

Termination Proofs for Systems Code

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Abstract

Program termination is central to the process of ensuring that reactive systems can always react. We describe a new interprocedural, path-sensitive and context-sensitive program termination prover that provides capacity for large program fragments (i.e. more than 20,000 lines of code) together with support for programming language features such as arbitrarily nested loops, pointers, function-pointers, side-effects, etc. We also present experimental results on device driver dispatch routines from the Windows operating system. The most distinguishing aspect of our tool is how it shifts the balance between the two tasks of *constructing* and respectively *checking* the termination argument. Checking becomes the hard step. In this paper we show how we solve the corresponding challenge of *checking* with *binary reachability analysis*.

1. Introduction

Reactive systems (e.g. operating systems, web servers, mail servers, database engines, etc) are usually constructed from a set of components that we expect will always terminate. Cases where these functions unexpectedly do not return to their calling context leads to non-responsive systems and system crashes. Device driver dispatch routines, for example, must always eventually return to their caller. Consider the function in Figure 1 which is called from several dispatch routines within the Windows serial enumeration device driver. This code calls other serial-based device drivers by passing I/O request packets via the kernel routine `IoCallDriver` (line 50, `pIrp` is a pointer to the request packet and `FdoData->TopOfStack` is the pointer to another serial-based device driver). In the case where the other device driver returns a return-value that indicates success, but does not place a positive value in `PIoStatusBlock->Information`, the serial enumeration driver will fail to increment the value pointed to by `nActual` (line 66), possibly causing the driver to infinitely execute this loop and not return to its calling context. The consequence of this error is that the computer's serial devices could become non-responsive. Worse yet, depending on what actions the other device driver takes, this loop may cause repeated acquiring and releasing of kernel resources (memory, locks, etc) at high priority and excessive physical bus activity. This extra work stresses the operating system, the other drivers, and the user applications

running on the system, which may cause them to crash or become non-responsive too.

This example demonstrates how a notion of termination is central to the process of ensuring that reactive systems can always react. Until now no automatic interprocedural, path-sensitive and context-sensitive termination tool has ever been able to provide a capacity for large program fragments (>20,000 lines) together with accurate support for programming language features such as arbitrarily nested loops, pointers, function-pointers, side-effects, etc. In this paper we describe such a tool, called TERMINATOR.

TERMINATOR's most distinguishing aspect, with respect to previous methods and tools for proving program termination, is how it shifts the balance between the two tasks of *constructing* and respectively *checking* the termination argument. The classical method is to construct an expression defining the *rank* of a state and to then check that its value decreases in every transition from a reachable state to another one. The construction of the ranking function is the hard part and forms a task that needs to be applied to the whole program. The checking part is relatively easy. In our method, the task of constructing ranking functions is the relatively easy part; they are constructed on demand based on the examination of only a few selected paths through the program.

Furthermore, TERMINATOR is not required to construct only *one* correct termination argument but rather a set of guesses of possible arguments, some of which may be bad guesses. That is, this set need not be the exact set of the 'right' ranking functions but only a *superset*. We find the same monotonicity as with iterative abstraction refinement (the set of predicates need not be the exact set of 'right' predicates but only a superset).

Checking the termination argument is the hard part of our method. This is because the termination argument is a set of ranking functions—meaning that it is not sufficient to show that at least one of them decreases in every transition from a reachable state to another one. Instead we must show that at least one of them decreases in every *sequence of transitions*, of any length. We call this step *binary reachability analysis*. Previously, it was not known whether binary reachability analysis could be made practical. It was our challenge (i.e. the challenge raised by our approach) to show that this is indeed the case.

In this paper, we show that one can make binary reachability analysis practical. Furthermore, through experiments with TERMINATOR on Windows device drivers, we demonstrate that it is effective at proving termination arguments for industrial systems code.

2. TERMINATOR

The algorithm underlying TERMINATOR is outlined in [12]. In this section we briefly describe TERMINATOR's design so as to explain the role that *binary reachability* plays in it.

