Turnaround Scheduling in Chemical Manufacturing

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1 Introduction

In chemical manufacturing and petroleum refining, large-scale maintenance activities must be conducted on a regular basis. Entire production units are shut down for disassembly and comprehensive inspection and renewal. Such a process is called *Shutdown and Turnaround* (or turnaround for short). It is an essential process but causes high out-of-service cost. Therefore a good schedule for the turnaround is of high priority to the manufacturer.

In a turnaround a huge number of interdependent operations must be executed by maintenance groups of different specializations (different resources). Scheduling these is already a complex task if working shifts must be respected. However, in this particular problem another issue increases the complexity drastically: the duration of an operation is flexible in the sense that it is determined by the number of resources that is assigned to it. Technical reasons restrict the choice to a range between a maximum and minimum duration. Clearly, due to communication delays, the duration of an operation does not decrease with the same rate as the number of assigned resources increases but we can assume a non-increasing discrete function.

Each resource unit causes a certain cost for each time unit it is used. It can be assumed that the number of available resources is unbounded because a presumably arbitrary number of external resources can be acquired. However, for some resource types resource units cannot be obtained for arbitrary small time periods but must be kept and paid for a certain minimal period. That means that the resource requirement of each resource type (or a subset) must be leveled over at least such a time period – in a simplification this is the entire turnaround period.

A feasible schedule consist of a feasible temporal assignment of resource units to operations and a feasible sequencing for all of them. Now, the goal is to find feasible schedules of minimum length for different budgets. In fact, we have a generalized version of the *Discrete Time-Cost Tradeoff Problem*. In case that the out-of-service cost for taking the production unit off-stream are given per time unit, the objective changes to finding a schedule that minimizes the total cost, i.e. the sum of out-of-service cost and the cost for acquiring resources over the turnaround period.

Related work. The turnaround scheduling problem can be interpreted as a generalization of the *Time-Cost Tradeoff* Problem and the problem of scheduling *malleable jobs*. In fact, relaxing the goal of leveling the resource usage for each type over the project duration leads to

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the *Discrete Time-Cost Tradeoff* problem. It is known to be $\mathcal{NP}$-hard [1] and approximation algorithms have been investigated; see Skutella [10]. In case that cost functions are continuous linear functions, the Time-Cost Tradeoff Problem can be solved optimally by combinatorial algorithms as has been shown independently by Fulkerson [2] and Kelley [4]. Later, Phillips and Dessouky [8] gave an improved version of the original algorithms in which iterative cut computations in a graph of critical jobs yield the time-cost tradeoff curve. The running time is polynomial in the number of breakpoints of the optimal time-cost curve. In this model (with linear or discrete cost functions), individual deadlines for jobs are not considered, the number of available resources is not restricted, and the resource usage is not leveled.

The problem of scheduling *malleable jobs* can be seen as a step towards combining time-cost optimization with resource leveling. Here, resource capacities are predefined in the form of parallel machines. Precedence constraints are given and the duration of a job is determined by the number of machines on which it is processed. Recent work by Lepére et al. [5] and Jansen and Zhang [3] provide approximation algorithms with constant performance guarantees.

2 Our contribution

In this talk, we introduce the problem of turnaround scheduling in chemical manufacturing and give an overview on related problems. We report on our experience with solving a real-world turnaround scheduling problem within a feasibility study. We describe the particular problem setting and describe a two-phase solution method.

1. In the first phase an extended Linear Time-Cost Tradeoff Problem, where deadlines must be respected, is solved optimally based on iterative minimum cost flow computations. The number of iterations is bounded by a polynomial in the number of breakpoints of the optimal curve.

   It is not always desirable to compute the exact time-cost curve since the number of breakpoints can be exponential [10] in the input data. In such cases it suffices to compute a solution arbitrarily close to optimality in polynomial time. The approximated convex time-cost curve can be found by a parametric search algorithm in the spirit of *Sandwich Algorithms* to approximate convex functions; see Rote [9].

   Based on that exact or approximated solution, a deadline T, i. e., a maximum total turnaround duration, is fixed either by the user or by minimizing the total cost in case that out-of-service costs are given per time unit.

2. In the second stage, a feasible schedule is found such that it obeys the turnaround deadline T and the resource usage (for certain resource types) is leveled over the turnaround duration interval. The latter goal translated into the objective to minimize the cost for the maximum resource usage at any time times the turnaround duration T summed up for all resource types.

   We solve this problem heuristically by combining binary search on resource capacities with different List scheduling algorithms.

We solve problems with $100.000 - 150.000$ jobs within a few minutes and yield good solutions as a comparison with integer linear programming solutions proves. The formulation we use is time-indexed and thus hopelessly large for problem sizes as they are typical in turnarounds for chemical manufacturing.

Finally, we would like to emphasize another interesting line of research that our ongoing project reveals – that is, dealing with uncertainty in turnaround scheduling. One question addresses the uncertainty of a determined schedule. Here, techniques by Meilijson and Nadas [6] and Weiss [11] can be applied to determine an upper bound on the expected lateness $\mathrm{E}[\max\{t - C_{\max}, 0\}]$ for a given project schedule with makespan $C_{\max}$ as a function of the completion time t. Interestingly, the computation of this function is strongly related to the Linear Time-Cost Tradeoff Problem.

Nevertheless, this method applied subsequently to an optimization process does not suggest good scheduling policies for jobs with uncertain durations. The uncertainty of maintenance activities should be incorporated into the optimization process in the flavor of classical stochastic scheduling; see, e. g., [7].

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