.2015

KIT – Institute for Theoretical Computer Science



Part III

Program Verification with Dynamic Logic

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Syntax and Semantics

Syntax

- Basis: Typed first-order predicate logic
- Modal operators $\langle p \rangle$ and $[p]$ for each (JAVA CARD) program p
- Class definitions in background (not shown in formulas)

Semantics (Kripke)

Modal operators allow referring to the final state of p:

- `[`p`]` F: If p terminates, then F holds in the final state (partial correctness)
- `<`p`>` F: p terminates and F holds in the final state (total correctness)

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Why Dynamic Logic?

- Transparency wrt target programming language
- Encompasses Hoare Logic
- More expressive and flexible than Hoare logic
- Symbolic execution is a natural interactive proof paradigm

- Programs are “first-class citizens”
- Real Java syntax

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Hoare triple $\{\psi\}\ \alpha\ \{\phi\}$ equiv. to DL formula ψ `->` $[\alpha]\ \phi$

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Not merely partial/total correctness:

- can employ programs for specification (e.g., verifying program transformations)
- can express security properties (two runs are indistinguishable)
- extension-friendly (e.g., temporal modalities)

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Dynamic Logic Example Formulas

$(\texttt{balance} >= c \;\&\; \texttt{amount} > 0)$ ->
`<charge(amount);>` $\texttt{balance} > c$

`<x = 1;>([while (true) {}] false)`

- Program formulas can appear nested

$\backslash\texttt{forall}\; int\; val; ((\texttt{<p>}\, \texttt{x} \doteq val) \;\texttt{<->}\; (\texttt{<q>}\, \texttt{x} \doteq val))$

- p, q equivalent relative to computation state restricted to x

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Dynamic Logic Example Formulas

```
  a != null
->
  <
    int max = 0;
    if ( a.length > 0 ) max = a[0];
    int i = 1;
    while ( i < a.length ) {
      if ( a[i] > max ) max = a[i];
      ++i;
    }
  >
    (
      \forall int j; (j >= 0 & j < a.length -> max >= a[j])
      &
      (a.length > 0 ->
        \exists int j; (j >= 0 & j < a.length & max = a[j]))
    )
```

Variables

Logical variables disjoint from program variables

- No quantification over program variables
- Programs do not contain logical variables
- “Program variables” actually non-rigid functions

Rigid and Flexible Terms

Example

$\texttt{<int i;>}\backslash\texttt{forall}\ \mathit{int}\ x; (\texttt{i} + 1 \doteq x \texttt{ -> } \texttt{<i++;>}(\texttt{i} \doteq x))$

- Interpretation of `i` depends on computation state $\Rightarrow$ flexible
- Interpretation of x and $+$ do not depend on state $\Rightarrow$ rigid

Locations are always flexible

Logical variables, standard functions are always rigid

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A JAVA CARD DL formula is valid iff it is true in all states.

We need a calculus for checking validity of formulas

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Sequents and their Semantics

Syntax

$$\underbrace{\psi_1, \ldots, \psi_m}_{Antecedent} \implies \underbrace{\phi_1, \ldots, \phi_n}_{Succedent}$$

where the ϕ_i, ψ_i are formulae (without free variables)

Semantics

Same as the formula

$$(\psi_1 \ \& \ \cdots \ \& \ \psi_m) \ \ {-}{>} \ \ (\phi_1 \ | \ \cdots \ | \ \phi_n)$$

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Sequent Rules

General form

$$\text{rule\_name}\ \frac{\overbrace{\Gamma_1 \Longrightarrow \Delta_1 \quad \cdots \quad \Gamma_r \Longrightarrow \Delta_r}^{\text{Premisses}}}{\underbrace{\Gamma \Longrightarrow \Delta}_{\text{Conclusion}}}$$

($r = 0$ possible: closing rules)

