

Verification of security protocols: from confidentiality to privacy

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- 12 academic departments: mathematics, computer science, chemistry, social sciences, . . .
- 13 research laboratories

Laboratoire Spécification & Vérification

Verification of critical software and systems

Goal: develop the mathematical and algorithmic **foundations** to the development of tools for **automatically** proving correctness and detecting flaws.

Applications: computerized systems, databases, **security protocols**



LSV in figures

- Founded in 1997
- Around 25 permanents + 15 PhD students
- 5 research teams

Security of Information Systems

- 4 permanents: David Baelde, H. Comon-Lundh, S. Delaune, et J. Goubault-Larrecq.



- 1 engineer + 1 postdoc
- 3 phd students

Cryptographic protocols everywhere !



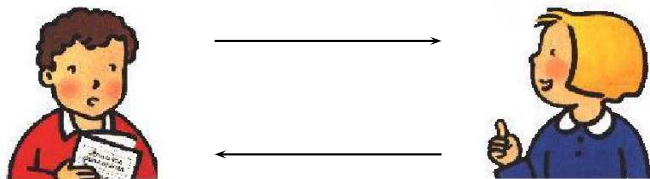
Goal: they aim at securing communications over public/insecure networks

Some security properties

- **Secrecy**: May an intruder learn some secret message between two honest participants?
- **Authentication**: Is the agent **Alice** really talking to **Bob**?
- **Anonymity**: Is an attacker able to learn something about the identity of the participants who are communicating?
- **Non-repudiation**: **Alice** sends a message to **Bob**. **Alice** cannot later deny having sent this message. **Bob** cannot deny having received the message.
- ...

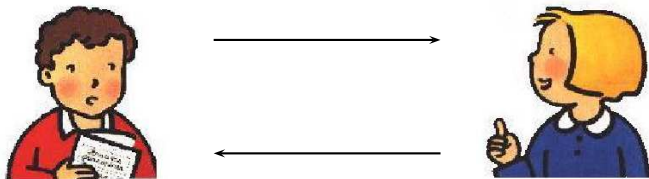
How does a cryptographic protocol work (or not)?

Protocol: small programs explaining how to exchange messages



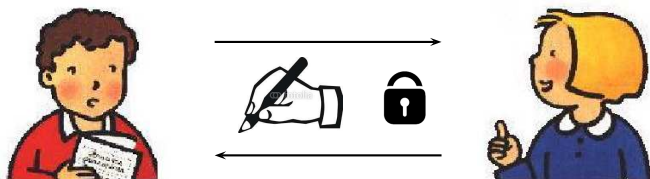
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Cryptographic: make use of cryptographic primitives

Examples: symmetric encryption, asymmetric encryption, signature, hashes, ...



What is a symmetric encryption scheme?

Symmetric encryption

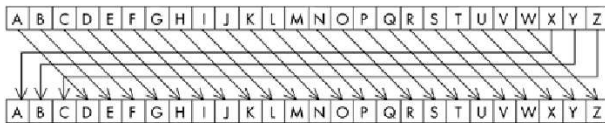


What is a symmetric encryption scheme?

Symmetric encryption



Example: This might be as simple as shifting each letter by a number of places in the alphabet (e.g. Caesar cipher)



Today: DES (1977), AES (2000)

A famous example

Enigma machine (1918-1945)

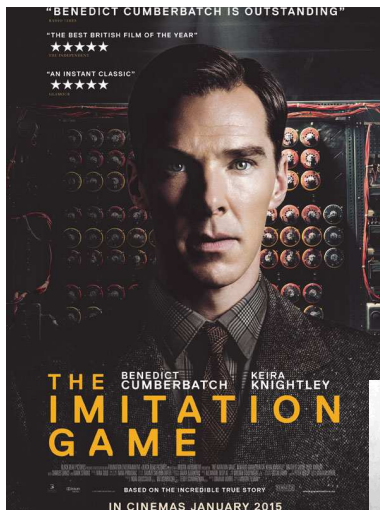
- electro-mechanical rotor cipher machines used by the German to encrypt during World War II
- permutations and substitutions



A bit of history

- 1918: invention of the Enigma machine
- 1940: Battle of the Atlantic during which **Alan Turing's** Bombe was used to test Enigma settings.

→ Everything about the breaking of the Enigma cipher systems remained secret **until the mid-1970s.**



What is an asymmetric encryption scheme?

