Universität des Saarlandes / Max-Planck-Institut für Informatik
Problem Set 3
Approximation Algorithms
WS 2008/09

Problem 1 **1 point**

Consider the following problem

INPUT: A ground set $E = \{e_1, e_2, \ldots, e_n\}$ of n elements, each element e in E has a cost $c(e) \geq 0$. Also a collection of subsets of E, namely $T_1, \ldots T_m \subseteq E$.

OUTPUT: A subset $A \subseteq E$ of minimum total cost, such that $A \cap T_i \neq \emptyset$, for all $i = 1, \ldots, m$.

(a) Give an IP formulation for the problem. Obtain the LP-relaxation and its dual.

(b) Let $k = \max\{|T_i| : i = 1 \ldots, m\}$. Show how to use the Primal-Dual framework to derive a k-approximation algorithm for this problem.

(c) Explain how the algorithm of part (b) implies a 2-approximation algorithm for the Vertex Cover problem.

Problem 2 **1 point**

Consider the following problem, also known as *Max k-Cut*: We are given an undirected graph $G = (V, E)$, with non-negative edge costs, and an integer k. The objective is to partition V into k sets so that the total cost of edges between these sets (i.e., edges of the form (u, v), with $u \in S_i$ and $v \in S_j$, and $i \neq j$), is maximized.

(a) Give a $(1 - \frac{1}{k})$-approximation randomized algorithm, that is, an algorithm for which the expected cost is at least $OPT \cdot (1 - \frac{1}{k})$.

(b) Show how to derandomize this algorithm, using the method of conditional expectations.

Problem 3 **2 points**

Consider the standard IP formulation of the Set Cover problem, and its LP-relaxation. Consider an algorithm that picks all sets associated with non-zero values in the optimal fractional solution. Show that this algorithm achieves a factor of f, where f is the frequency of the most frequent element, as defined in class.

Hint: Use the primal complementary slackness conditions to prove this.