Soundness

If all premisses are valid, then the conclusion is valid

Use in practice

Goal is matched to conclusion

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not_left $$\frac{\Gamma \Longrightarrow A, \Delta}{\Gamma, !\, A \Longrightarrow \Delta}$$

imp_left $$\frac{\Gamma \Longrightarrow A, \Delta \qquad \Gamma, B \Longrightarrow \Delta}{\Gamma, A \;\text{->}\; B \Longrightarrow \Delta}$$

close_goal $$\frac{}{\Gamma, A \Longrightarrow A, \Delta}$$

close_by_true $$\frac{}{\Gamma \Longrightarrow \mathrm{true}, \Delta}$$

all_left $$\frac{\Gamma, \backslash\mathtt{forall}\; t\; x; \phi,\; \{x/e\}\phi \Longrightarrow \Delta}{\Gamma, \backslash\mathtt{forall}\; t\; x; \phi \Longrightarrow \Delta}$$

where e var-free term of type $t' \prec t$

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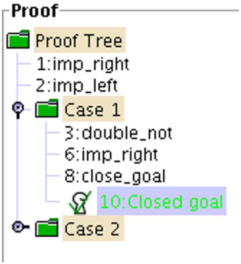
Sequent Calculus Proofs

Proof tree

- Proof is tree structure with goal sequent as root
- Rules are applied from conclusion (old goal) to premisses (new goals)
- Rule with no premiss closes proof branch
- Proof is finished when all goals are closed



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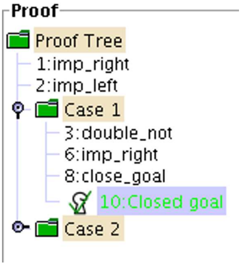

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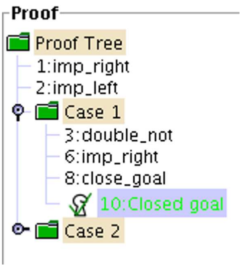

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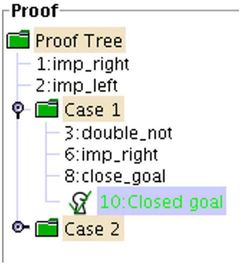

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- Sequent rules for program formulas?
- What corresponds to top-level connective in a program?

The Active Statement in a Program

- Sequent rules execute symbolically the active statement

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Rules for Symbolic Program Execution

If-then-else rule

$$\frac{\Gamma, B = \mathit{true} \Longrightarrow \texttt{<}p \;\; \omega\texttt{>}\,\phi, \Delta \qquad \Gamma, B = \mathit{false} \Longrightarrow \texttt{<}q \;\; \omega\texttt{>}\,\phi, \Delta}{\Gamma \Longrightarrow \texttt{<if (}B\texttt{) \{ }p\texttt{ \} else \{ }q\texttt{ \} }\omega\texttt{>}\,\phi, \Delta}$$

Complicated statements/expressions are simplified first, e.g.

$$\frac{\Gamma \Longrightarrow \texttt{<v=y; y=y+1; x=v; }\omega\texttt{>}\,\phi, \Delta}{\Gamma \Longrightarrow \texttt{<x=y++; }\omega\texttt{>}\,\phi, \Delta}$$

Simple assignment rule

$$\frac{\Gamma \Longrightarrow \{loc := val\}\texttt{<}\omega\texttt{>}\,\phi, \Delta}{\Gamma \Longrightarrow \texttt{<}loc\texttt{=}val\texttt{; }\omega\texttt{>}\,\phi, \Delta}$$

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Extending DL by Explicit State Updates

Updates

explicit syntactic elements in the logic

Elementary Updates

$$\{loc := val\}\, \phi$$

where (roughly)

- loc a program variable x, an attribute access $o.attr$, or an array access $a[i]$
- val is same as loc, or a literal, or a logical variable

Parallel Updates

$$\{loc_1 := t_1 \mid\mid \cdots \mid\mid loc_n := t_n\}\, \phi$$

no dependency between the n components (but 'right wins' semantics)

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Updates are:

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- *eagerly parallelised* + simplified

Advantages

- no renaming required
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Symbolic Execution with Updates

(by Example)