Asymmetric encryption



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Asymmetric encryption



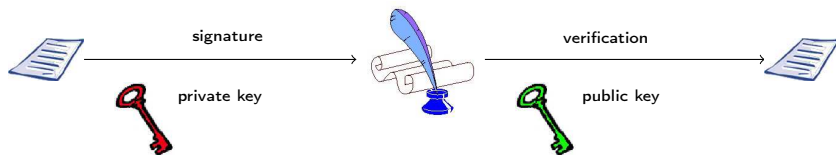
Examples:

- 1976: first system published by W. Diffie, and M. Hellman,
 - 1977: RSA system published by R. Rivest, A. Shamir, and L. Adleman.
- > their security relies on well-known **mathematical problems** (e.g. factorizing large numbers, computing discrete logarithms)

Today: those systems are still in use

What is a signature scheme?

Signature



Example:

The RSA cryptosystem (in fact, most public key cryptosystems) can be used as a signature scheme.

How does a cryptographic protocol work (or not)?

Example: A simplified version of the Denning-Sacco protocol (1981)

$A \rightarrow B$: $\text{aenc}(\text{sign}(k, \text{priv}(A)), \text{pub}(B))$

$B \rightarrow A$: $\text{senc}(s, k)$

What about secrecy of s ?

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What about secrecy of s ?

Consider a scenario where A starts a session with C who is **dishonest**.

1. $A \rightarrow C$: $\text{aenc}(\text{sign}(k, \text{priv}(A)), \text{pub}(C))$

C knows the key k

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2. $C(A) \rightarrow B$: $\text{aenc}(\text{sign}(k, \text{priv}(A)), \text{pub}(B))$

3. $B \rightarrow A$: $\text{senc}(s, k)$ **Attack !**

Exercise

We propose to fix the Denning-Sacco protocol as follows:

Version 1

$$\begin{aligned} A \rightarrow B & : \text{aenc}(\langle A, B, \text{sign}(k, \text{priv}(A)) \rangle, \text{pub}(B)) \\ B \rightarrow A & : \text{senc}(s, k) \end{aligned}$$

Version 2

$$\begin{aligned} A \rightarrow B & : \text{aenc}(\text{sign}(\langle A, B, k \rangle, \text{priv}(A))), \text{pub}(B)) \\ B \rightarrow A & : \text{senc}(s, k) \end{aligned}$$

Which version would you prefer to use?

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Which version would you prefer to use? Version 2

→ Version 1 is still vulnerable to the aforementioned attack.

What about protocols used in real life ?





Serge Humpich case - “ **Yescard** ” (1997)





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Step 1: A **logical flaw** in the protocol allows one to copy a card and to use it without knowing the PIN code.

→ not a real problem, there is still a bank account to withdraw



Credit Card payment protocol



Serge Humpich case - “ **Yescard** ” (1997)

Step 1: A **logical flaw** in the protocol allows one to copy a card and to use it without knowing the PIN code.

→ not a real problem, there is still a bank account to withdraw



Step 2: **breaking encryption** via factorisation of the following (96 digits) number: 213598703592091008239502270499962879705109534182
6417406442524165008583957746445088405009430865999

→ now, the number that is used is made of **232** digits



Lots of bugs and attacks, with fixes every month

FREAK attack discovered by Baraghavan et al (Feb. 2015)

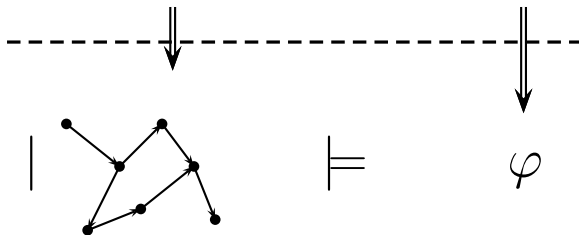
- 1 a logical flaw that allows a **man in the middle attacker** to downgrade connections from 'strong' RSA to 'export-grade' RSA;
- 2 **breaking encryption** via factorisation of such a key can be easily done.

→ 'export-grade' were introduced under the pressure of US governments agencies to ensure that they would be able to decrypt all foreign encrypted communication.

This talk: formal methods for protocol verification

Does the **protocol** *satisfy* a **security property**?

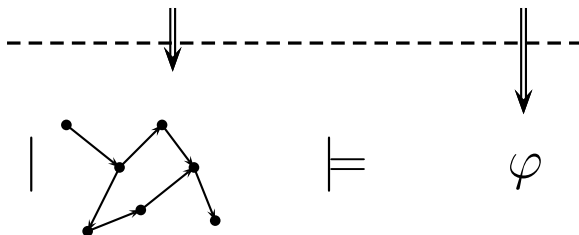
Modelling



This talk: formal methods for protocol verification

Does the protocol satisfy a security property?

Modelling



Two main tasks

- 1 Modelling cryptographic protocols and their security properties
- 2 Designing verification algorithms

Modelling messages
and
Deciding knowledge
(in a simple setting)

Symbolic model

→ Various models (e.g. [Dolev & Yao, 81]) having some common features

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Messages

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Messages

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The attacker



Symbolic model

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Messages

They are abstracted by **terms**.