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```

1 NTSTATUS
2 Serenum_ReadSerialPort(CHAR * PReadBuffer, USHORT Buflen,
3                       ULONG Timeout, USHORT * nActual,
4                       IO_STATUS_BLOCK * PIoStatusBlock,
5                       const FDO_DEVICE_DATA * FdoData)
6 {
7     NTSTATUS status;
8     IRP * pIrp;
9     LARGE_INTEGER startingOffset;
10    KEVENT event;
11    SERIAL_TIMEOUTS timeouts;
12    ULONG i;
13
14    startingOffset.QuadPart = (LONGLONG) 0;
15    //
16    // Set the proper timeouts for the read
17    //
18
19    timeouts.ReadIntervalTimeout = MAXULONG;
20    timeouts.ReadTotalTimeoutMultiplier = MAXULONG;
21    timeouts.ReadTotalTimeoutConstant = Timeout;
22    timeouts.WriteTotalTimeoutMultiplier = 0;
23    timeouts.WriteTotalTimeoutConstant = 0;
24
25    KeInitializeEvent(&event, NotificationEvent, FALSE);
26
27    status = Serenum_IoSyncIoctlEx(IOCTL_SERIAL_SET_TIMEOUTS, FALSE, FdoData->TopOfStack,
28                                &event, &timeouts, sizeof(timeouts), NULL, 0);
29
30    if (!NT_SUCCESS(status)) {
31        return status;
32    }
33
34    Serenum_KdPrint(FdoData, SER_DBG_SS_TRACE, ("Read pending...\n"));
35
36    *nActual = 0;
37
38    while (*nActual < Buflen) {
39        KeClearEvent(&event);
40
41        pIrp = IoBuildSynchronousFsdRequest(IRP_MJ_READ, FdoData->TopOfStack,
42                                          PReadBuffer, 1, &startingOffset,
43                                          &event, PIoStatusBlock);
44
45        if (pIrp == NULL) {
46            Serenum_KdPrint(FdoData, SER_DBG_SS_ERROR, ("Failed to allocate IRP\n"));
47            return STATUS_INSUFFICIENT_RESOURCES;
48        }
49
50        status = IoCallDriver(FdoData->TopOfStack, pIrp);
51
52        if (status == STATUS_PENDING) {
53
54            status = KeWaitForSingleObject(&event, Executive, KernelMode, FALSE, NULL);
55
56            if (status == STATUS_SUCCESS) {
57                status = PIoStatusBlock->Status;
58            }
59        }
60
61        if (!NT_SUCCESS(status) || status == STATUS_TIMEOUT) {
62            Serenum_KdPrint(FdoData, SER_DBG_SS_ERROR, ("IO Call failed with status %x\n", status));
63            return status;
64        }
65
66        *nActual += (USHORT)PIoStatusBlock->Information;
67        PReadBuffer += (USHORT)PIoStatusBlock->Information;
68    }
69
70    return status;
71 }

```

Figure 1. Utility function containing a termination bug. This function is used by several dispatch routines in the Windows serial enumeration device driver, SERENUM.SYS

$T := \emptyset$ (* termination argument: union of well-founded relations *)

repeat

(* check binary reachability for T *)

if $R_I^+ \subseteq T$ **then**

report “Terminating”

else

$\rho :=$ a binary relation such that $\rho \subseteq R_I^+$ but $\rho \not\subseteq T$

(* find rank function for ρ if it exists *)

if ρ is not well-founded relation **then**

report “Not Terminating”

else

(* construct termination argument *)

$W :=$ a ranking relation, i.e. $W \supseteq \rho$, W well-founded

$T := T \cup W$

end.

Figure 2. Algorithm underlying TERMINATOR. The binary reachability analysis to check the inclusion $R_I^+ \subseteq T$ is described in Section 3. The binary relation ρ is represented by a sequence of statements with a loop. The construction of a ranking function and of the corresponding ranking relation W is implemented via techniques explained in [20].

TERMINATOR iterates between two procedures, its *binary reachability analysis* in order to check a termination argument, and a method of rank function synthesis in order to incrementally construct a termination argument. See Figure 2.

We assume a program P with a transition relation R and a set of initial states I . We define the binary reachability relation R_I^+ as the transitive closure of R restricted to reachable states; it consists of the pairs of states (s_1, s_2) such that s_2 is reachable from s_1 in at least one step and s_1 itself is reachable from an initial state s_0 . Formally,

$$R_I^+ \triangleq \{(s_1, s_2) \mid \exists s_0 \in I. (s_0, s_1) \in R^* \wedge (s_1, s_2) \in R^+\}$$

Binary reachability analysis is a procedure that checks whether R_I^+ is contained in a given binary relation T :

$$R_I^+ \subseteq T.$$

In the case that the inclusion does not hold, there exists a non-empty sequence of statements $\tau_1, \dots, \tau_i, \dots, \tau_n$ with an execution sequence $s_0 \rightsquigarrow_{\tau_1} s_1 \dots s_{i-1} \rightsquigarrow_{\tau_i} s_i \dots s_{n-1} \rightsquigarrow_{\tau_n} s_n$ such that the pair of states (s_i, s_n) does not lie in T , formally $(s_i, s_n) \in R_I^+$ but $(s_i, s_n) \notin T$. By the form of T in our setting, the two states s_i and s_n will always have the same program location; thus, the statements $\tau_{i+1}, \dots, \tau_n$ form a loop. The relation ρ consists of all pairs of states (s_i, s_n) that lie on an execution sequence of the form described above (induced by the sequence of statements $\tau_1, \dots, \tau_i, \dots, \tau_n$). That is, the relation ρ is a subset of R_I^+ but not of T .

The relation ρ is a well-founded relation iff the sequence of statements $\tau_1, \dots, \tau_i, \dots, \tau_n$ is—when thought of as a program with a single while loop—a terminating program, which is iff a rank function exists. A rank synthesis tool based on [20] constructs a ranking function and the corresponding ranking relation, which consists simply of the pairs of states with decreasing, positive rank. Thus, the constructed ranking relation W contains the relation ρ as a subset and is well-founded. The union T of ranking relations, however, is in general not well-founded. This is why it would not be sufficient to show the inclusion $R \subseteq T$, and why we must instead prove $R_I^+ \subseteq T$.

Example. Consider Figure 3. In order to prove this program terminating, TERMINATOR incrementally constructs a relation T that must eventually contain the binary reachability relation of that program. It is sufficient to specify (and to construct) the subrelations T^ℓ with pairs of states at the same program location ℓ , i.e. $T^\ell = \{(s, t) \in T \mid s(\text{pc}) = t(\text{pc}) = \ell\}$, where ℓ is one of 7, 12, 28 and 33. These locations form a set of cutpoints, in the terminology of Floyd [14].

In order to show that the binary reachability relation R_I^+ is contained in a union of well-founded relations (the overall proof goal), it is sufficient to show that this is the case for its restriction to pairs of states at the same program location ℓ , for each cutpoint ℓ .

The relations T^ℓ constructed by TERMINATOR are listed in Figure 4. Note that $T^{28} = T_0^{28} \cup T_1^{28} \cup T_2^{28}$ is a union of well-founded relations but is itself not well-founded.

TERMINATOR starts with T_0^ℓ , the empty relation denoted by `false`, and adds ranking relations T_1^ℓ, T_2^ℓ etc. until there are no more counterexamples for ℓ . This means that the relation T^ℓ formed by their union contains every pair (s_i, s_n) of states at location ℓ such that s_i is reachable from a state s_1 at the start of `main` and s_n is reachable from s_i by any non-empty sequence of execution steps.

TERMINATOR’s binary reachability analysis is designed to support programs with pointer aliasing, as found in Figure 3. Notice that the termination argument at program location 28, T^{28} , does not specify the relationships between `x`, `y`, `p`, and `q`. This is an example of the separation of concerns in TERMINATOR. Binary reachability analysis tracks the aliasing of `*p` but the termination argument does not specify the aliasing relationships. Furthermore, the construction of the termination argument does not need to track them.

In order to prove that T^{33} is complete with respect to pairs of states at location 33, the binary reachability analysis must derive and prove that `b=true` is a program invariant at location 36. It thus subsumes the synthesis of program invariants, the task of standard reachability analysis. Again, this is an example of the separation of concerns in TERMINATOR, since the construction of T^{33} does not involve deriving and proving the invariant.

If we comment out the code at line 11 in Figure 3 then TERMINATOR reports that the termination proof has failed, and produces the following path through the program, denoted by line numbers (notice that the call to `Ack` is not reachable in the first iteration of the loop).

