Infinite Random Graphs

Timo Kötzing

Max Planck Institute for Informatics

May 24, 2011

Finite Random Graphs

- Consider a set of vertices V.
- Suppose, for now, V is finite, say, of size n.
- Pick two graphs on V uniformly at random.
- Erdős and Rényi 1963:

  What is the probability that those two graphs are isomorphic?
- Let p_n be that probability.

Probability of Isomorphy

How does p_n behave?

- Erdős and Rényi show:

$$\lim_{n\to\infty} p_n = 0.$$

- In fact: When drawing a graph G on n vertices u.a.r., then G has no automorphism with probability $1 - o(1)$.

Infinite Random Graphs

> *Now at last I get to write about one of my favourite topics in mathematics.* —Peter J. Cameron

- Consider an infinite set of vertices V; in fact, consider $V = \mathbb{N}$.
- We draw random graphs on V by deciding for each edge independently at random whether it is included with fixed probability $0 < p < 1$.
- Pick two graphs on V at random as above.
- Let p_∞ be the probability that those two graphs are isomorphic.
- Erdős and Rényi:

$$p_\infty = 1.$$

Key Lemma

Lemma

- Let G be a random graph on $\mathbb{N}$.
- Let U, W be disjoint finite sets of vertices.
- Then there is a $z \in \mathbb{N} \setminus (U \cup W)$ such that
  1. there is an edge in G between z and any member of U;
  2. there is no edge in G between z and any member of W.

Any two graphs with the above property are isomorphic.

The Infinite Random Graph

The infinite random graph R has a number of interesting properties.

- Every finite graph is an induced subgraph of R.
- Every countable graph is an induced subgraph of R.
- R has a cyclic automorphism.
- R has continuum-many different cyclic automorphisms.
- If we partition the vertices of R in two sets, then one of these sets induces a graph isomorphic to R (pigeonhole property).
- Concerning countably infinite graphs, only R, the empty and the complete graph have the pigeonhole property.

How to get more

Read Peter Cameron's blog on it. Google for (fourth hit):

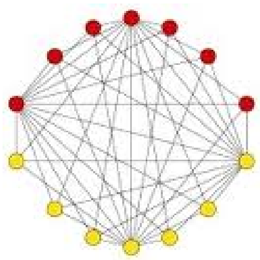
Peter Cameron random Graph

Read the paper:

Asymmetric Graphs. Erdős and Rényi, 1963.

Read a text book:

Chapter 11 of *Graph Theory* by Reinhard Diestel.



Thank You.