The attacker

- may **read** every message sent on the network,
- may **intercept** and **send** new messages according to its deduction capabilities.
→ only **symbolic** manipulations on terms.



Messages as terms

→ It is important to have a tight modelling of messages

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Terms

They are built over a **signature** \mathcal{F} , and an infinite set of **names** \mathcal{N} .

$$\begin{array}{l} t ::= n \quad \text{name } n \in \mathcal{N} \\ \quad | f(t_1, \dots, t_k) \quad \text{application of symbol } f \in \mathcal{F} \end{array}$$

- Names are used to model **atomic data**
→ e.g. keys, nonces, agent names, ...
- Function symbols are used to model **cryptographic primitives**
→ e.g. encryption, signature, ...

A typical signature

Standard primitives

$$\mathcal{F} = \{\text{senc}, \text{aenc}, \text{sk}, \text{sign}, \langle \rangle\}$$

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Going back to the Denning Sacco protocol

$A \rightarrow B$: $\text{aenc}(\text{sign}(k, \text{priv}(A)), \text{pub}(B))$

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These messages can be modelled as follows:

- 1 $\text{aenc}(\text{sign}(k, \text{sk}(a)), b);$
- 2 $\text{senc}(s, k)$

Symbolic manipulation on terms

He may **build** new messages following **deduction rules**



Pairing

$$\frac{x \quad y}{\langle x, y \rangle} \quad \frac{\langle x, y \rangle}{x} \quad \frac{\langle x, y \rangle}{y}$$

Symmetric encryption

$$\frac{x \quad y}{\text{senc}(x, y)} \quad \frac{\text{senc}(x, y) \quad y}{x}$$

Asymmetric encryption

$$\frac{x \quad y}{\text{aenc}(x, y)} \quad \frac{\text{aenc}(x, y) \quad \text{sk}(y)}{x}$$

Signature

$$\frac{x \quad \text{sk}(y)}{\text{sign}(x, \text{sk}(y))} \quad \frac{\text{sign}(x, \text{sk}(y))}{x}$$

We say that u is **deducible from** T if there exists a proof tree such that:

- 1 each leaf is labeled by v with $v \in T$;
- 2 for each node labeled by v_0 and having n sons labeled by v_1, \dots, v_n , there exists a deduction rule R such that

$$\frac{v_1 \quad \dots \quad v_n}{v_0} \quad \text{is an instance of } R$$

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Exercise - Going back to the Denning Sacco protocol

Let $T = \{a, b, c, \text{sk}(c), \text{aenc}(\text{sign}(k, \text{sk}(a)), c), \text{senc}(s, k)\}$.

Is s deducible from T ?

Exercise - Going back to the Denning Sacco protocol

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Is s deducible from T ?

Answer: Of course, Yes !

$$\begin{array}{c}
 \text{aenc}(\text{sign}(k, \text{sk}(a)), c) \quad \text{sk}(c) \\
 \hline
 \text{sign}(k, \text{sk}(a)) \\
 \hline
 \text{senc}(s, k) \qquad \qquad \qquad k \\
 \hline
 s
 \end{array}$$

1. $A \rightarrow C : \text{aenc}(\text{sign}(k, \text{priv}(A)), \text{pub}(C))$
2. $C(A) \rightarrow B : \text{aenc}(\text{sign}(k, \text{priv}(A)), \text{pub}(B))$
3. $B \rightarrow A : \text{senc}(s, k)$ Attack !

Exercise (continued)

Let $T_0 = \{a, b, c, \text{sk}(c), \text{aenc}(\text{sign}(k, \text{sk}(a)), c)\}$.

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Denning Sacco protocol

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Is $\text{aenc}(\text{sign}(k, \text{sk}(a)), b)$ deducible from T_0 ?

Answer: Of course, Yes !

$$\frac{\frac{\text{aenc}(\text{sign}(k, \text{sk}(a)), c) \quad \text{sk}(c)}{\text{sign}(k, \text{sk}(a))} \quad b}{\text{aenc}(\text{sign}(k, \text{sk}(a)), b)}$$

The deduction problem

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Output: Is u deducible from T ?

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Proposition

The deduction problem is decidable in PTIME.

Deciding deduction (in this simple setting)

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Proposition

The deduction problem is decidable in PTIME.

Algorithm

- 1 **Saturation** of T with terms in $St(T \cup \{u\})$ that are deducible in one step;
- 2 if u is in the saturated set then return **Yes** else return **No**.