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              ⋮
x < y  ==>  {x:=y || y:=x}<> y < x
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$\vdots$

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$$\texttt{x} < \texttt{y} \Longrightarrow \{\texttt{t:=x} \parallel \texttt{x:=y}\}\{\texttt{y:=t}\}\texttt{<>}\ \texttt{y} < \texttt{x}$$

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$$\texttt{x} < \texttt{y} \Longrightarrow \{\texttt{t:=x}\}\texttt{<x=y; y=t;>}\ \texttt{y} < \texttt{x}$$

$\vdots$

$$\Longrightarrow \texttt{x} < \texttt{y} \texttt{ -> <int t=x; x=y; y=t;>}\ \texttt{y} < \texttt{x}$$

Symbolic Execution with Updates
(by Example)

```
x < y  ==>  x < y
       ⋮
x < y  ==>  {x:=y || y:=x}<> y < x
       ⋮
x < y  ==>  {t:=x || x:=y || y:=x}<> y < x
       ⋮
x < y  ==>  {t:=x || x:=y}{y:=t}<> y < x
       ⋮
x < y  ==>  {t:=x}{x:=y}<y=t;> y < x
       ⋮
x < y  ==>  {t:=x}<x=y; y=t;> y < x
       ⋮
       ==>  x < y -> <int t=x; x=y; y=t;> y < x
```

Symbolic Execution with Updates

(by Example)

$$x < y \;\Longrightarrow\; x < y$$

$$\vdots$$

$$x < y \;\Longrightarrow\; \{\texttt{x:=y} \,\|\, \texttt{y:=x}\}\texttt{<>}\ y < x$$

$$\vdots$$

$$x < y \;\Longrightarrow\; \{\texttt{t:=x} \,\|\, \texttt{x:=y} \,\|\, \texttt{y:=x}\}\texttt{<>}\ y < x$$

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Symbolic Execution with Updates
(by Example)

$$\texttt{x} < \texttt{y} \implies \texttt{x} < \texttt{y}$$

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Program State Representation

Local program variables

Modeled as non-rigid constants

Heap

Modeled with theory of arrays:

$$
\begin{aligned}
&heap: && \to Heap \text{ (the heap in the current state)}\\
&select: && Heap \times Object \times Field \to Any\\
&store: && Heap \times Object \times Field \times Any \to Heap
\end{aligned}
$$

Heap axioms (excerpt)

$$select(store(h, o, f, x), o, f) = x$$

$$select(store(h, o, f, x), u, f) = select(h, u, f) \text{ if } o \neq u$$

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Handling Abrupt Termination

- Abrupt termination handled by program transformations
- Changing control flow = rearranging program parts

Example

TRY-THROW

$$\frac{\Gamma \Longrightarrow \left\langle \begin{array}{l}\texttt{if (exc instanceof T)} \\ \quad \texttt{\{try \{e=exc; r\} finally \{s\}\}} \\ \quad \texttt{else \{s throw exc;\}} \quad \omega \end{array} \right\rangle \phi, \Delta}{\Gamma \Longrightarrow \texttt{<try\{throw exc; q\} catch(T e)\{r\} finally\{s\}} \ \omega\texttt{>} \phi, \Delta}$$

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Part III

Program Verification with Dynamic Logic

Part III

Program Verification with Dynamic Logic

Supported Java Features

- method invocation with polymorphism/dynamic binding
- object creation and initialisation
- arrays
- abrupt termination
- throwing of NullPointerExceptions, etc.
- bounded integer data types
- transactions

All JAVA CARD language features are fully addressed in KeY

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Java—A Language of Many Features

Karlsruhe Institute of Technology

Ways to deal with Java features

- Program transformation, up-front
- Local program transformation, done by a rule on-the-fly
- Modeling with first-order formulas
- Special-purpose extensions of program logic

Pro: Feature needs not be handled in calculus
Contra: Modified source code
Example in KeY: Very rare: treating inner classes

Java—A Language of Many Features

Karlsruhe Institute of Technology

Ways to deal with Java features

- Program transformation, up-front
- Local program transformation, done by a rule on-the-fly
- Modeling with first-order formulas
- Special-purpose extensions of program logic

Pro: Flexible, easy to implement, usable
Contra: Not expressive enough for all features
Example in KeY: Complex expression eval, method inlining, etc., etc.

