

Decision procedures for logical theories

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Motivation

Long-term goal of AI and automated theorem proving research

Computers intelligent assistants in mathematics, informatics, engineering

Efficient reasoning on large data extremely important in applications

- First-order logic is undecidable

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are often decidable or even tractable

– Single theories occur very seldom in applications
extensions or combinations of theories need to be considered

+ ... the theory combinations often still easy to tackle

Example: Lift controller



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data types:

floor, rf := \mathbb{Z} ; time := \mathbb{R}

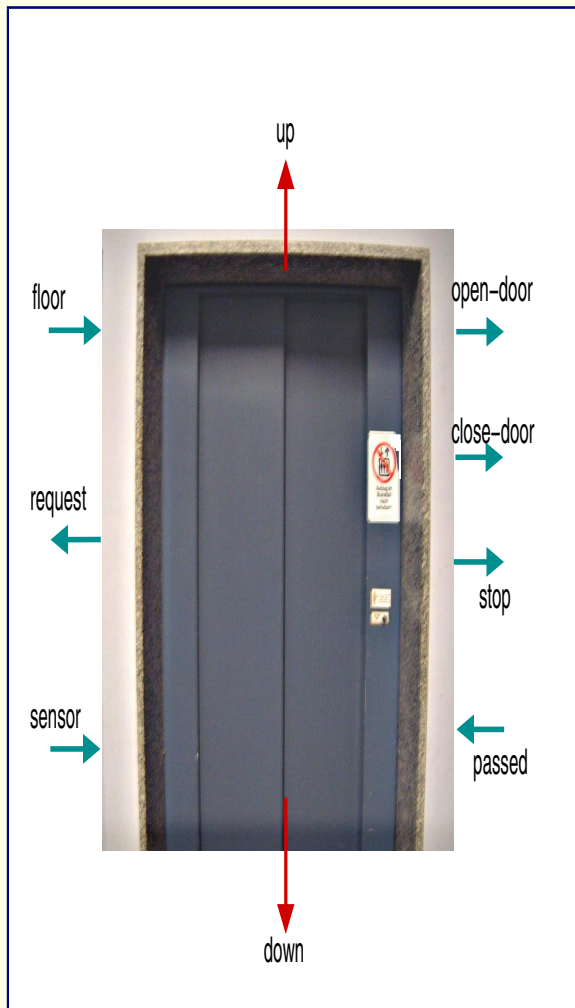
reqfloors := $\mathbb{P}(\mathbb{Z})$ [lists(\mathbb{Z})]

doors := {0(open), 1(closed)}

Theories

- integers
- real numbers
- Booleans
- sets of integers
- or lists of integers,
- or arrays of integers

Example: Lift controller



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reqfloors := $\mathbb{P}(\mathbb{Z})$

door := {0(open), 1(closed)}

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Sample formula [acknowl. P. Maier, J. Hoenicke]

$$rf' \neq rf \rightarrow ((\neg \text{door} \wedge rf' = \text{floor}) \rightarrow \text{reqfloors}' = \text{reqfloors}) \wedge$$
$$((\text{door} \vee rf' \neq \text{floor}) \rightarrow \text{reqfloors}' = \text{reqfloors} \cup \{rf'\})$$

Behavior

Events, Transitions

pre- and post-conditions

Other examples

- Program analysis (need to refer to various data structures)
- Compiler validation
- Knowledge representation:
 - Relational databases, terminological databases.
 - Non-classical logics
- Mathematics
-

Goal of the seminar

- Identify decidable/ tractable fragments of 1st-order logic
- Discuss methods for proving decidability of logical theories
- Identify application domains where decision procedures are used.

Overview

- Reasoning about standard datatypes
- Reasoning in theory extensions
- Reasoning in combinations of theories

Important:

- identify decidable/tractable fragments

Reasoning about standard datatypes

- Numbers
 - natural numbers, integers, reals, rationals
- Data structures
 - theories of lists
 - theory of acyclic lists
 - theory of arrays
 - theories of sets, multisets
- Algebraic theories
 - lattices, semilattices
 - distributive lattices
 - Boolean algebras
 - groups, rings, ...

Reasoning in theory extensions

- Numbers
 - integers, reals, rationals
- Data structures
 - theories of lists of integers, reals, ...
 - theory of acyclic lists of integers, reals, ...
 - theory of arrays of integers, reals, ...
 - theories of sets of integers, reals, ...
 - + functions (free, rec. def.) e.g : length, card
- Algebraic theories
 - total orderings with monotone functions
 - lattices, semilattices with operators
 - distributive lattices with operators
 - Boolean algebras with operators
 - fields with operators

Extensions & combinations of theories

Elevator example:	Reason about sets (or lists) of integers
Program verification:	Reason about lists (or arrays) over some data
Mathematics:	Reason about properties of real functions

Ideally: Use a prover for the base theory as a black-box

Extensions & combinations of theories

Elevator example:	Reason about sets (or lists) of integers
Program verification:	Reason about lists (or arrays) over some data
Mathematics:	Reason about properties of real functions

Ideally: Use a prover for the base theory as a black-box

Program verification:	Reason about reals, lists, and (free) functions Reason about lists and arrays over some data
Logic:	Reason in combinations of modal logics

Ideally: Use provers for the components as black-boxes

Structure of the seminar

Topics

- List of suggested topics + literature
- Additional possible topics

Structure

- General comments: talks and presentations
- Background: first-order logic, resolution theorem proving, superposition, combinations of theories (generalities, motivation)
- Presentations: 45 min + discussions, slides, written abstract

Further possibilities

- continue with a FoPra on the topic of the talk, or on a related topic.