A Quantum Computational Approach to Correspondence Problems on Point Sets

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Introduction

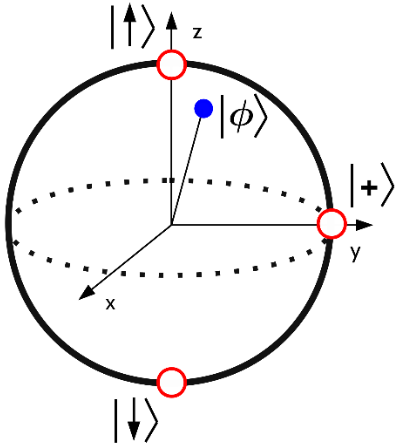
Since the proposal of a quantum computer by Y. Manin and R. Feynman in early 1980s, quantum computing hardware significantly progressed.

Adiabatic quantum computers (AQC) are already used to solve difficult combinatorial optimisation problems in various domains of science.

As of 2020, AQC are becoming mature for machine learning as well as CVPR.

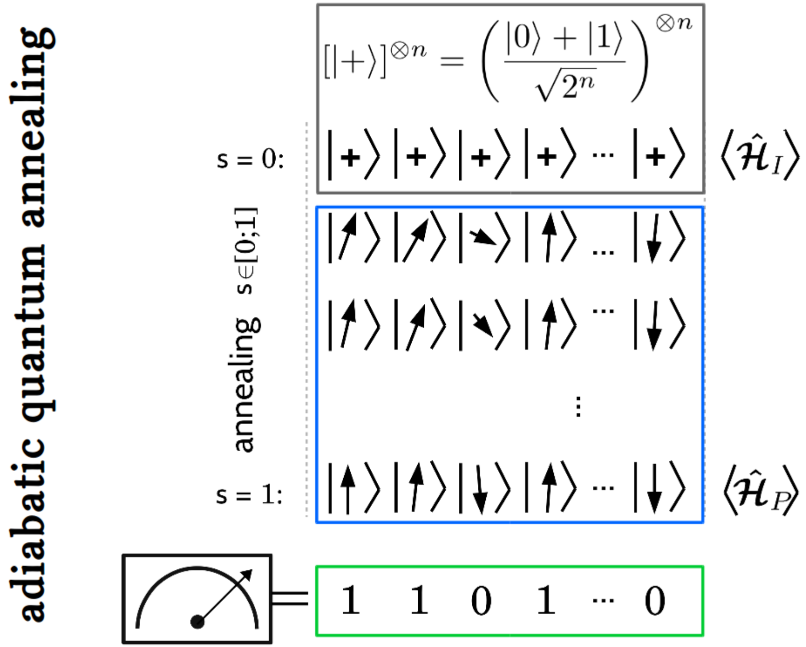
In this paper, we show how to formulate transformation estimation and point set alignment so that they can run on AQC, *i.e.*, as a *quadratic unconstrained binary optimisation problem* (QUBOP).

The Principle of Quantum Annealing



$$|\phi\rangle = \alpha|0\rangle + \beta|1\rangle$$
$$|\alpha|^2 + |\beta|^2 = 1$$

model of a qubit (Bloch sphere)



classical bitstring (ready for unembedding)

initialisation (initial default Hamiltonian)

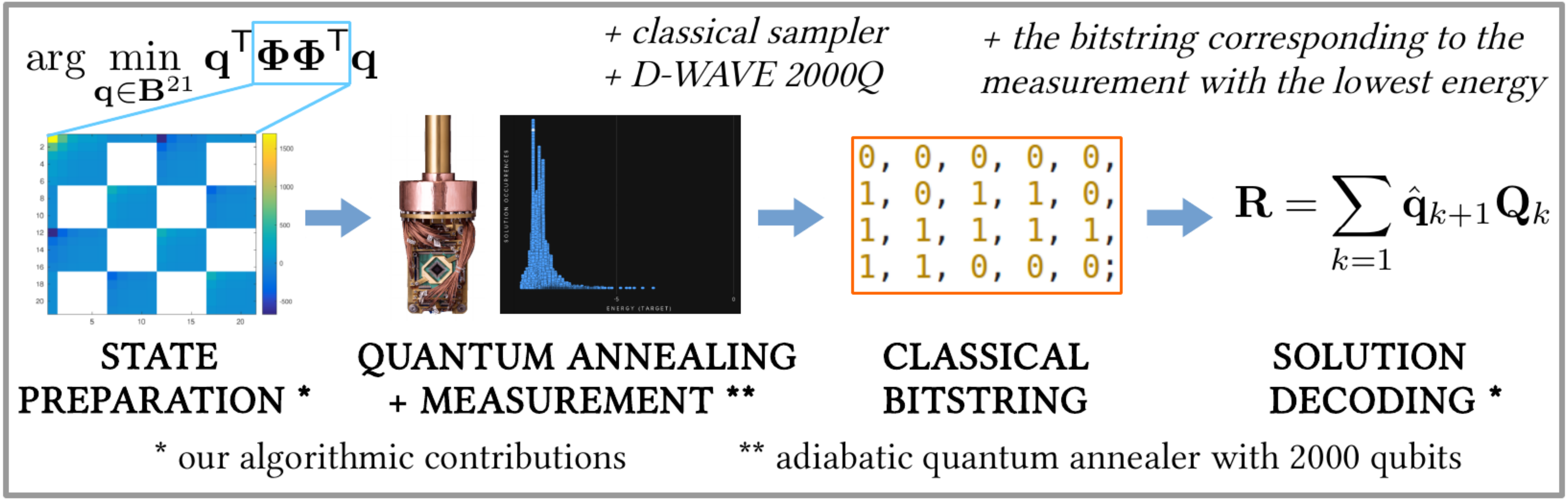
annealing (20 μs.)

