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Geometric Modeling

Assignment Sheet 8

Assignment 1 [2 points] (Complexity)

Propose an $O(n)$ algorithm to compute the convex hull of a non-convex polygon given by the list vertices enumerated in the counter-clockwise direction.

Assignment 2 [1 points] (Incorrect algorithm)

Assume that P is a closed polyline in which no three consecutive vertices lie on the same line. The following algorithm to check if P is the boundary of a convex polygon does not work:

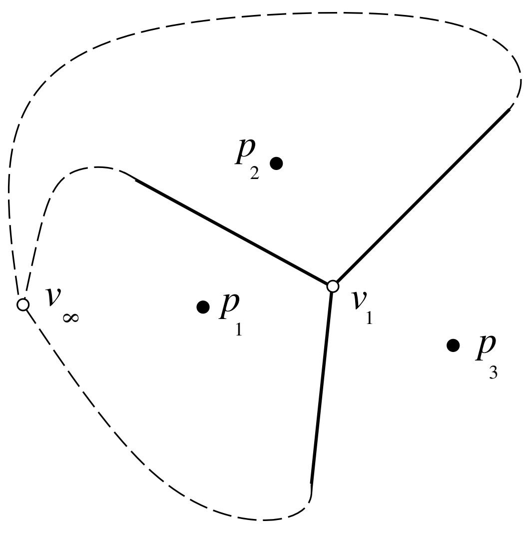
`If every consecutive sequence of three vertices along` P `form a left-turn, then say that` P `is convex. Otherwise, say that P is not convex.`

Draw an example polyline for which the algorithm incorrectly decides that it is the boundary of a convex polygon.

Assignment 3 [2 points] (Voronoi diagram)

Consider a set of n points (sites) P on the plane. Prove that for $n \geq 3$ the number of vertices in the Voronoi diagram of P, Vor(P), is at most $2n - 5$.

HINT: Prove the statement using Euler's formula. Since Euler's formula cannot be applied directly to Vor(P), because Vor(P) has infinite edges, add one extra vertex $\boldsymbol{v}_\infty$ "at infinity" to the set of sites P and connect all half-infinite edges of Vor(P) to $\boldsymbol{v}_\infty$. Apply Euler's formula to the obtained graph.



Assignment 4 [2 points] (Point on circle)

Given four points $\boldsymbol{p}_0$, $\boldsymbol{p}_1$, $\boldsymbol{p}_2$, $\boldsymbol{p}_3$ in the plane. Prove that point $\boldsymbol{p}_0$ lies on the circle through $\boldsymbol{p}_1$, $\boldsymbol{p}_2$, $\boldsymbol{p}_3$ if and only if

$$\begin{vmatrix} x_0 & y_0 & x_0^2 + y_0^2 & 1 \\ x_1 & y_1 & x_1^2 + y_1^2 & 1 \\ x_2 & y_2 & x_2^2 + y_2^2 & 1 \\ x_3 & y_3 & x_3^2 + y_3^2 & 1 \end{vmatrix} = 0$$

Assignment 5 [3 points] (Programming assignment, discrete Gaussian curvature)

Write a program that computes the number of inner mesh vertices $\mathbf{p}$ for which $|K(\mathbf{p})| < T$ for a given positive number T Here $K(\mathbf{p})$ is the discrete Gaussian curvature estimated via the angle deficit at $\mathbf{p}$. Test your program for $T = 1$ and $T = 2$ on the models available at
`http://www.mpi-inf.mpg.de/~ag4-gm/programming.html#off`
and report the results.

Due date: Thursday, July 13, *before* class