Java—A Language of Many Features

Karlsruhe Institute of Technology

Ways to deal with Java features

- Program transformation, up-front
- Local program transformation, done by a rule on-the-fly
- Modeling with first-order formulas
- Special-purpose extensions of program logic

Pro: No logic extensions required, enough to express most features
Contra: Creates difficult first-order POs, unreadable antecedents
Example in KeY: Dynamic types and branch predicates

Java—A Language of Many Features

Karlsruhe Institute of Technology

Ways to deal with Java features

- Program transformation, up-front
- Local program transformation, done by a rule on-the-fly
- Modeling with first-order formulas
- Special-purpose extensions of program logic

Pro: Arbitrarily expressive extensions possible
Contra: Increases complexity of all rules
Example in KeY: Method frames, updates

Components of the Calculus

1. Non-program rules
   - first-order rules
   - rules for data-types
   - first-order modal rules
   - induction rules
2. Rules for reducing/simplifying the program (symbolic execution)
   Replace the program by
   - case distinctions (proof branches) and
   - sequences of updates
3. Rules for handling loops
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Part III

Program Verification with Dynamic Logic

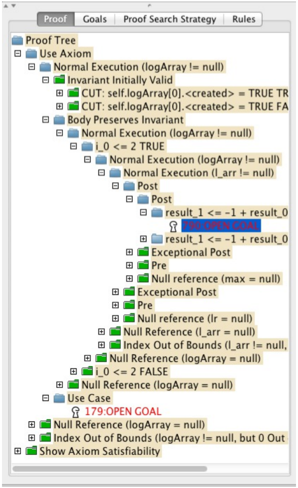
Part III

Program Verification with Dynamic Logic

Taclets: KeY's Rule Description Language

Taclets ...

- represent sequent calculus rules in KeY
- use a simple text-based format
- are descriptive, but with operational flavor
- are *not* a tactic metalanguage



Taclet Syntax

$$\text{andLeft}\ \frac{\Gamma, A, B \Longrightarrow \Delta}{\Gamma, A \,\&\, B \Longrightarrow \Delta}$$

Taclet

```
andLeft {
    \find ( A & B ==> )
    \replacewith ( A, B ==>)
};
```

- Unique name
- Find expression:
  - Formula (Term) to be modified
  - Sequent arrow ==> formula must occur top level and on the corresponding side of the sequent.
- Goal Description: describes new sequent

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Taclet Syntax

Some rules are only sound in a certain context

$$\text{modusPonens}\ \frac{\Gamma, A, B \Longrightarrow \Delta}{\Gamma, A, A \;\texttt{->}\; B \Longrightarrow \Delta}$$

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```
modusPonens {
  \assumes ( A ==> )
  \find ( A -> B ==> )
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Taclet Syntax

Karlsruhe Institute of Technology

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Taclet Syntax

Proof Splitting: andRight

$$\frac{\Gamma \Longrightarrow A, \Delta \quad \Gamma \Longrightarrow B, \Delta}{\Gamma \Longrightarrow A \;\&\; B, \Delta}$$

```
andRight {
  \find ( ==> A & B )
  \replacewith (==> A );
  \replacewith (==> B )
};
```

Variable Conditions: allRight

$$\frac{\Gamma \Longrightarrow \{x/c\}\Phi, \Delta}{\Gamma \Longrightarrow \forall T\; x; \Phi, \Delta}, \text{c new}$$

```
allRight {
  \find ( ==> \forall x;phi )
  \varcond(\new(c,\dependingOn(phi)))
  \replacewith ( ==> {\subst x;c}phi )
};
```

