Exercise 1 *Let $B^{(1)}$, $B^{(2)}$, $\ldots$ be an unknown sequence of bits. We have n experts. The i-th expert predicts $b_i^{(1)}$, $b_i^{(2)}$, $\ldots$. Whom should we follow?*

Consider the following weighted majority rule:

let $w_i^{(1)} =$ *for all i,* $1 \leq i \leq n$.
for $t = 1, 2, \ldots$ ***do***
  let $W^{(t)} = \sum_i w_i^{(t)}$
  let $b^{(t)} =$ *nearest integer to* $\sum_i w_i^{(t)} b_i^{(t)} / W^{(t)}$;
  predict $b^{(t)}$
  learn $B^{(t)}$
  for $1 \leq i \leq n$ ***do***
    if $B^{(t)} \neq b_i^{(t)}$ ***then***
      $w_i^{(t+1)} = (1-\varepsilon) w_i(t)$
    end if
  end for
end for

1. *Let $m_i^{(t)}$ be the number of mistakes made by the i-th expert in rounds 1 to t, and let $m^{(t)}$ be the number of mistakes made by the majority rule. Prove*
   - $w_i^{(t+1)} = (1-\varepsilon)^{m_i^{(t)}}$
   - *if $b^{(t)} \neq B^{(t)}$, then $W^{(t+1)} \leq (1-\varepsilon/2) W^{(t)}$.*
   - $W^{(t+1)} \leq (1-\varepsilon)^{m^{(t)}}$.
2. *Use the previous item to bound $m^{(t)}$.*
3. *Interpret the result.*

Exercise 2 *Consider a network of k parallel links with lengths $L_1 < L_2 < \ldots < L_k$. Let D_i be the diameter of the i-th link.*

1. *$R_i = L_i/D_i$ is the resistance of the i-th link. Compute the effective resistance of the network.*
2. *Compute the potential difference between source and sink and the current on the i-th link.*
3. *Let $D = \sum_i D_i$. Determine the derivative $\dot{D}$ of D.*
4. *Let $x_i = D_i/D$. Determine the derivative $\dot{x}_i$ of x_i. Answer: $\dot{x}_i = (1/D)(L/L_i - 1)x_i$, where L is defined in the next item.*
5. *Let L be such that $1/L = \sum_i x_i/L_i$. Show that $L_1 \leq L \leq L_k$. Show that L is decreasing.*
6. *Conclude that at all times there is an index $k(t)$ such that D_i increases for $i \leq k(t)$ and grows for $i > k(t)$. Moreover, $k(t)$ is a decreasing function of time.*