Soundness, completeness, and termination

Soundness If the algorithm returns **Yes** then u is indeed deducible from T . \longrightarrow easy to prove

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Completeness If the term u is deducible from T , then the algorithm returns **Yes**. Otherwise, it returns **No**.
 \longrightarrow this relies on a **locality property**

Locality lemma

Let T and u be such that $T \vdash u$. There exists a proof tree witnessing this fact for which all the nodes are labeled by some v with $v \in St(T \cup \{u\})$.

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Let P be a proof tree witnessing the fact that $T \vdash u$ having a **minimal size** (number of nodes). We show by **induction on P** that:

- if P ends with root labeled by v then P only contains terms in $St(T \cup \{v\})$;

Locality lemma

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We first split the deduction rules into **two categories**:

- 1 **composition rules**: encryption, signature, and pairing
- 2 **decomposition rules**: decryption, projections, ...

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- if P ends with root labeled by v then P only contains terms in $St(T \cup \{v\})$;
- if P ends with a **decomposition rule** then P only contains terms in $St(T)$.

→ this is left as an exercise

Exercise

Consider the following set of deduction rules:

$$\frac{x \quad sk(y)}{\text{sign}(x, sk(y))} \quad \frac{\text{sign}(x, sk(y)) \quad vk(y)}{x} \quad \frac{y}{vk(y)}$$

- 1 Give an example showing that these deduction rules are **not local**.
- 2 Extend the notion of subterms to restore the locality property, and show that the deduction problem is decidable.

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- 1 Give an example showing that these deduction rules are **not local**.
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Solution

- 1 Let $T = \{\text{sign}(s, sk(a)); a\}$ and $u = s$.
- 2 $St^+(T) = St(T) \cup \{vk(u) \mid sk(u) \in St(T)\}$.
→ the locality proof is left as an exercise

Exercise

Consider the following set of deduction rules:

$$\frac{x \quad y}{\langle x, y \rangle} \quad \frac{\langle x, y \rangle}{x} \quad \frac{\langle x, y \rangle}{y} \quad \frac{x \quad y}{\text{senc}(x, y)} \quad \frac{\text{senc}(x, y) \quad y}{x}$$

In order to decide whether a term u is deducible from a set of terms T , we propose the following algorithm:

- 1 Starting from T , apply as much as possible the decryption and the projection rules. This leads to a set of terms called $\text{Decomposition}(T)$.
- 2 Check whether u can be obtained by applying the composition rules on top of terms in $\text{Decomposition}(T)$.
- 3 In case of success, the algorithm returns **Yes**. Otherwise, it returns **No**.

Questions

What about termination, soundness, and completeness?

Modelling messages
and
Deciding knowledge
(in a richer setting)

More cryptographic primitives

We may want to consider a richer term algebra and rely on an **equational theory E** to take into account the properties of the primitives

Exclusive or operator:

$$\begin{aligned}(x \oplus y) \oplus z &= x \oplus (y \oplus z) & x \oplus x &= 0 \\ x \oplus y &= y \oplus x & x \oplus 0 &= x\end{aligned}$$

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Blind signature (used in evoting protocol)

$$\begin{aligned}\text{check}(\text{sign}(x, y), \text{vk}(y)) &= x \\ \text{unblind}(\text{blind}(y, y), y) &= x \\ \text{unblindsign}(\text{sign}(\text{blind}(x, y), z), y) &= \text{sign}(x, z)\end{aligned}$$

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Homomorphic encryption:

$$\begin{aligned} \text{enc}(\langle x, y \rangle, z) &= \langle \text{enc}(x, z), \text{enc}(y, z) \rangle & \text{sdec}(\text{senc}(x, y), y) &= x \\ \text{dec}(\langle x, y \rangle, z) &= \langle \text{dec}(x, z), \text{dec}(y, z) \rangle & \text{proj}_1(\langle x, y \rangle) &= x \\ & & \text{proj}_2(\langle x, y \rangle) &= y \end{aligned}$$

Going back to the Denning Sacco protocol

$A \rightarrow B$: $\text{aenc}(\text{sign}(k, \text{priv}(A)), \text{pub}(B))$

$B \rightarrow A$: $\text{senc}(s, k)$

What function symbols and equations do we need to model this protocol?