Taclet Syntax

Proof Splitting: andRight

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Taclets for Program Transformations

$$\frac{\Gamma \Longrightarrow \left\langle \begin{array}{l} \pi\ \texttt{if (exc == null) \{} \\ \quad \texttt{try\{ throw new NPE(); catch(T e) \{r\};} \\ \texttt{\} else if (exc instanceof T) \{e=exc; r\}} \\ \texttt{else throw exc;}\ \omega \end{array} \right\rangle \phi}{\Gamma \Longrightarrow \texttt{<}\pi\ \texttt{try\{throw exc; q\} catch(T e)\{r\};}\ \omega\texttt{>}\phi}$$

```
\find ( <.. try { throw #se; #slist }
            catch ( #t #v0 ) { #slist1 } ...> post )
\replacewith (
  <.. if (#se == null) {
        try { throw new NullPointerException(); }
        catch (#t #v0) { #slist1 }
      } else if (#se instanceof #t) {
        #t #v0 = (#t) #se;
        #slist1
      } else throw #se; ...> post )
```

Part IV

Verifying Information-Flow Properties

Part IV

Verifying Information-Flow Properties

12 Information Flow

13 Formalisation in DL

14 Objects and Information Flow

Language-based Information Security

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Secret and public information

Partitioning of the set of program variables into

- variables which contain confidential information (“high variables”)
  – NOT observable by the attacker –
- variables which contain non-confidential information (“low variables”)
  – observable by the attacker –

Informal definition of non-interference

A program is secure, if the initial values of the high variables do not interfere with the final values of the low variables.

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Examples

Note

Sequential Java programs
Termination not considered

Which methods are secure?

```
void m_1() {
  low = high;
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void m_3() {
  if (high > 0) {low = 1;}
  else          {low = 2;};
}
```

