CDM: Assignment 3

Due on Wednesday June 10

Section I

Answer all questions in this section. Make sure to explain your solutions completely to receive full credit.

1. Use symmetric chain decomposition theorem to prove Sperner's theorem.

2. Consider the following generalization of Erdös-Ko-Rado theorem. Assume that the size of the universe is $n \geq 2k$. Suppose that $\mathbb{F}$ is an intersecting family, none of the two members in $\mathbb{F}$ contain each other, and each member of $\mathbb{F}$ is of size at most k. Prove that $|\mathbb{F}| \leq \binom{n-1}{k-1}$.

   Hint. *Use symmetric chain decomposition theorem.*

3. Suppose that we add a number of incoming edges with any capacity to the source and similarly a number of outgoing edges with any capacity to the destination. Prove that the maximum flow value remains unaltered.

4. We are given a road-network, where the lieutenants and soldiers are distributed among the vertices. Each edge has a cost which reflects the efforts needed to demolish the traffic on that road. Our goal is to cut the communication between all lieutenants and all their soldiers. Design a polynomial time algorithm to find the minimal cost feasible solution.

Section II

Answer at least two of the following four questions. Make sure to explain your solutions completely to receive full credit.

1. Let $x_1, x_2, \cdots, x_n$ be real numbers and each of them $\geq$ 1.Let S be the set of all number s, which can be obtained as a linear combination of $\sum_{i=1}^{n} \alpha_i x_i$, where each $\alpha_i \in \{-1, 1\}$. Let $I = [a, b)$ be any interval (in the real line) of length $b - a = 2$. Show that $|I \cap S| \leq \binom{n}{\lfloor n/2 \rfloor}$.

   Hint. *Use Sperner's theorem.*

2. When $n = 2k$, in class, we have shown a simple way of constructing a k-uniform intersecting family of size $\binom{n-1}{k-1}$. Could you find another way of constructing such a family?

3. Prove the following version of Menger's theorem. Given an *undirected* graph, the minimum number of edges required to be removed to disconnect s and t is the same as the maximum number of edge-disjoint $s - t$ paths.

4. A town has r residents, q clubs, and p political parties. Each resident is a member of at least one club and can belong to exactly one political party. Each club must nominate one of its members to represent it on the town's governing council so that the number of council members belonging to the political party P_k is at most u_k. Design an algorithm to find a "balancing" council.

   Hint. *Use flow.*