Speeding up Evolutionary Algorithms by Restricted Mutation Operators

TECHNISCHE FAKULTÄT DER CHRISTIAN-ALBRECHTS UNIVERSITÄT ZU KIEL

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Motivation

Applications show that specific variation operators can help to speed up computations

Aim here: Show this for a natural example by a rigorous runtime analysis

Eulerian Cycle Problem

Problem: Given an undirected Eulerian graph G with m edges. Find a permutation π that is a cycle.

Fitness Function
$\mathrm{path}(\pi) :=$ length of the path p that can be build up starting with the first edge in π.

General jump: $\mathrm{jump}(i,j)$ jump element on position i to position j
Restricted jump: $\mathrm{jump}(i) = \mathrm{jump}(i,1)$

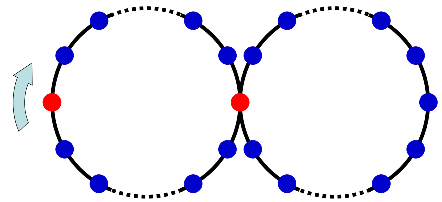
(1+1) EAs

General/Restricted (1+1) EA
1.) Choose $\pi \in S_m$ uniformly at random.
2.) Choose s due to a Poisson distribution with mean 1 and perform sequentially $s+1$ general/restricted jump operations.
3.) Replace π by π' if $\mathrm{path}(\pi') \geq \mathrm{path}(\pi)$.
4.) Repeat Steps 2 and 3 forever.

Runtime: Expected number of fitness evaluations

Previous Result: (Neumann, CEC 2004)
General jumps instead of restricted jumps
Upper bound: $O(m^5)$

The Restricted Operator



Upper Bound $O(m^3)$

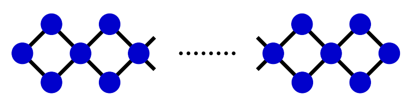
Crucial situation: Current path is a cycle.

Start vertex performs directed walk in one direction.

At each possibility a constant probability to lengthen the path

Expected time $O(m^2)$ for an improvement

$O(m)$ improvements => result



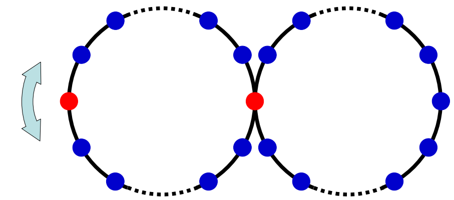
Lower Bound $\Omega(m^3)$

Turning point: A constant probability to “close” the current cycle.

$\Theta(m)$ improvements necessary. (Chernoff)

$\Theta(m^2)$ steps for an improvement. => result

The General Operator



Lower Bound $\Omega(m^4)$

Probability $1/3$ to obtain one single cycle.

Start vertex has distance at least $m/4$ to the vertex v of degree 4 with constant probability.

Cycle is revolved in both directions.
Start vertex of π performs a random walk on a cycle of length $\Theta(m)$. Thus, $\Theta(m^2)$ accepted steps to lenghthen the path.

Waiting time for an accepted step $\Omega(m^2)$.
Expected time $\Omega(m^4)$ integrate the second cycle.

Conclusion

Comparison of a general and a restricted operator by a rigorous runtime analysis

Specific operators can lead to faster algorithms

Restricted mutation operator changes structure of the plateau

Upper bound reduces from $O(m^5)$ to $O(m^3)$. Safe at least a factor of m.