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Which methods are secure?

```
void m_4() {
  high = 0;
  low  = high;
}
```

```
void m_5() {
  low = high;
  low = low-high;
}
```

```
void m_6() {
  if (false) low = high;
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```

Examples

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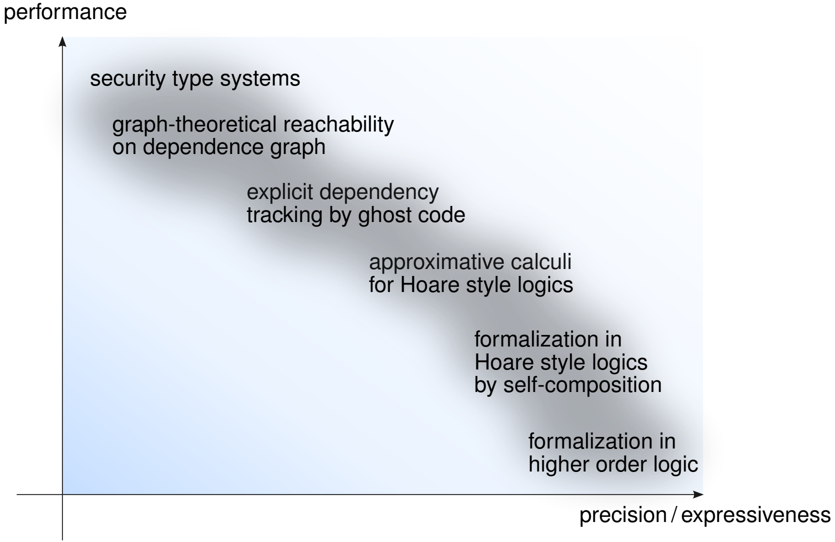
SECURE

Information-flow Analysis Approaches



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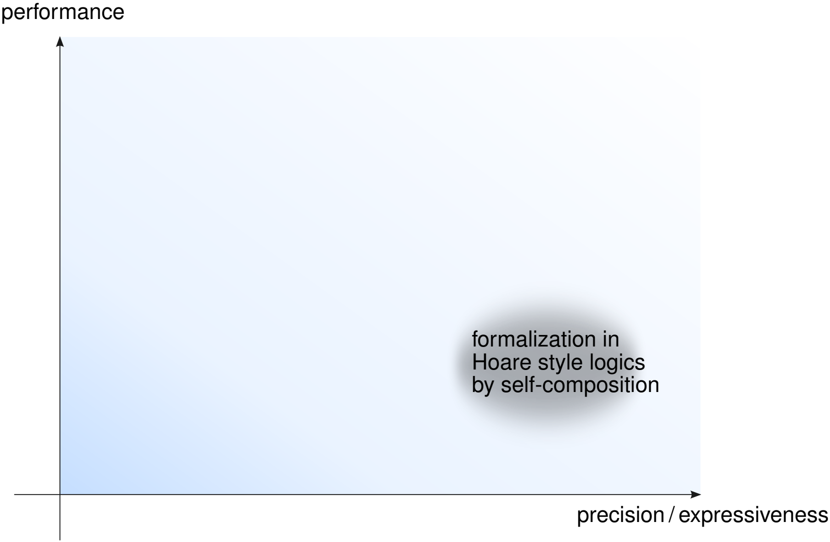

Non-Interference

Definition (Low-equivalence on states)

Two states are low-equivalent if they assign the same values to low variables.

Definition (Non-interference)

Starting P in two arbitrary low-equivalent states results in two final states that are also low-equivalent.

Non-Interference

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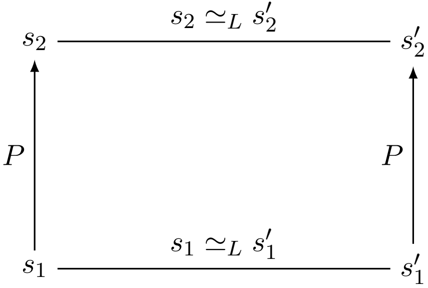
Non-interference

- P a program
- L the set of low variables



where

$$s_i \simeq_L s'_i \quad \Leftrightarrow \quad \forall\, v \in L \; (v^{s_i} = v^{s'_i})$$

Non-interference

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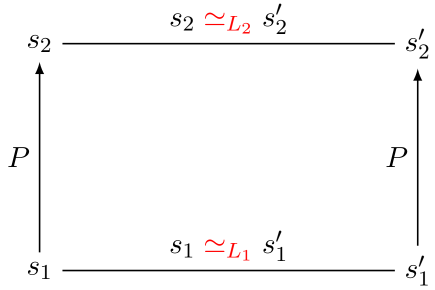
- P a program
- L_1, L_2 sets of low variables



where

$$s_i \simeq_{L_i} s'_i \quad \Leftrightarrow \quad \forall\, v \in L_i \; (v^{s_i} = v^{s'_i})$$

Non-Interference in JavaDL (Version 1)

Encoding with alternating quantifiers

For all low input values in_l, there exist low output values r such that for all high input values in_h, if we assign the values in_l to the program variables low and in_h to the program variables $high$, then after execution of P the values of low are r.

$$\forall in_l \exists r \forall in_h (\{low := in_l \mid\mid high := in_h\}[P]low = r)$$

Problem

Not suitable for automatic verification
$\rightsquigarrow$ instantiation of existential quantifier difficult.

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Problem