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① symmetric encryption: $\text{senc}(\cdot, \cdot), \text{sdec}(\cdot, \cdot)$

$\longrightarrow \text{sdec}(\text{senc}(x, y), y) = x$

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② asymmetric encryption: $\text{aenc}(\cdot, \cdot), \text{adec}(\cdot, \cdot), \text{pk}(\cdot)$

$$\longrightarrow \text{adec}(\text{aenc}(x, \text{pk}(y)), y) = x$$

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 $\longrightarrow \text{sdec}(\text{senc}(x, y), y) = x$
- 2 asymmetric encryption: $\text{aenc}(\cdot, \cdot)$, $\text{adec}(\cdot, \cdot)$, $\text{pk}(\cdot)$
 $\longrightarrow \text{adec}(\text{aenc}(x, \text{pk}(y)), y) = x$
- 3 signature: $\text{sign}(\cdot, \cdot)$, $\text{check}(\cdot, \cdot)$
 $\longrightarrow \text{check}(\text{sign}(x, y), \text{pk}(y)) = x$

Deduction in this more general setting

Deduction rules are as follows:

$$\frac{u_1 \quad \cdots \quad u_k}{f(u_1, \dots, u_k)} \quad f \in \mathcal{F} \qquad \frac{u}{u'} \quad u =_{\mathcal{E}} u'$$

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$$\frac{u_1 \quad \cdots \quad u_k}{f(u_1, \dots, u_k)} \quad f \in \mathcal{F}$$

$$\frac{u}{u'} \quad u =_{\mathbf{E}} u'$$

Example: Let $\mathbf{E} := \text{sdec}(\text{senc}(x, y), y) = x$ and $T = \{\text{senc}(\text{secret}, k), k\}$.
We have that $T \vdash \text{secret}$.

$$\frac{\frac{\text{senc}(\text{secret}, k) \quad k}{\text{sdec}(\text{senc}(\text{secret}, k), k)} \quad \text{sdec} \in \mathcal{F}}{\text{secret}} \quad \text{sdec}(\text{senc}(x, y), y) = x$$

The deduction problem: is u deducible from ϕ ?

We consider a signature \mathcal{F} and an equational theory E .

The deduction problem

Input A sequence $\phi = \{w_1 \triangleright v_1, \dots, w_n \triangleright v_n\}$ of terms and a term u

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Characterization of deduction

$T \vdash u$ if, and only if, there exists a term R such that $R\phi =_E u$.

\longrightarrow such a term R is a **recipe** of the term u .

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Example: Let $\phi = \{w_1 \triangleright \text{pk}(ska); w_2 \triangleright \text{pk}(skb); w_3 \triangleright skc;$
 $w_4 \triangleright \text{aenc}(\text{sign}(k, ska), \text{pk}(skc)); w_5 \triangleright \text{senc}(s, k)\}$.

We have that:

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Characterization of deduction

$T \vdash u$ if, and only if, there exists a term R such that $R\phi =_E u$.

→ such a term R is a **recipe** of the term u .

Example: Let $\phi = \{w_1 \triangleright \text{pk}(ska); w_2 \triangleright \text{pk}(skb); w_3 \triangleright \text{skc}; w_4 \triangleright \text{aenc}(\text{sign}(k, ska), \text{pk}(skc)); w_5 \triangleright \text{senc}(s, k)\}$.

We have that:

- k is deducible from ϕ using $R_1 = \text{check}(\text{adec}(w_4, w_3), w_1)$,

The deduction problem: is u deducible from ϕ ?

We consider a signature \mathcal{F} and an equational theory E .

The deduction problem

Input A sequence $\phi = \{w_1 \triangleright v_1, \dots, w_n \triangleright v_n\}$ of terms and a term u

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We have that:

- k is deducible from ϕ using $R_1 = \text{check}(\text{adec}(w_4, w_3), w_1)$,
- s is deducible from ϕ using $R_2 = \text{sdec}(w_5, R_1)$.

Proposition

The **deduction problem** is decidable for the equational theory modelling the DS protocol (and actually any subterm convergent equational theory).

Algorithm:

- 1 **saturation** of ϕ with its deducible **subterm**; we get ϕ^+
- 2 does there exist a recipe R such that $R\phi^+ = s$ (syntactic equality)

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Some other equational theories

Blind signature

$$\begin{aligned}\text{check}(\text{sign}(x, y), \text{vk}(y)) &= x \\ \text{unblind}(\text{blind}(y, y), y) &= x \\ \text{unblindsign}(\text{sign}(\text{blind}(x, y), z), y) &= \text{sign}(x, z)\end{aligned}$$

Decidability can be shown in a similar fashion **extending the notion of subterm**.

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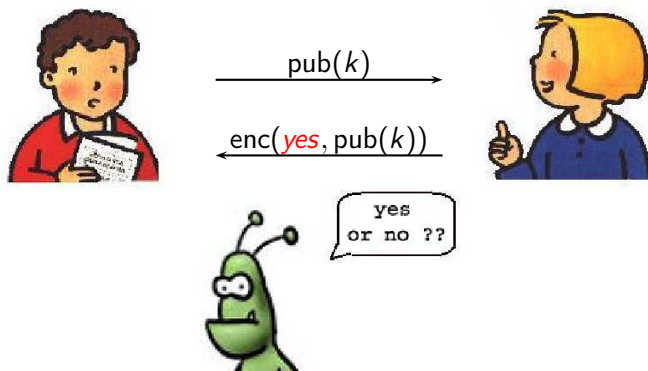
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Exclusive or

$$\begin{aligned}(x \oplus y) \oplus z &= x \oplus (y \oplus z) & x \oplus x &= 0 \\ x \oplus y &= y \oplus x & x \oplus 0 &= x\end{aligned}$$

The deduction problem can be reduced to the problem of **solving systems of linear equations over $\mathbb{Z}/2\mathbb{Z}$** .