```
stem = 20 → 21 → 23 → 24 → 25 → 27 → 29 → 39 → 40 →
       41 → 42 → 44 → 27 → 29 → 30 → 3 → 5 → 9 →
cycle = 12 → 3 → 5 → 6 → 7 → 3 → 5 → 9 → 12
```

The path is a lasso consisting of two parts, a stem and a cycle; the cycle is iterated forever if it is entered by a state that results from executing the stem. Notice that in general, the stem and the cycle can contain the unrolling of one or more loops of the program.

3. Binary reachability analysis

In this section we design an implementation of binary reachability analysis. We proceed with describing the following steps:

1. We give a characterization of R_I^+ by the least fixpoint of a function F .
2. Guided by the structure of F , we show a transformation on P . This transformation creates a new program \hat{P} that implements F . The least fixpoint of F is equivalent to the set of reachable states in \hat{P} .
3. We describe a transformation of \hat{P} that creates \hat{P}_T . An error location in \hat{P}_T is not reachable iff the inclusion $R_I^+ \subseteq T$ holds.

```

1 int Ack(int x, int y)
2 {
3     if (x>0) {
4         int n;
5         if (y>0) {
6             y--;
7             n = Ack(x,y);
8         } else {
9             n = 1;
10        }
11        x--;
12        return Ack(x,n);
13    } else {
14        return y+1;
15    }
16 }
17
18 void main()
19 {
20     int x = nondet();
21     int y = nondet();
22
23     int * p = &y;
24     int * q = &x;
25     bool b = true;
26
27     while(x<100 && 100<y && b)
28     {
29         if (p==q) {
30             int k = Ack(nondet(),nondet());
31             (*p)++;
32             while((k--)>100)
33             {
34                 if (nondet()) {p = &y;}
35                 if (nondet()) {p = &x;}
36                 if (!b) {k++;}
37             }
38         } else {
39             (*q)--;
40             (*p)--;
41             if (nondet()) {p = &y;}
42             if (nondet()) {p = &x;}
43         }
44         b = nondet();
45     }
46 }

```

Figure 3. Example program. `nondet()` is used to represent non-deterministic chosen integers and Booleans

4. Finally, we describe how TERMINATOR transforms error paths in \widehat{P}_T to input for TERMINATOR’s rank function synthesis engine.

3.1 Fixpoint characterization

We first define a function F whose least fixpoint is R_I^+ . The domain of F consists of binary relations over states of the given program. The least element is the transition relation restricted to initial states.

$$\perp \triangleq \{(s_1, s_2) \mid s_1 \in I \wedge (s_1, s_2) \in R\}$$

Before formalizing F , we define an auxiliary function $\text{id}_{(2)}$ that restricts the identity relation over the program states to the image of its input relation, formally,

$$\text{id}_{(2)}(X) \triangleq \{(s_2, s_2) \mid \exists s_1. (s_1, s_2) \in X\}.$$

ℓ	well-founded binary relations T_k^ℓ
7	$T_0^7(s, t) \triangleq \text{false}$ $T_1^7(s, t) \triangleq t(y) > -1 \wedge t(y) < s(y)$
12	$T_0^{12}(s, t) \triangleq \text{false}$ $T_1^{12}(s, t) \triangleq t(x) > -1 \wedge t(x) < s(x)$
28	$T_0^{28}(s, t) \triangleq \text{false}$ $T_1^{28}(s, t) \triangleq t(y) > 100 \wedge t(y) < s(y)$ $T_2^{28}(s, t) \triangleq t(x) < 100 \wedge t(x) > s(x)$
33	$T_0^{33}(s, t) \triangleq \text{false}$ $T_1^{33}(s, t) \triangleq t(k) > 100 \wedge t(k) < s(k)$

Figure 4. The termination argument (a union of well-founded binary relations T_k^ℓ between states at the same cutpoint location ℓ) incrementally constructed and then checked by TERMINATOR, thus proving the termination of the program in Figure 3. TERMINATOR starts with T_0^ℓ , the empty relation denoted by `false`.

Let \circ denote the relational composition operator:

$$X \circ Y \triangleq \{(s_1, s_3) \mid \exists s_2. (s_1, s_2) \in X \wedge (s_2, s_3) \in Y\}.$$

The function F takes a binary relation X as input. It returns the relational composition of the union of X and the identity relation restricted to the second component, with the transition relation R of the program.

$$F(X) \triangleq (X \cup \text{id}_{(2)}(X)) \circ R$$

In effect F either copies the righthand component of X into the left before passing it to R , or else it simply passes X itself to R .

THEOREM 1. *The binary reachability relation R_I^+ of the program P is equal to the least fixpoint of the function F on the domain of binary relations with the least element \perp :*

$$R_I^+ = \text{lfp}(F, \perp).$$

3.2 Reachability characterization

We define a transformation \widehat{P} of the program P and an equivalence relation \simeq between pairs of states in P and states of \widehat{P} . The transformation reflects the structure of the function F , expressed in P ’s programming language. We will establish a connection between the least fixpoint of F over \perp and the set of reachable states of \widehat{P} .

Let $V = \{v_1, \dots, v_n, \text{pc}\}$ be the set of program variables in P , including the program counter. The set of variables \widehat{V} of transformed program \widehat{P} contains V and the corresponding pre-versions $\text{'}V = \{\text{'}v_1, \dots, \text{'}v_n, \text{'}pc\}$ (We will discuss how TERMINATOR create the pre-versions for pointer variables and records in Section 4.2). We say that a state \widehat{s} in \widehat{P} is equivalent to a pair of states (s_1, s_2) in P , written $\widehat{s} \simeq (s_1, s_2)$, if the following conditions hold.

$$\begin{array}{ll}
\widehat{s}(\text{'}v_1) = s_1(v_1) & \widehat{s}(v_1) = s_2(v_1) \\
\vdots & \vdots \\
\widehat{s}(\text{'}v_n) = s_1(v_n) & \widehat{s}(v_n) = s_2(v_n) \\
\widehat{s}(\text{'}pc) = s_1(pc) & \widehat{s}(pc) = s_2(pc)
\end{array}$$

Let \widehat{I} be the set of initial states of \widehat{P} defined as follows.

$$\widehat{I} \triangleq \{\widehat{s} \mid \exists s_0 \in I. \widehat{s} \simeq (s_0, s_0)\}$$

