Data Structures and Algorithms
FR 6.2 Informatik

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http://www.mpi-sb.mpg.de/~sanders/courses/algdat03/index.html

Up to **four** *people can work on an exercise together. But each of you should be able to explain the solutions to the TA (Bremser). Write your names* **and** *the name of your group (time, TA) on the sheets. Staple them together*

Assignment 12

Deadline: February 16, 2004

Solve at least two of the following four exercises.

Exercise 1

Formulate the *maximum multicommodity flow* problem as an LP: Let $G = (V, E)$ be a directed graph with edge capacities $u : E \to \mathbb{R}^+$, a set of commodities $\mathcal{K} = \{1, 2, \ldots, k\}$, where commodity $i \in \mathcal{K}$ is given by a source vertex $s_i \in V$ and a sink vertex $t_i \in V$, a "multicommodity flow" f consists of $s_i - t_i$ flows f_i for $i \in \mathcal{K}$. The flow f satisfies the capacity constraints "$f \leq u$", if $f(e) = \sum_{i=1}^{k} f_i(e) \leq u(e)$, for every $e \in A$. The value of a multicommodity flow f is the $k-$tuple $(|f_1|, |f_2|, \ldots, |f_k|)$; its total value is $|f| = \sum_{i=1}^{k} |f_i|$.

If in addition, a set of demands $(d_1, \ldots, d_k) \in \mathbb{R}_+^k$ is given for each commodity $i \in \mathcal{K}$, a multicommodity flow f satisfies the demands if $|f_i| = d_i$ for all $i \in \mathcal{K}$.

The *maximum multicommodity flow problem* asks for a multicommodity flow f maximizing $|f|$. On the other hand, the *multicommodity flow problem* asks for a multicommodity flow f satisfying the given demands $(d_1, \ldots, d_k)$.

Exercise 2

Linear time fractional knapsacks.

Explain how to solve the fractional knapsack problem (the linear relaxation of the knapsack problem, i.e. items can be selected fractionally) in linear expected time. Hint: use a similar idea as in the well known quicksort-like median selection algorithm.

Exercise 3

Formulate the following *set covering* problem as an ILP: Given a set $M = \{1, \ldots, m\}$, n subsets $M_i \subseteq M$ for $1 \leq i \leq n$ and a cost c_i for set M_i. Assume $\bigcup_{i=1}^{n} M_i = \{1, \ldots, m\}$. Select $F \subseteq \{1, \ldots, n\}$ such that $\bigcup_{i \in F} M_i = \{1, \ldots, m\}$ and $\sum_{i \in F} c_i$ is minimized.

Exercise 4

Making Change.

Suppose you have to program a vending machine that should give exact change using a minimum number of coins.

1. Develop an optimal greedy algorithm that works in the Euro zone with coins worth 1, 2, 5, 10, 20, 50, 100, and 200 cents and in the dollar zone with coins worth 1, 5, 10, 25, 50, and 100 cents.
2. Show that this algorithm would not be optimal if there were a 4 cent coin.
3. Develop a dynamic programming algorithm that gives optimal change for any currency system.