Selected Topics in Algorithms
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Exercise 4

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We will discuss this exercise sheet on May 29^{th}.

Motivation

We fill in some details of de Pina's algorithm for minimum cycle basis.

de Pina's Algorithm

de Pina suggested the following algorithm.

$B := \emptyset$
while $|B| < m - (n-1)$ **do**
  compute a non-zero $S \in k^E$ such that $\langle C, S \rangle = 0$ for all $C \in B$.
  compute a minimum weight (isometric) circuit C with $\langle C, S \rangle \neq 0$.
  add C to B.
end while

Correctness: Show that both versions of the algorithm (with and without the adjective isometric) computes a minimum weight k-basis.

Finding a Minimum Weight Circuit For the field of two elements (undirected cycle basis), the following method computes a minimum weight circuit.
Set up an auxiliary graph G_A. For each vertex v of G, we have vertices $(v,0)$ and $(v,1)$ in G_A. For each edge $e = uv \in G$, we have the edges $((u,i),(v,i+S_e))$ for $i = 0,1$ in G_A. Here, addition is modulo two.

- Illustrate this definition by a small example.
- Consider a path in G_A from $(v,0)$ to $(v,1)$. Argue that is corresponds to a circuit C in G with $\langle C, S \rangle \neq 0$.
- Derive an alg for computing a minimum weight circuit with $\langle C, S \rangle \neq 0$.

Have fun with the solution!