```

L: stmt  ⇒
      L:  if (nondet()) {
            'v1 = v1;
            ...
            'vn = vn;
            'pc = L;
          } else {
            skip;
          }
      stmt

```

Figure 5. Transformation used to construct \widehat{P} from P .

See Figure 5. We construct \widehat{P} by applying this transformation on each statement of the program P . Let $\text{post}_{\widehat{P}}$ denote the post operator of the program \widehat{P} .

There is an analogy between transformed statements and the structure of the function F : first, the assignment statements in Figure 5 correspond to the application $\text{id}_{(2)}(X)$; secondly, the `if`-conditional with non-deterministic choice between the branches corresponds to the union $X \cup \text{id}_{(2)}(X)$; and finally, the relational composition with R is reflected by i) having the original statement of the program P after the conditional and ii) the connection between states \widehat{s} and pairs of states (s_1, s_2) . We formalize the analogy in the following theorem.

THEOREM 2. *The least fixpoint of the function F on the domain of binary relations with the least element \perp is equal to the set of states of the transformed program \widehat{P} reachable after at least one step:*

$$\text{lfp}(F, \perp) = \text{post}_{\widehat{P}}^+(\widehat{I}).$$

The equality holds under the assumption that we identify a state \widehat{s} with a pair of states (s_1, s_2) if they are \simeq -equivalent.

The statement of Theorem 2 involves the following technicalities:

- We assume that there are no statements in P whose destination location is the initial location of P .
- We assume that the operator $\text{post}_{\widehat{P}}$ treats the compound statements

```
L: if (nondet()) {...} stmt
```

of the program \widehat{P} as monolithic ones. This means that the intermediate states at locations of \widehat{P} added due to the transformation are not considered to be elements of $\text{post}_{\widehat{P}}^+(\widehat{I})$.

We can now implement the binary reachability analysis in three steps:

- Create the transformed program \widehat{P} ,
- Compute the set of reachable states $\text{post}_{\widehat{P}}^+(\widehat{I})$, and
- Check the inclusion between the computed set and T .

3.3 Non-reachability characterization

In practice, we would like to stop the reachability computation as soon as it becomes evident that the inclusion does not hold. In this section we describe an additional transformation, applied on the program \widehat{P} , that addresses this issue.

The additional transformation takes the program \widehat{P} and produces \widehat{P}_T by replacing each compound statement of \widehat{P} (except the initial statement):

```
L: if (nondet()) {...} stmt
```

by the statement below.