Deduction is not always sufficient



→ The intruder **knows** the values **yes** and **no** !

The real question

Is the intruder able to tell whether Alice sends **yes** or **no**?

The static equivalence problem

Input Two frames ϕ and ψ

$$\phi = \{w_1 \triangleright u_1, \dots, w_\ell \triangleright u_\ell\} \quad \psi = \{w_1 \triangleright v_1, \dots, w_\ell \triangleright v_\ell\}$$

Output Can the attacker distinguish the two frames, *i.e.* does there exist a **test** $R_1 \stackrel{?}{=} R_2$ such that:

$$R_1\phi =_E R_2\phi \text{ but } R_1\psi \neq_E R_2\psi \text{ (or the converse).}$$

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Example: Consider the frames:

- $\phi = \{w_1 \triangleright \text{pk}(sks); w_2 \triangleright \text{aenc}(\text{yes}, \text{pk}(sks))\}$; and
- $\psi = \{w_1 \triangleright \text{pk}(sks); w_2 \triangleright \text{aenc}(\text{no}, \text{pk}(sks))\}$.

They are **not** in static equivalence: $\text{aenc}(\text{yes}, w_1) \stackrel{?}{=} w_2$.

Exercise

Consider the equational theories:

- E_{senc} defined by $\text{sdec}(\text{senc}(x, y), y) = x$, and
- E_{cipher} which extends E_{senc} by the equation $\text{senc}(\text{sdec}(x, y), y) = x$.

Questions

Which of the following pairs of frames are statically equivalent ? Whenever applicable give the distinguishing test.

$$\begin{array}{ccc} \{w_1 \triangleright \text{yes}\} & \stackrel{?}{\sim}_{E_{\text{senc}}} & \{w_1 \triangleright \text{no}\} \\ \{w_1 \triangleright \text{senc}(\text{yes}, k)\} & \stackrel{?}{\sim}_{E_{\text{senc}}} & \{w_1 \triangleright \text{senc}(\text{no}, k)\} \\ \{w_1 \triangleright \text{senc}(n, k), w_2 \triangleright k\} & \stackrel{?}{\sim}_{E_{\text{senc}}} & \{w_1 \triangleright \text{senc}(n, k), w_2 \triangleright k'\} \\ \{w_1 \triangleright \text{senc}(n, k), w_2 \triangleright k\} & \stackrel{?}{\sim}_{E_{\text{cipher}}} & \{w_1 \triangleright \text{senc}(n, k), w_2 \triangleright k'\} \end{array}$$

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- 1 **saturation** of ϕ/ψ with their deducible **subterms** ϕ^+/ψ^+
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→ actually, we only need to consider **small tests**

Example

Consider the frames:

- $\phi = \{w_1 \triangleright \text{aenc}(\langle \text{yes}, r_1 \rangle, \text{pk}(sks)); w_2 \triangleright sks\}$; and
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→ **Conclusion:** ϕ^+ and ψ^+ are **not** in static equivalence: $w_4 \stackrel{?}{=} \text{yes}$.

Some other equational theories

Blind signature

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This can be done in a similar fashion **extending** a bit **the notion of subterm**
→ again $\text{sign}(m, k)$ will be considered as a subterm of $\text{sign}(\text{blind}(m, r), k)$.

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Exclusive or

$$\begin{aligned}(x \oplus y) \oplus z &= x \oplus (y \oplus z) & x \oplus x &= 0 \\ x \oplus y &= y \oplus x & x \oplus 0 &= x\end{aligned}$$

The static equivalence problem can be reduced in PTIME to the problem of deciding whether two systems of linear equations have the **same set of solutions** over $\mathbb{Z}/2\mathbb{Z}$.

