Master Thesis
Integral Cycle Bases
Structure, Algorithms, Complexity

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A cycle basis of a graph G is called *integral* if every cycle in G can be written as an integral linear combination of the cycles in B. See [KLM+09] for a survey on cycle basis; see also the notes at `www.mpi-inf.mpg.de/~mehlhorn/SelectedTopics/SelectedTopics.html`

Integral cycle bases are not well understood. They have an important application, namely the computation of cyclic time tables[1]. This application is briefly discussed in the survey and further references can be found there.

The goal of the thesis is to advance the state of the art on integral cycle basis along three axes.

Structural Results: What kind of circuits occur in minimum integral cycle bases? Find reduction rules. E.g., is the following true? Let G be a graph with a self-loop e. Then a minimum integral basis for G consists of a minimum integral basis for $G \setminus e$ plus the circuit consisting of the edge e.

Some structural results can be found in the survey.

Exact and Approximate Algorithms: For a graph G with total weight $W = \sum_{e \in E} w(e)$, there is always an integral basis of weight $O(W \log n)$. Such a basis can be constructed in polynomial time. Provide an efficient implementation of this algorithm. Here efficiency is meant in a double sense: Small running time and small constant in the weight bound.

There are efficient algorithms for computing minimum directed and undirected cycle bases. These algorithms are available at Dimitris Michail's web page. Experiment with these algorithms. For what graphs do they yield, do they not yield, integral basis.

The minimum integral cycle basis problem can be formulated as an integer program. The existing ILP formulation is inefficient and only tiny graphs can be solved with it. Provide a better ILP formulation. Explore other methods for computing optimal integral bases. For example, is there a systematic way for changing the edge weights such that a minimum basis with respect to the modified weights is (likely to be) integral.

[1] Public transportation system typically run according to a cyclic schedule

Complexity: Is the minimum integral cycle basis problem in P? Is it NP-complete?

Difficulty: The topic for a master thesis should pose questions at different levels. Some, the advisor should know how to solve, some the advisor should know how to attack, but not necessarily solve, and some the advisor should consider challenges that are beyond a master's thesis.

Here, the questions in the paragraph algorithms are of the first kind, the questions in the paragraph structure are of the second kind, and the question in the paragraph complexity is of the third kind.

References

[KLM+09] T. Kavitha, Ch. Liebchen, K. Mehlhorn, D. Michail, R. Rizzi, T. Ueckerdt, and K. Zweig. Cycle Bases in Graphs: Characterization, Algorithms, Complexity, and Applications. 78 pages, submitted for publication, available at `www.mpi-inf.mpg.de/~mehlhorn/ftp/SurveyCycleBases.pdf`, March 2009.