```
L: if ( ! T_L ) { ERROR: skip; }
    if (nondet()) {...} stmt
```

To construct the Boolean expression T_L we first assume that the relation T is represented by an assertion over variables ' V and ' V ', which denote the values of the program variables in the first and the second component of the pairs $(s_1, s_2) \in T$. Note that the program counter variable `pc` does not appear in the program text of C programs. Hence, we cannot insert the assertion T into the program text directly. To overcome this, we use the expression T_L that is obtained by substituting L for `pc` in T . For example, for $T = ((\text{'pc} = L12 \wedge \text{pc} = L12) \implies (x > -1 \wedge x < 'x))$ and location `L28` we obtain the following conditional:

```
L28: if ( ! ( ! ( 'pc==L12 && L28==L12 )
              || ( x>-1 && x<'x ) )
        )
    {
        ERROR: skip;
    }

```

We will revisit this example in Section 4.1.

THEOREM 3. *The inclusion $R_T^+ \subseteq T$ holds if and only if the location `ERROR` is not reachable in the program \widehat{P}_T .*

Now, we can apply a temporal safety checker on the program \widehat{P}_T to prove the non-reachability of the location `ERROR`.

3.4 Analyzing error paths of \widehat{P}_T

We assume that a temporal safety checker can produce an error path if the location `ERROR` is reachable. Next, we describe the interpretation of such an error path π in the context of `TERMINATOR`'s algorithm, which we have described in Section 2.

We need to extract a counterexample ρ to the inclusion $R_T^+ \subseteq T$ from the error path π . We observe that π must, at some point, traverse through the positive branch of the second conditional added by the program transformation in Figure 5. We split π at the latest appearance of such statement into *stem* and *cycle*. We then remove all statements that were added by the program transformation from the stem and the cycle. Let R_{stem} and R_{cycle} be the transition relations of the stem and the cycle, respectively. We produce the relation ρ , see Figure 2, as follows.

$$\rho \triangleq \{(s_2, s_3) \mid \exists s_1. (s_1, s_2) \in R_{stem} \wedge (s_2, s_3) \in R_{cycle}\}$$

The resulting relation ρ is represented as a conjunction of atomic assertions computed via a symbolic simulation of the path in P .

We assume that the safety checker also outputs an aliasing configuration between pointer variables that together with the error path π witnesses the reachability of the location `ERROR` in \widehat{P}_T . We encode this information into ρ by an additional conjunction.

If the ranking function synthesis fails for the relation ρ , the stem and the cycle constitute a possible counterexample to termination of the program P .

4. Optimizations

In this section we describe several optimizations that `TERMINATOR` applies during the program transformation described in Section 3. The first optimization exploits the fact that we can construct and check termination arguments for one location at a time. The remaining optimizations prune away executions of \widehat{P}_T that the temporal safety checker does not need to consider when trying to proving the non-reachability of the location `ERROR`. We implement these optimizations as additional program transformations and perform them during the construction of the program \widehat{P}_T .

4.1 Specialization of \widehat{P}_T

The termination argument T for the program P is a conjunction of termination arguments T^ℓ for each cutpoint ℓ . Each termination argument T^ℓ is of the form

$$(\text{'pc} = \ell \wedge \text{pc} = \ell) \implies (T_1^\ell \vee \dots \vee T_n^\ell).$$

Assume that TERMINATOR is processing the location ℓ . When the program transformation creates a statement of the program \widehat{P}_T at the location $\ell' \neq \ell$ then the corresponding expression \mathbb{T} is equivalent to `false`, since the conjunction $T^\ell \wedge \text{pc} = \ell'$ is valid. Thus, we can drop the statement

```
if ( ! T_L ) { ERROR: skip; }
```

at all locations different from ℓ (see the example in Section 3.3). Furthermore, we can also drop the statement

```
if ( nondet() ) { ... }
```

at these locations, since the states \widehat{s} that are created by taking the positive branch of the above conditional at location $\ell' \neq \ell$ cannot cause the computation to reach the location `ERROR`, because $\widehat{s} \notin T^\ell$.

We apply this optimization on all examples in the remainder of this section. See Figure 6 for an example of specialization of \widehat{P}_T .

4.2 Pre-variables for C programs

The example program in Figure 3 demonstrates that the correct handling of pointer variables is important when building a tool such as TERMINATOR.

The program transformation described in Section 3 implicitly requires that we create a pre-variable for every heap and stack location addressable using C expressions from the program variables in scope. However, in practice this is impossible.

Assume that TERMINATOR is processing a termination argument for a given cutpoint. For each variable x of scalar type, *i.e.*, `int`, `char`, `long`, etc., in scope of the cutpoint we count the number n of dereference operators in the type definition of the variable x . For example, if x is defined as `int **x`; then the resulting number n is two. For each $0 \leq i \leq n + 1$ we create the following pre-variable.

$$\underbrace{\text{'p...p}x}_{i \text{ times}}$$

and we insert the assignment statement

$$\underbrace{\text{'p...p}x}_{i \text{ times}} = \underbrace{*...*x}_{i \text{ times}};$$

in the second conditional, see Figure 5, during the program transformation.

TERMINATOR also creates pre-variables for field access expressions, say `x->f`, that appear in the termination argument. It creates a pre-variable `'x_f` and the corresponding assignment statement `'x_f = x->f`;

4.3 Structured programs

When proving the termination argument for a cutpoint ℓ within a structured program it is not necessary to prove termination at ℓ for executions that leave the ℓ loop and then return—these executions will be covered when proving termination of the outer loop. To implement this optimization we insert

```
if ( 'pc == L ) { exit(); }
```

into the source code of the transformed program at exit points of the loop that ℓ represents.

We illustrate this optimization on Figures 7 and 8. Assume that

```
1 void main()
2 {
3     int ...;
4 L0:
5 L1: while (...) {
6     ...
7 L2:     while (...) {
8         ...
9     }
10    ...
11    }
12    if (...)
13        goto L0;
14 }
```

Figure 7. Example program with three cutpoints L0, L1, and L2.

we are in the process of inferring a termination argument for the cutpoint that corresponds to the `while`-loop at the location L1. By inserting the conditional statement at line 9.1 we exclude from consideration states \widehat{s} , where $\widehat{s}(\text{pc}) = \text{L2}$ that appear on computations leaving the inner loop and coming back to the location L2.