the system evolves to the ground state of the problem Hamiltonian

measurement

qubits collapse to one of the basis states

Overview of Quantum Approach (QA)



+ the state is a symmetric matrix with qubit biases and interaction weights between qubits
+ unembedding is the decoding of the solution to QUBOP $\arg\min_{\mathbf{q}\in\mathbf{B}^{21}} \mathbf{q}^\top \mathbf{\Phi}\mathbf{\Phi}^\top \mathbf{q}$ to the solution of the original alignment problem (derived from the state preparation).

Overview of Quantum Approach (QA)

The final QUBOP:

$$\arg\min_{\mathbf{q}\in\mathbf{B}^{21}} \mathbf{q}^\mathsf{T}\mathbf{\Phi}\mathbf{\Phi}^\mathsf{T}\mathbf{q} \qquad \mathbf{P} = \mathbf{\Phi}\mathbf{\Phi}^\mathsf{T}$$

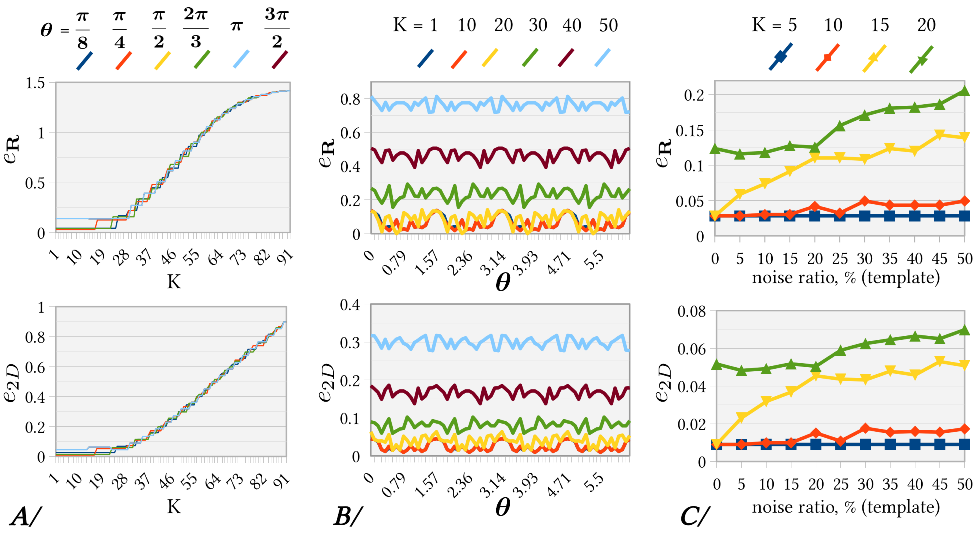
$$\mathbf{\Phi} = \begin{bmatrix} \mathbf{x}_1^\mathsf{T} & \mathbf{x}_2^\mathsf{T} & \dots & \mathbf{x}_N^\mathsf{T} \\ -[\mathbf{Q}_1\mathbf{y}_1]^\mathsf{T} & -[\mathbf{Q}_1\mathbf{y}_2]^\mathsf{T} & \dots & -[\mathbf{Q}_1\mathbf{y}_N]^\mathsf{T} \\ -[\mathbf{Q}_2\mathbf{y}_1]^\mathsf{T} & -[\mathbf{Q}_2\mathbf{y}_2]^\mathsf{T} & \dots & -[\mathbf{Q}_2\mathbf{y}_N]^\mathsf{T} \\ \vdots & \vdots & \ddots & \vdots \\ -[\mathbf{Q}_K\mathbf{y}_1]^\mathsf{T} & -[\mathbf{Q}_K\mathbf{y}_2]^\mathsf{T} & \dots & -[\mathbf{Q}_K\mathbf{y}_N]^\mathsf{T} \end{bmatrix} \quad \textbf{(2D)}$$

Basis for $\mathbf{R}$:

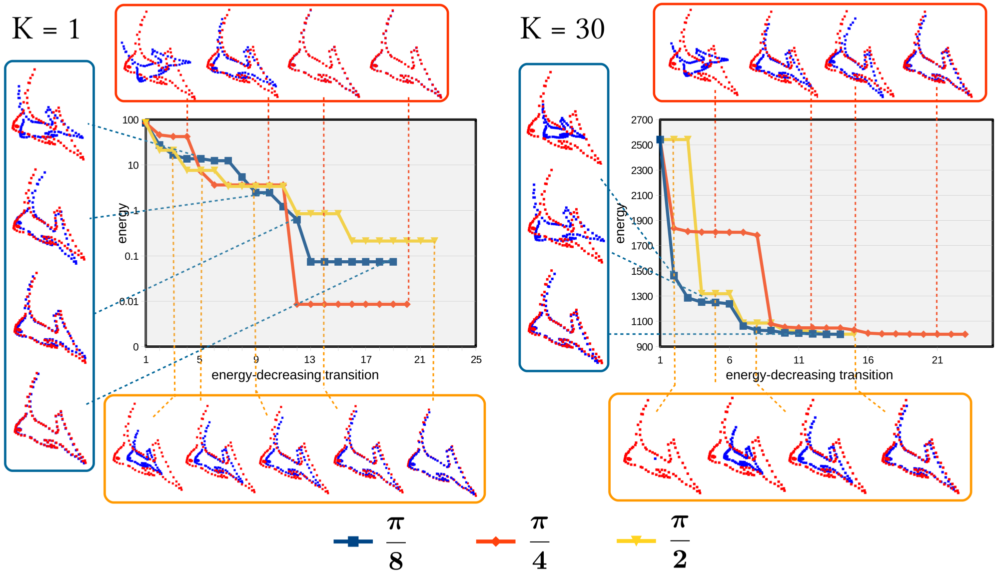
$$\Big\{\mathbf{Q}_k = \omega\,\mathbf{C} \in \mathbb{R}^{2\times 2}, \forall\omega \in \{0.5, 0.2, 0.1, 0.1, 0.05\}, \forall\mathbf{C} \in \{\mathbf{I}, \mathbf{M}, -\mathbf{I}, -\mathbf{M}\}\Big\}.$$

+ <u>the objective</u>: minimise the distances between the transformed template points and the corresponding reference points. The basis for $\mathbf{R}$ is suitable for quantum annealing.

Experiments on a Classical Sampler and Spectral Gap Analysis



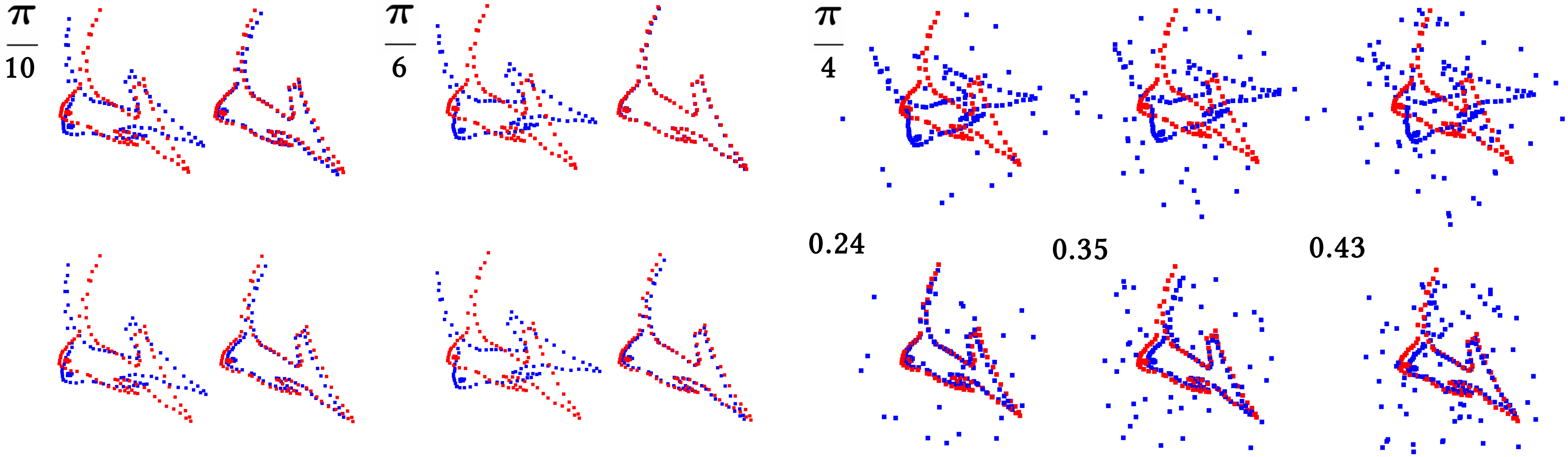
The metrics as the functions of A/: the size of the point interaction region parametrised by K; B/: the angle of initial misalignment θ; C/: the template noise ratio.



The sequences of energy-decreasing transitions and the corresponding energy values observed in our sampler.

+ K is the number of interacting template points, for every point of the reference

Experiments on D-WAVE 2000Q (2D)



K = 20 (top row), 30 (bottom row) **K = 30, noisy template**

+ it is foreseeable that future generations of quantum annealers will support QA for 3D data (this requires an improved AQC architecture and more physical qubits).

Thank You