



Dr. Michael Hemmer, Dr. Michael Sagraloff, Michael Kerber

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Exercises for Non-linear Computational Geometry

Assignment 1

Due 31.10.2008

Exercise 1 (ϵ Points)

Write an email to `mkerber[at]mpi-sb.mpg.de` with subject “Non-linear Computational Geometry” containing your name and Matrikelnummer.

Exercise 2 (5 Points)

- a) Consider the circles

$$C_1 = V((x-1)^2 + y^2 - 2)$$

$$C_2 = V(x^2 + (y+2)^2 - 3)$$

$$C_3 = V(x^2 + y^2 - 4.1599)$$

How many different pairwise intersection points exist? Is $C_1 \cap C_2 \cap C_3 = \emptyset$? (proof!)

- b) For which values of λ is

$$C_1 \cap C_2 \cap C^{(\lambda)} \neq \emptyset,$$

where $C^{(\lambda)} = V(x^2 + y^2 - \lambda)$?

Exercise 3 (5 Points)

For this exercise, we recommend to use the web demo at <http://exacus.mpi-sb.mpg.de>.

Consider the parameterized curve

$$C^{(\lambda_1, \dots, \lambda_6)} := V(y^4 - y^3 + 2x^2y^2 + 3x^2y + x^4 + \lambda_1x^2y + \lambda_2xy + \lambda_3x + \lambda_4)$$

- a) Start with $\lambda_1 = \dots = \lambda_4 = 0$. Where does $C^{(0, \dots, 0)}$ have a singular point (self-intersection)?
- b) Set $\lambda_1 = 0.001$, is there a significant change ?
- c) Set $\lambda_2 = 0.001$ as well. What is the number of singular points ? Is there a significant change at all ?
- d) What happens if we set $\lambda_3 = 0.001$ as well ?
- e) Set $\lambda_4 = 0.001$. Is there a self intersection ?

PLEASE TURN OVER

Exercise 4 (5 Points)

We consider the *orientation predicate* for three planar points:

$$\text{orient}(p, q, r) = \text{sgn}((q_x - p_x)(r_y - p_y) - (q_y - p_y)(r_x - p_x))$$

a) Consider the points

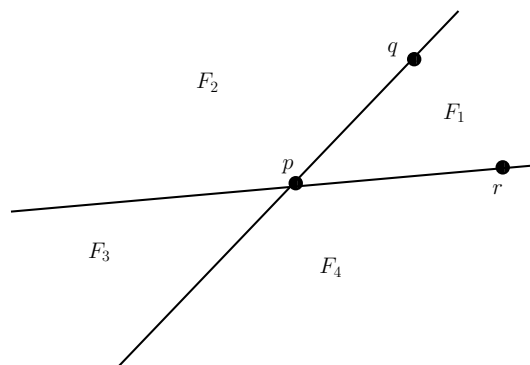
$$p^{(x,y)} := \begin{pmatrix} 0.5 + 2^{-64}x \\ 0.5 + 2^{-64}y \end{pmatrix} \quad q := \begin{pmatrix} 12 \\ 12 \end{pmatrix} \quad r := \begin{pmatrix} 24 \\ 24 \end{pmatrix}$$

Write a C++ program using **long double** arithmetic that produces a 256×256 pixel image. For $0 \leq i, j < 256$, the pixel (i, j) is drawn red if $\text{orient}(p^{(i,j)}, q, r) = 1$, it is drawn yellow if $\text{orient}(p^{(i,j)}, q, r) = 0$, and it is drawn blue if $\text{orient}(p^{(i,j)}, q, r) < 0$.

We suggest that the picture is stored in the very simple ppm-format (Portable Pixmap). The format is supported by, e.g., *gimp* (GNU Image Manipulation Program). A short documentation of the format can be found at: http://de.wikipedia.org/wiki/Portable_Pixmap.

Send the source code and the resulting picture to `mkerber[at]mpi-sb.mpg.de`.

b) Consider three points p, q, r in the plane. The lines pq and pr subdivide the plane into 4 regions F_1, \dots, F_4 .



Define a predicate to test whether a point $s := (s_x, s_y)$ lies inside F_1 .