Not suitable for automatic verification
$\leadsto$ instantiation of existential quantifier difficult.

Non-Interference in JavaDL (Version 2)

Encoding with self-composition

Running two instances of P on the same low values but on arbitrary high values results in low variables which have the same values.

$$
\begin{aligned}
&\forall in_l \forall in_h^1 \forall in_h^2 \forall out_l^1 \forall out_l^2 \; \{low := in_l\}(\\
&\quad \{high := in_h^1\}[P]out_l^1 = low \\
&\quad \land \{high := in_h^2\}[P]out_l^2 = low \\
&\quad \rightarrow out_l^1 = out_l^2 \\
&)
\end{aligned}
$$

Declassification

Intuition

Let $T(high, low)$ be a term.

The only thing the attacker is allowed to learn about the secret inputs is the value of T in the initial state.

Definition (Non-interference with declassification)

Starting P in two arbitrary low-equivalent states coinciding in the value of T results in two final states that are also low-equivalent.

Declassification

Intuition

Let $T(high, low)$ be a term.

The only thing the attacker is allowed to learn about the secret inputs is the value of T in the initial state.

Definition (Non-interference with declassification)

Starting P in two arbitrary low-equivalent states coinciding in the value of T results in two final states that are also low-equivalent.

Declassification in JavaDL

Encoding non-interference with declassification

Running two instances of P on the same low values and arbitrary high values coinciding on T results in low variables which have the same values.

$$
\begin{aligned}
&\forall in_l \forall in_h^1 \forall in_h^2 \forall out_l^1 \forall out_l^2 \ \{low := in_l\}(\\
&\qquad \{high := in_h^1\}T = \{high := in_h^2\}T \\
&\quad \land \{high := in_h^1\}[P]out_l^1 = low \\
&\quad \land \{high := in_h^2\}[P]out_l^2 = low \\
&\quad \to out_l^1 = out_l^2 \\
&)
\end{aligned}
$$

DEMO

Verifying Information-flow Properties with the KeY Tool

Object-Sensitive Non-interference

Leakage by aliasing