```
1 void main()
2 {
3     int ...;
4 L0:
5 L1: while (...) {
6     ...
7 L2:     while (...) {
7.1         if ( ! T_L ) { ... }
7.2         if ( nondet() ) { ...; 'pc = L2; }
8         ...
9     }
9.1     if ( 'pc == L2 ) { exit(); }
10    ...
11    }
12    if (...)
13        goto L0;
14 }
```

Figure 8. Transformed program for cutpoint L2 with the `return`-statement at line 9.1 added due to optimization.

4.4 Weak binary reachability

In large programs, paths to a cutpoint can be very long. These paths may perform a computation that is not relevant to the termination analysis at the cutpoint. We observed examples of such paths on some dispatch routines when applying TERMINATOR. TERMINATOR can abstract away the prefixes of such paths in two different ways:

1. Do not consider all program statements that are executed between the start of the main function of the program and the call to the function containing the cutpoint under consideration.
2. Ignore all program statements within the function containing the cutpoint that appear between the first statement and the statement that corresponds to the cutpoint.

We introduce two approximations R_1^+ and R_2^+ of the binary reachability relation R_I^+ that correspond to the above abstractions. Then, we define the corresponding program transformations that given a

Program P		Program \widehat{P}_T	
		-1	int 'x, 'y, 'pc;
		0	
1	void main()	1	void main()
2	{	2	{
3	int x, y;	3	int x, y;
4		4	
5	L0: if (y>=1) {	4.1	'x = x;
6	while (x>=0)	4.2	'y = y;
7	{	4.3	'pc = L0;
8	L: x = x+y;	5	L0: if (y>=1) {
9	}	6	while (x>=0)
10	}	7	{
11	}	8.1	L: if ('pc==L && L==L && ! (x>=0 && x<='x-1)) {
		8.2	ERROR: ;
		8.3	}
		8.4	if (nondet()) {
		8.5	'x = x;
		8.6	'y = y;
		8.7	'pc = L;
		8.8	}
		8	x = x-y;
		9	}
		10	}
		11	}

Figure 6. Example of the program transformation for checking the binary reachability query $(\text{'pc} = L \wedge \text{pc} = L) \implies (x \geq 0 \wedge x \leq \text{'x} - 1)$ at the location L.

program \widehat{P}_T create programs \widehat{P}_T^1 and \widehat{P}_T^2 which implement the first and the second approximation respectively.

Let ℓ_{entry} be the entry location of the function containing the cutpoint that we are analyzing. We define the first approximation R_1^+ as follows.

$$R_1^+ \triangleq \{(s_1, s_2) \mid \exists s_0. s_0(\text{pc}) = \ell_{\text{entry}} \wedge (s_0, s_1) \in R^* \wedge (s_1, s_2) \in R^+\}$$

The second approximation R_2^+ is the transitive closure of R without any restrictions. Note that $R_1^+ \subseteq R_2^+ \subseteq R^+$.

The program \widehat{P}_T^1 , which represents the approximation R_1^+ , is obtained from \widehat{P} by inserting a call to the function containing the cutpoint as the first instruction in the function main. We illustrate \widehat{P}_T^1 in Figure 9.

The program \widehat{P}_T^2 , which represents the approximation R_2^+ , is obtained from \widehat{P}_T^1 inserting a goto-statement at the beginning of the function containing the cutpoint. The destination label of the goto-statement is the cutpoint location. We illustrate \widehat{P}_T^2 in Figure 10.

TERMINATOR first analyzes \widehat{P}_T^2 . If \widehat{P}_T^2 's error location is not reachable then TERMINATOR proceeds with the next cutpoint. If, however, an error path is found, then TERMINATOR switches its focus to \widehat{P}_T^1 . If \widehat{P}_T^1 's error location is not reachable then, again, TERMINATOR proceeds with the next cutpoint. If an error path in \widehat{P}_T^1 is found then TERMINATOR must revert to the original \widehat{P}_T . Note that predicates found by a predicate-abstraction based temporal safety checker during the analysis of \widehat{P}_T^2 can be reused with \widehat{P}_T^1 , etc.

This optimization does not affect TERMINATOR's precision: TERMINATOR produces counterexamples to termination only when

<pre> 1 void main() 2 { 3 ... 4 goo(); 5 ... 6 } 7 8 void goo() 9 { 10 ... 11 foo(); 12 ... 13 } 14 15 void foo() 16 { 17 ... 18 L1: while(a<b) { 19 ... 20 }</pre> <p style="text-align: center;">(a)</p>	<pre> 1 void main() 2 { 2.1 foo(); 2.2 exit(); 3 ... 4 goo(); 5 ... 6 } 7 8 void goo() 9 { 10 ... 11 foo(); 12 ... 13 } 14 15 void foo() 16 { 17 ... 18 L1: while(a<b) { 19 ... 20 }</pre> <p style="text-align: center;">(b)</p>
---	--

Figure 9. (a) Example program with a cutpoint at location L1 in the function foo. (b) Transformed program \widehat{P}_T^1 for cutpoint L1. The safety checker does not consider the function goo.

analyzing \widehat{P}_T . This optimization speeds up the analysis of programs that terminate, and slows down the checking of loops and recursive functions that do not guarantee termination.