Existing decidability/complexity results and tools

Theory E	Deduction	Static Equivalence
subterm convergent	PTIME decidable [Abadi & Cortier, TCS'06]	
blind sign., addition, homo. encryption		
ACU	NP-complete	PTIME
Exclusive Or Abelian Group	PTIME	PTIME
ACUNh/AGh	PTIME	decidable
[D., IPL'05; Cortier & D., JAR'12]		

→ A **combination** result for disjoint theories [Cortier & D., JAR'12]

→ **Automatic tools for checking static equivalence**: YAPA M. Baudet (2006); KISS S. Ciobaca (2010); and FAST B. Conchinha (2011)

Modelling protocols and security properties

Applied pi calculus

[Abadi & Fournet, 01]

basic programming language with constructs for **concurrency** and **communication**

→ based on the π -calculus [Milner *et al.*, 92], and in some ways similar to the spi-calculus [Abadi & Gordon, 98]

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Some advantages:

- allows us to model **cryptographic primitives**
- both **reachability** and **equivalence**-based specification of properties

Protocols as processes - syntax and semantics

Syntax :	P, Q	$:=$	0	null process
			$\text{in}(c, x).P$	input
			$\text{out}(c, u).P$	output
			$\text{if } u = v \text{ then } P \text{ else } Q$	conditional
			$P \mid Q$	parallel composition
			$!P$	replication
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Semantics \rightarrow :

Comm	$\text{out}(c, M).P \mid \text{in}(c, x).Q \rightarrow P \mid Q\{M/x\}$
Then	$\text{if } M = N \text{ then } P \text{ else } Q \rightarrow P \text{ when } M =_E N$
Else	$\text{if } M = N \text{ then } P \text{ else } Q \rightarrow Q \text{ when } M \neq_E N$

closed by **structural equivalence** (\equiv) and application of **evaluation contexts**.

Going back to Denning Sacco protocol

$A \rightarrow B$: $\text{aenc}(\text{sign}(k, \text{priv}(A)), \text{pub}(B))$

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Alice and Bob as processes:

$P_A(sk_a, pk_b) = \text{new } k. \text{out}(c, \text{aenc}(\text{sign}(k, sk_a), pk_b)).$
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One possible scenario:

$P_{DS} = \text{new } sk_a, sk_b. (P_A(sk_a, pk(sk_b)) \mid P_B(sk_b, pk(sk_a)))$

Going back to Denning Sacco protocol

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$P_{DS} = \text{new } sk_a, sk_b. (P_A(sk_a, pk(sk_b)) \mid P_B(sk_b, pk(sk_a)))$

$\rightarrow \text{new } sk_a, sk_b, k. (\text{in}(c, x_a). \text{let } y_a = \text{sdec}(x_a, k) \text{ in} \dots$
 $\mid \text{let } y_b = k \text{ in new } s. \text{out}(c, \text{senc}(s, y_b)))$

Going back to Denning Sacco protocol

$A \rightarrow B$: $\text{aenc}(\text{sign}(k, \text{priv}(A)), \text{pub}(B))$

$B \rightarrow A$: $\text{senc}(s, k)$

Alice and Bob as processes:

$P_A(sk_a, pk_b) = \text{new } k. \text{out}(c, \text{aenc}(\text{sign}(k, sk_a), pk_b)).$
 $\text{in}(c, x_a). \text{let } y_a = \text{sdec}(x_a, k) \text{ in} \dots$

$P_B(sk_b, pk_a) = \text{in}(c, x_b). \text{let } y_b = \text{check}(\text{adec}(x_b, sk_b), pk_a) \text{ in}$
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One possible scenario:

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$\rightarrow \text{new } sk_a, sk_b, k, s. (\text{let } y_a = \text{sdec}(\text{senc}(s, k), k) \text{ in} \dots \mid 0)$

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→ this simply models a **normal execution** between two **honest** participants

Confidentiality for process P w.r.t. secret s

For **all processes** A such that $A \mid P \rightarrow^* Q$, we have that Q is not of the form $C[\text{out}(c, s).Q']$ with c public.

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Some difficulties:

- we have to consider **all** the possible executions in presence of an **arbitrary adversary** (modelled as a process)
- we have to consider **realistic** initial configurations
 - replications to model an **unbounded** number of sessions,
 - reveal public keys and private keys to model **dishonest** agents,
 - P_A/P_B may play with other (and perhaps) dishonest agents, ...

Going back to the Denning Sacco protocol

$A \rightarrow B : \text{aenc}(\text{sign}(k, \text{priv}(A)), \text{pub}(B))$

$B \rightarrow A : \text{senc}(s, k)$

The aforementioned attack

1. $A \rightarrow C : \text{aenc}(\text{sign}(k, \text{priv}(A)), \text{pub}(C))$

2. $C(A) \rightarrow B : \text{aenc}(\text{sign}(k, \text{priv}(A)), \text{pub}(B))$

3. $B \rightarrow A : \text{senc}(s, k)$

The “minimal” initial configuration to retrieve the attack is:

$\text{new } sk_a. \text{new } sk_b. (\text{out}(c, \text{pk}(sk_b)) \mid P_A(sk_a, \text{pk}(sk_c)) \mid P_B(sk_b, \text{pk}(sk_a)))$

Going back to the Denning Sacco protocol

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Exercise: Exhibit the process A (the behaviour of the attacker) that witnesses the aforementioned attack.

