Basic Algorithms and Data Structures
Khabarovsk State University of Technology

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http://www.mpi-sb.mpg.de/~rdementi/courses/algdat04/index.html

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Up to **two** *people can work on an assignment together. But each of you should be able to explain the solutions. Write your names* **and** *your student book number on the sheets. Staple them together.*

Assignment 5

Deadline: May 25, 2004

Exercise 1 : 10 Points
Explain how to modify **DFS** so that it computes a spanning forest of an undirected graph in time $\mathcal{O}(m+n)$. In addition, your algorithm should select a *representative* node for each connected component of the graph and assign a value $component[v]$ to each node that identifies this representative. Note that isolated nodes are simply connected components of size one.

Exercise 2 : 15 Points
Nesting property of DFS numbers and finishing times

Show that $\nexists u, v \in V : dfsNum[u] < dfsNum[v] < finishTime[u] < finishTime[v]$.

Exercise 3 : 25 Points
Show that the MST is uniquely defined if all edge weights are different. Show that in this case the MST does not change if each edge weight is replaced by its rank among all edge weights.

Exercise 4 : 30 Points
Streaming MST

Suppose the edges of a graph are presented to you only once (for example over a network connection) and you do not have enough memory to store all of them. The edges do *not* necessarily arrive in sorted order. Outline an algorithm that nevertheless computes an MST using $\mathcal{O}(|V|)$ space. Prove the correctness of your algorithm. What is the worst case running time of your algorithm?

Exercise 5 : 20 Points
Give an example for an n node graph with $\mathcal{O}(n)$ edges where a naive implementation of the union-find data structure without balancing and path compression would lead to quadratic execution time for Kruskal's algorithm. Give an explanation.