```

1 void main()          1 void main()
2 {                   2 {
3     ...              2.1     foo();
4     goo();           2.2     exit();
5     ...              3     ...
6 }                   4     goo();
7                     5     ...
8 void goo()          6 }
9 {                   7
10    ...              8 void goo()
11    foo();           9 {
12    ...              10    ...
13 }                   11    foo();
14                     12    ...
15 void foo()         13 }
16 {                   14
17    ...              15 void foo()
18 L1: while(a<b) {   16 {
19    ...              16.1     goto L1;
20 }                   17     ...
                       18 L1: while(a<b) {
                       19     ...
                       20 }

```

(a) (b)

Figure 10. (a) Example program with a cutpoint at location L1 in the function `foo`. (b) Transformed program \hat{P}_T^2 for cutpoint L1. The safety checker does not consider the function `goo` and the initial part of `foo`.

5. Experimental results

We have integrated TERMINATOR into the Static Driver Verifier (SDV) [19] formal verification tool, which is distributed as a part of the Microsoft Windows Device Driver Development Kit. SDV currently uses the SLAM software model checker [2] to prove temporal safety properties of device drivers. SDV provides a set of safety properties together with an abstract model of the environment in which device drivers execute. We were able to reuse SDV’s environment model in the new integration after two minor modifications. The new property that our SDV/TERMINATOR integration proves of a device driver is that the dispatch routines, when called, always return back to the environment’s call-site. The environment uses non-deterministic choices to model the possibility of calling any of the device driver’s dispatch routines, providing coverage for all of the dispatch routines in the device driver.

We applied SDV/TERMINATOR to the standard 23 Windows OS device drivers used within Microsoft to test SDV. Each of these 23 drivers provides from 5 to 10 dispatch routines.

The results of these experiments are displayed in Figure 11. The results indicate the scalability of TERMINATOR to programs with up to 35,000 lines of code. In practice TERMINATOR spends effectively 100% of its time in the binary reachability analysis. For this reason, the results in Figure 11 also demonstrate the accuracy and scalability of TERMINATOR’s binary reachability analysis.

The termination violations reported by TERMINATOR are split into two categories in Figure 11: *true bugs* and *false bugs*. The false bugs are due to inaccuracies in TERMINATOR’s analysis, which can be categorized accordingly:

Heaps: A majority of the false bugs were caused by loops in which the program is walking a linked-list data structure. For example:

