CDM: Definitions and Theorems from Lectures 8 & 9

The following is a list of definitions, theorems, and notation from lectures 8 and 9.

Notation 1. Let n be some natural number. Then the set $[n] = \{1, \ldots, n\}$. For some set $X = [n]$, the notation 2^X denotes the powerset of X, i.e. the set of all subsets of X.

Theorem 1 (Sperner's Theorem)**.** *Let $X = [n]$. Then the largest antichain in the inclusion POSET 2^X is of size $\binom{n}{\lfloor n/2 \rfloor}$.*

Theorem 2 (Lubell-Yamamoto-Meshalkin (LYM) Inequality)**.** *Let $X = [n]$ and let $\mathcal{A} = (a_1, a_2, \cdots)$ denote some antichain in 2^X. If $|a_i|$ denotes the size of a member in $\mathcal{A}$, then*

$$\sum_{a_i \in \mathcal{A}} \frac{1}{\binom{n}{|a_i|}} \leq 1.$$

Definition 1. We say that a chain $\mathcal{C}_1 \subset \mathcal{C}_2 \subset \cdots \subset \mathcal{C}_k$ is *symmetric* if and only if

1. $|\mathcal{C}_1| + |\mathcal{C}_k| = n = |X|$, and
2. $|\mathcal{C}_{i+1}| = |\mathcal{C}_i| + 1$, for all $1 \leq i < k$.

Theorem 3 (Symmetric Chain Decomposition)**.** *For $X = [n]$, the poset 2^X can be decomposed into $\binom{n}{\lfloor n/2 \rfloor}$ symmetric chains.*

Theorem 4 (Bollobás Theorem)**.** *Let $A_1, \ldots, A_m, B_1, \ldots, B_m \subseteq [n]$ such that $A_i \cap B_j = \emptyset$ if and only if $i = j$. Then*

$$\sum_{i=1}^{m} \frac{1}{\binom{a_i + b_i}{a_i}} \leq 1$$

where $a_i = |A_i|$ and $b_i = |B_i|$.