Security properties - authentication

This can be expressed as a **correspondence property**:

if B finishes a session, thinking he has talked to A then A has also finished a session, thinking she has talked to B (+ possibly agreement on some values).

Enriched syntax for processes:

P, Q	$:=$	0	null process
		$\text{in}(c, x).P$	input
		\dots	
		$\text{event } p(u_1, \dots, u_n).P$	event

Authentication properties with agreement on some values:

$$\forall x. \text{EndB}(a, b, x) \Rightarrow \text{EndA}(a, b, x)$$

confidentiality for an unbounded number of sessions

- **undecidable** in general [Even & Goldreich, 83; Durgin *et al*, 99]

▶ More details

- some **decidability results** for some restricted fragment, e.g. one variable per protocol's rule [Comon & Cortier, 03]

- **ProVerif**: A tool that does not correspond to any decidability result but works well in practice. [Blanchet, 01]

▶ More details

confidentiality for a bounded number of sessions

- a **decidability** result (NP-complete)
[Rusinowitch & Turuani, 01; Millen & Shmatikov, 01]
- result extended to deal with various cryptographic primitives.

→ various automatic tools, e.g. **AVISPA platform** [Armando *et al.*, 05]
More details about this tomorrow !

Challenge

Would you be able to find the attack on the well-known
Needham-Schroeder protocol?

$$\begin{aligned} A \rightarrow B &: \{A, N_a\}_{\text{pub}(B)} \\ B \rightarrow A &: \{N_a, N_b\}_{\text{pub}(A)} \\ A \rightarrow B &: \{N_b\}_{\text{pub}(B)} \end{aligned}$$


To help you:

<http://www.lsv.ens-cachan.fr/~delaune/VTSA/proverif.pdf>

Questions ?

See you tomorrow !

Post Correspondence Problem

Input A sequence of tiles $(u_0, v_0) (u_1, v_1) \dots (u_n, v_n)$ with $u_i, v_i \in \{a, b\}^*$.

Output Does there exist $k \geq 1$, and $1 \leq i_1, \dots, i_k \leq n$ such that

$$u_{i_1} \dots u_{i_k} = v_{i_1} \dots v_{i_k}$$

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Example:

u_1	u_2	u_3	u_4	v_1	v_2	v_3	v_4
<i>aba</i>	<i>bbb</i>	<i>aab</i>	<i>bb</i>	<i>a</i>	<i>aaa</i>	<i>abab</i>	<i>babba</i>

A solution is **1431**. Indeed, we have that:

$$u_1.u_4.u_3.u_1 = aba.bb.aab.aba = a.babba.abab.a = v_1.v_4.v_3.v_1$$

No solution if we remove the tile (u_4, v_4) .

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Proposition: The PCP is undecidable.

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We built a protocol that admits an attack (s is revealed) if, and only if, PCP has a solution.

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We encode words and concatenation using pairs

- $babba$ is encoded as $\langle\langle\langle\langle b, a \rangle, b \rangle, b \rangle, a \rangle$,
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Building words

- $! \text{in}(\text{senc}(\langle x, y \rangle, k)).\text{out}(\text{senc}(\langle x \cdot u_1, y \cdot v_1 \rangle, k))$
- ...
- $! \text{in}(\text{senc}(\langle x, y \rangle, k)).\text{out}(\text{senc}(\langle x \cdot u_1, y \cdot v_1 \rangle, k))$

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Revealing the secret s : $\text{in}(\text{senc}(\langle z, z \rangle, k)).\text{out}(s)$

ProVerif is a verifier for cryptographic protocols that may **prove** that a protocol is secure or **exhibit attacks**.

- Online demo available at: <http://proverif.rocq.inria.fr/>
- Sources available on Bruno Blanchet's webpage

Advantages

- fully automatic, and quite efficient
- A rich process algebra: replication, else branches, ...
- Handles many cryptographic primitives
- Proves various security properties: secrecy, correspondences, equivalences

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No miracle

Termination is not guaranteed and sometimes the tool is not able to conclude.

→ still, ProVerif works well in practice.

Protocol	Result	ms
Needham-Schroeder shared key	Attack	52
Needham-Schroeder shared key corrected	Secure	109
Denning-Sacco	Attack	6
Denning-Sacco corrected	Secure	7
Otway-Rees	Secure	10
Otway-Rees, variant of Paulson98	Attack	12
Yahalom	Secure	10
Simpler Yahalom	Secure	11
Main mode of Skeme	Secure	23

Pentium III, 1 GHz.