

Up to **four** people can work on an exercise together. But each of you should be able to explain the solutions to the TA (Bremser). Write your names **and** the name of your group (time, TA) on the sheets. Staple them together

Assignment 3

Deadline: November 17, 2003

Solve at least two of the following three exercises completely (or more of them partially)

Exercise 1

Arbitrage is the use of discrepancies in currency exchange rates to transform one unit of a currency into more than one unit of the same currency. For example, suppose that 1 U.S. dollar buys 0.7 British pound, 1 British pound buys 9.5 French francs, and 1 French franc buys 0.16 U.S. dollar. Then by converting currencies, a trader can start with 1 U.S. dollar and buy $0.7 \times 9.5 \times 0.16 = 1.064$ U.S. dollars, thus turning a profit of 6.4 percent.

Suppose that we are given n currencies c_1, c_2, \dots, c_n and $n \times n$ table R of exchange rates, such that one unit of currency c_i buys $R[i, j]$ units of currency c_j .

- (a) Give an efficient algorithm to determine whether or not there exists a sequence of currencies $\langle c_{i_1}, c_{i_2}, \dots, c_{i_k} \rangle$ such that

$$R[i_1, i_2] \cdot R[i_2, i_3] \cdots R[i_{k-1}, i_k] \cdot R[i_k, i_1] > 1.$$

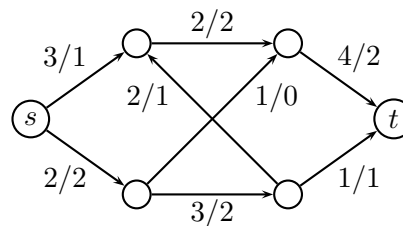
Analyze the running time of your algorithm.

- (b) Give an efficient algorithm to print out such a sequence if one exists. Analyze the running time of your algorithm.

Hint: $\prod_{i=1}^k x_i = e^{\sum_{i=1}^k \ln x_i}$. Furthermore, $\prod_{i=1}^k x_i > 1$ iff $1 / \prod_{i=1}^k x_i < 1$.

Exercise 2

Consider the graph G and the flow f given on the right.



- (a) Is f a blocking flow?
- (b) Give the residual graph G_f .
- (c) Give the layered subgraph L_f of G_f .
- (d) Find an augmenting path and give the resulting augmented flow. Repeat until the flow is maximum.
- (e) Give a saturated (s, t) -cut for the maximum flow.

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Exercise 3

Which of the following claims are true and which are false. Justify your answer by giving either a (short) proof or a counterexample.

- (a) In any maximum flow there are no cycles that carry positive flow. (A cycle $\langle e_1, \dots, e_k \rangle$ carries positive flow iff $f(e_1) > 0, \dots, f(e_k) > 0$.)
- (b) There always exists a maximum flow without cycles carrying positive flow.
- (c) If all edges in a graph have distinct capacities, there is a unique maximum flow.
- (d) In a directed graph with at most one edge between each pair of vertices, if we replace each directed edge by an undirected edge, the maximum flow value remains unchanged.
- (e) If we multiply all edge capacities by a positive number λ , the minimum cut remains unchanged.
- (f) If we add a positive number λ to all edge capacities, the minimum cut remains unchanged.
- (g) If we add a positive number λ to all edge capacities, the maximum flow increases by a multiple of λ .