```
do { f(p); p = p->next } while (p != NULL);
```

Driver	Run-time (seconds)	True bugs found	False bugs reported	Lines of code	Cutpoint set size
1	12	0	1	1K	3
2	8	0	0	1K	8
3	410	0	1	8K	26
4	1475	0	1	7.5K	24
5	123292	1	11	5.5K	50
6	196	1	3	5K	29
7	4174	0	0	8K	23
8	210	0	11	5K	27
9	1294	0	5	6K	38
10	158	0	0	8K	21
11	13	0	0	2.5K	6
12	204	0	0	2.5K	16
13	257	1	1	7.5K	26
14	5	0	0	1K	2
15	141	0	1	6.5K	18
16	22	0	0	1.5K	2
17	800	1	6	4K	35
18	1503	1	0	6.5K	31
19	209	0	3	3K	28
20	4099	0	2	10K	63
21	1461	1	4	16K	56
22	114762	0	5	34K	65
23	158746	2	10	35K	75

Figure 11. Results of experiments using an integration of TERMINATOR with the Windows Static Driver Verifier[19] product (SDV) on the standard 23 Windows OS device drivers used to test SDV. Each device driver exports from 5 to 10 dispatch routines, all of which must be proved terminating.

TERMINATOR currently does not have a rank function synthesis mechanism for operations occurring in the program that modify the shape of the heap. TERMINATOR’s rank function synthesis module can determine that a counter pointed to by a pointer is decremented, but it is unable to reason effectively about the effects of instructions on heap-sizes.

Note that, in cases where the drivers are using high-level operations on kernel-level data-structures (such as queues and stacks), these were not reported as bugs by TERMINATOR. This is due to the fact that we were able to model the size of the structures using arithmetic in the SDV environment model.

Bit operations: Because our implementation of binary reachability analysis uses SLAM, it does not accurately treat the C bit operations such as `&` (see [11] for more information on recent extensions to SLAM to support this). Several cases occurred in which the termination condition required more precision from these operations.

Note that in most cases the false bugs caused by linked-lists and bit operations are easily recognizable by the developer of the driver. Another interesting aspect is that we saw no false bugs due to inaccuracies in the environment model. This gives us hope that, with improvements to the handling of bit-vectors and heaps, TERMINATOR could achieve an unprecedented level of accuracy for automatic program verification.

The true bugs in Figure 11 are termination counterexamples found by TERMINATOR that appear to be reproducible in a running system. The bug in driver number 5 is the example used in Section 1 (Figure 1). In this case TERMINATOR returned a path with 2531 steps¹. The path was from the start of the environment model’s `main` function through the driver and into the loop in Figure 1.

We were not able to compare TERMINATOR to other termination proof tools as there are no other termination provers that support the required mix of features for device drivers: pointers/function pointers, side-effects, arbitrarily nested loops, large code-size, automation, and accuracy.

6. Related work

Automatic program termination is a research topic dating back to Turing [21] and before. Techniques have been developed in a number of contexts, including term rewriting (e.g. [10, 15]) and logic and functional programming (e.g. [8, 17, 18]). Inspired by the success of automatic program verification for temporal safety properties (e.g. [1, 4, 16]) researchers have found a renewed interest in techniques for proving program termination of imperative programs ([6, 9, 13] for example).

In summary, our work differs from the previous research in two ways:

- We are not aware of reported experiments with interprocedural, path-sensitive and context-sensitive termination provers that support the features found in realistic industrial code (i.e. large imperative programs with pointers, callbacks, side-effects, etc). To the best of our knowledge, the experimental results in Section 5 represent the first known successful application of automatic program termination analysis to industrial systems code using a tool that supports real language features.
- We are also not aware of methods or tools that facilitate a powerful method of checking rich combinations of termination arguments, as TERMINATOR does. Binary reachability takes the burden off of the termination argument, resulting in a new form of termination argument that is easier to understand and easier to refine. In the previous approaches for proving the

¹ A step is an assignment statement or a conditional check

termination of imperative programs, the search and construction of a ranking function is the hard part (and forms a task that needs to be applied to the whole program) while proving that the rank function actually decreases in every computation step of the program is fairly easy.

One of the earliest abstraction-based verification tools, SYNTOX [5] can check termination of Pascal programs using greatest-fixpoint iteration. This is an alternative approach with a very different flavor. The problem here is to develop practical over-approximation techniques for greatest-fixpoint iteration.

There are other abstraction-based methods for termination that are targeted at more specialized settings (e.g. [22] or [17]). These approaches do not implement abstraction refinement, meaning that they lose information that cannot be recovered. While probably (much) faster than TERMINATOR, the analysis that these tools implement are less precise.

We finally come to the comparison with model checking for *finite-state* systems. It has been observed that for a given finite-state system, termination can be reduced to a safety property².

The algorithm in [3] makes this explicit using a program transformation, but this fact implicitly underlies all model checking algorithms for termination in finite-state systems. The idea is to search for a repeated occurrence of the same state on some trace. Interestingly, there exists a way in which our method generalizes the model checking algorithm for termination of finite-state systems. Namely, we can phrase the algorithm as the binary reachability analysis that checks the inclusion $R_T^+ \subseteq T$ for one particular binary relation T , fixed independently of the finite-state system; T consists of all pairs of different states.

$$T = \{(s_1, s_2) \mid s_1 \neq s_2\} = \bigcup_{s_1 \neq s_2} \{(s_1, s_2)\}$$

The relation T is a finite union of well-founded relations (finite because of the limitation to finite-state systems; in fact, T is the largest relation with this property). Hence, termination can be shown by checking the inclusion of the binary reachability relation of the finite-state system in T , i.e. $R_T^+ \subseteq T$.

7. Conclusion

In this paper we have introduced TERMINATOR, which is the first program termination prover to support the following mixture of features:

Scalability. As the experimental results demonstrate, TERMINATOR is able to prove the termination of device driver dispatch routines with up to 35,000 lines of code. This is due to the fact that TERMINATOR’s binary reachability analysis implements a form of counterexample-guided abstraction refinement that leverages the locality of each binary reachability query.

Applicability. TERMINATOR supports most of the language features required in at least one application area (device drivers): arbitrary nesting, side-effects and aliasing, function-pointers, etc. This is due to binary reachability which tracks these details independently.

Automation. TERMINATOR is completely automatic. It does not require the user to provide ranking functions or proof hints. This is due to TERMINATOR’s counterexample-guided argument refinement mechanism, which leverages binary reachability.

Precision. TERMINATOR implements a precise interprocedural path-sensitive and context-sensitive analysis. As in unary reach-

² In contrast, for an infinite-state system, termination can only be reduced to the *existence* of a safety property. The difficulty of an automatic proof is that the tool has to find that safety property.

ability, binary reachability computes a representation of the set of reachable states at every program location, which is then used when proving termination.

Counterexample generation. TERMINATOR provides counterexamples to failed termination proofs. This is, again, a feature that is a direct consequence of TERMINATOR's binary reachability analysis.

TERMINATOR achieves this milestone by shifting the burden away from the construction of termination arguments and to the checking of termination arguments. TERMINATOR constructs termination arguments which are the disjunction of (possibly many) simple well-founded relations. Each of these relations is drawn, on demand, from a simple and fast analysis on a single path through the program. TERMINATOR's binary reachability analysis, on the other hand, must perform the arduous task of actually checking that the disjunction of arguments covers over all possible pairs of states within all possible traces through the program. The experiments that we have performed with TERMINATOR show that (at least for one application area) this task can be solved with satisfying accuracy and scalability.

Future work In the future we would like to investigate ways in which binary reachability and TERMINATOR's method of refinement for termination arguments can be used when proving liveness properties of concurrent programs. We would also like to investigate methods of accelerating the production of counterexamples in TERMINATOR's binary reachability analysis using tools such as CBMC [7].

TERMINATOR could potentially be used in ways beyond simply proving termination. For example, SYNTAX [5] is used to derive debugging information, namely, to derive states that must inevitably reach the error state, a property that can be phrased in terms of termination. TERMINATOR's analysis could potentially be used in a similar fashion.

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