```
void m() {
    C c1 = new C();    // new obj
    C c2 = c1;         // alias
    c2.x = high;
    low  = c1.x;
}
```

Object-Sensitive Non-interference

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    low  = c1.x;
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NOT SECURE

Object-Sensitive Non-interference

Object creation and object identity

```
if (high>0) {
    low1 = new C();
    low2 = new C();
} else {
    low2 = new C();
    low1 = new C();
}
```

Assumption

- References are opaque
- Only comparison of objects by == is observable

Object creation and object identity

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low1 = new C();
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if (high>0) { low1 = low2; }
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```
if (high>0) { low = new C() }
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Object creation and object identity

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Object-Sensitive Non-interference

Idea

ISOMORPHIC object structures in low variables
IDENTITY NOT required

Instead of

$$s_i \simeq_{L_i} s_i' \quad \Leftrightarrow \quad \forall\, v \in L_i\ (v^{s_i} = v^{s_i'})$$

use

$$s_i \simeq_{L_i}^{\pi_i} s_i' \quad \Leftrightarrow \quad \forall\, v \in L_i\ (\pi_i(v^{s_i}) = v^{s_i'})$$

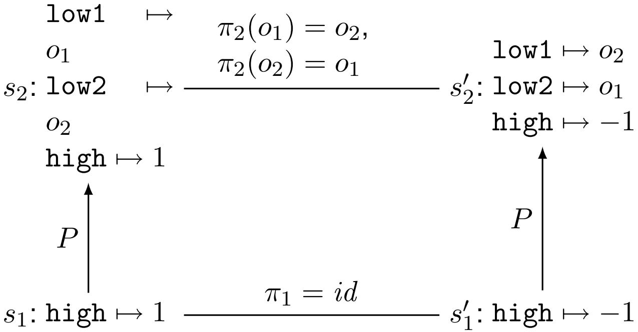
where

π_1, π_2 are compatible

i.e.

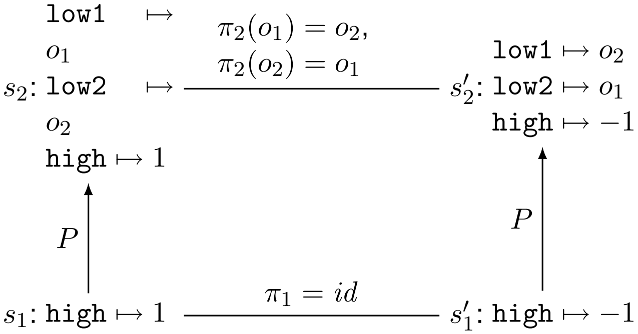
$\pi_1(o) = \pi_2(o)$ if o observable in both s_1 and s_2

Object-Sensitive Non-interference



Secure because o_1, o_2 not observable in s_1

Object-Sensitive Non-interference



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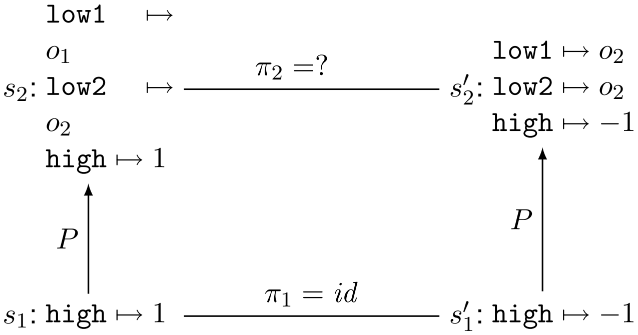
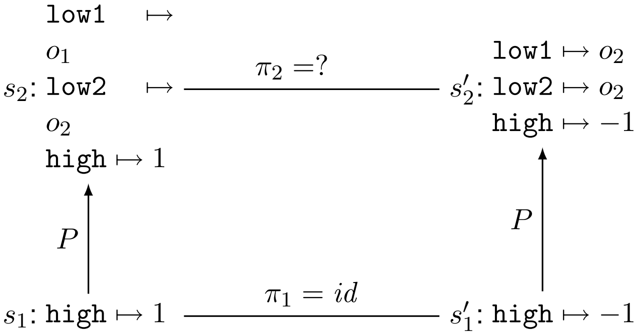
Object-Sensitive Non-interference



Not secure because no suitable π_2 exists

Object-Sensitive Non-interference



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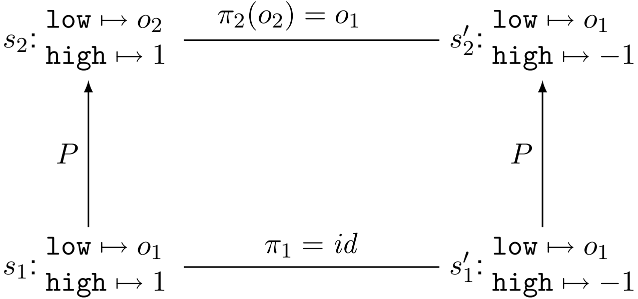
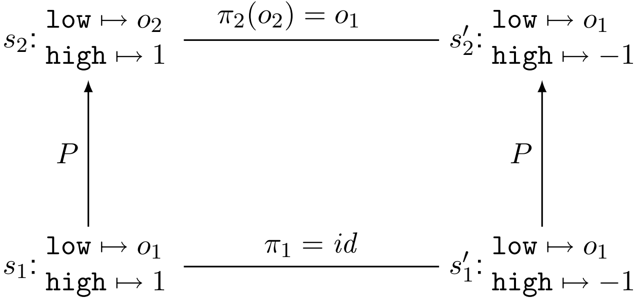


Not secure because o_1 observable in s_1 and $\pi_1(o_1) \neq \pi_2(o_1)$

Object-Sensitive Non-interference



Not secure because o_1 observable in s_1 and $\pi_1(o_1) \neq \pi_2(o_1)$

Object-Sensitive Non-interference

Optimisations

- L_1, L_2 sequences of low terms (instead of sets of variables)
- π_1 can be fixed to be id

DEMO

Objects and Information-flow with the KeY Tool

Part V

Wrap Up

Part V

Wrap Up

15 Further Usage of Verification Technology

16 Directions of Current Research in KeY

Further Usage of Verification Technology

- Verification performs deep *Program Analysis*
- Information in (partial) proofs usable for other purposes

Further Usage: Verification-Driven Test Generation

- Specification- and code-based approach
- Achieve strong hybrid coverage criteria
- Exploit strong correspondence:
  proof branches $\leftrightarrow$ program execution paths
- Each leaf of (partial) proof branch contains
  constraint on inputs
  resulting in
  corresponding path condition

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Directions of Current Research with KeY

Topics

- Scalability (combine with light-weight techniques)
- Usability (support user in understanding proof state)
- Concurrency and distribution
- Information-flow / security properties
- Application: eVoting

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THE END

(for now)