Dense Batch Non-Rigid Structure from Motion in a Second

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Abstract

We show how to minimise a quadratic function on a set of orthonormal matrices using an efficient semidefinite programming solver with application to dense non-rigid structure from motion. Thanks to the proposed technique, a new form of the convex relaxation for the Metric Projections (MP) algorithm is obtained. The modification results in an efficient single-core CPU implementation enabling dense factorisations of long image sequences with tens of thousands of points into camera pose and non-rigid shape in seconds, i.e., at least two orders of magnitude faster than the runtimes reported in the literature so far. The proposed implementation can be useful for interactive or real-time robotic and other applications, where monocular non-rigid reconstruction is required...

	# points	# frames	environment	runtime, sec
T&K	~82000	60	C++	1
AMP (ours)	~9100	45	C++	1
MP [2]	37	74	matlab	30
Vicente&Agapito [7]	540	50		720
VA [3]	~82000	60	C++/CUDA C	100
Dense NRSfM [8]	~78000	90		600
AMP (ours)	~50700	202	C++	30

Contributions:

- we show how to formulate a quadratic optimization problem as a Semi-Definite Programming (SDP) problem and solve it with an efficient SDP solver (such as CSDP)
- the methodology is applied to dense non-rigid surface factorisation – on example of Metric Projections [2]; it enables a speedup which makes our implementation – Accelerated MP – one of the fastest NRSfM methods.

Method overview

$$\mathbf{W}_{2f\times p} = [\mathbf{W}_1 \mathbf{W}_2 \dots \mathbf{W}_f]^\top$$

measurement matrix

$$\mathbf{S}_i = \sum_{d=1}^{k} l_{id}\mathbf{B}_d \leftarrow$$

k basis shapes

f shapes; k weights

f frames,
p points p. fr.

camera pose

$$\mathbf{W}_i = [l_{i1}\mathbf{R}_i \;\; \dots \;\; l_{ik}\mathbf{R}_i]\,[\mathbf{B}_1 \;\; \dots \;\; \mathbf{B}_k]^\top$$

projection of M onto the motion manifold:

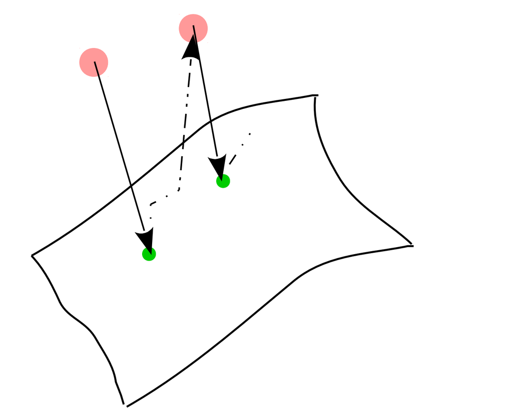
$$\min_{\mathbf{R}_i, l_{i1},\dots,l_{ik}} \|\mathbf{M}_i - [l_{i1}\mathbf{R}_i| \dots |l_{ik}\mathbf{R}_i]\|^2_{\mathcal{F}}$$

quadratic form:

$$\min_{\mathbf{R}_i} \mathbf{r}_i^\top \left[-\sum_{d=1}^{k} \mathbf{m}_{id}\mathbf{m}_{id}^\top\right] \mathbf{r}_i \quad \text{so that}$$

$$\mathbf{R}_i\mathbf{R}_i^\top = \mathbf{I}_{2\times 2}$$

update $\quad l_{id} = \frac{1}{2}\,\mathrm{tr}\,[\mathbf{M}_{id}^\top \mathbf{R}_i]$



projection onto the motion manifold (augmented Lee group) [2]

Quadratic optimisation problem

$$\min_{\mathbf{q}=vec(Q)} \mathbf{q}^\top\mathbf{E}\mathbf{q}, \quad \mathrm{E} \in \mathbb{R}^{6\times 6} \text{ and } Q \in \mathbb{R}^{3\times 2}$$

$$\min_{\mathbf{q}=vec(Q)} \mathrm{tr}(\mathbf{E}\mathbf{q}\mathbf{q}^\top) = \min_{\mathbf{X}\in co(S)} \mathrm{tr}(E\mathbf{X}),$$

$$X \in \mathbb{R}^{6\times 6} = \begin{pmatrix} A & B \\ B^\top & C \end{pmatrix},$$

$$A \in \mathbb{R}^{3\times 3},\; X \succcurlyeq 0,\; \mathrm{tr}(A) = \mathrm{tr}(C) = 1,\; \mathrm{tr}(B) = 0,$$

$$Y = \begin{pmatrix} I_3 - A - C & \mathbf{w} \\ \mathbf{w}^\top & 1 \end{pmatrix} \succcurlyeq 0,\; \mathbf{w} = \begin{pmatrix} b_{23} - b_{32} \\ b_{31} - b_{13} \\ b_{12} - b_{21} \end{pmatrix}.$$

Constraints in the unified form

Semi-Definite Program

$$\max\; \mathrm{tr}(C'U),$$
$$\text{subject to } \left\{\mathcal{A}(U) = a;\; U \succcurlyeq 0\right\},$$

where

$$\mathcal{A}(U) = [\mathrm{tr}(A_1U) \quad \mathrm{tr}(A_2U) \quad \dots \quad \mathrm{tr}(A_mU)]^\top$$

* the matrices C', A_1, A_2, . . . , A_m as well as the vector a are known, and the operator *A* maps the unknown PSD matrix U to a vector.

Proposition about a block-PSD matrix

Proposition 1 *If a matrix* $U = \begin{pmatrix} U_1 & 0 \\ 0 & U_2 \end{pmatrix}$ *has a block structure, then* U *is PSD if both* U_1 *and* U_2 *are PSD. Conversely, if* U *is PSD, then the principal submatrices* U_1 *and* U_2 *are PSD.*

* a principal submatrix is a square submatrix obtained by removing k rows and columns with the same indexes (whenever i-th row is removed from U , the i-th column is also removed)

Coefficient splitting

$$c_{11}z_{11} + c_{12}z_{12} + \dots + c_{1n}z_{1n} + \dots + c_{nn}z_{nn} = b$$

$$\sum_{i=1}^{n} c_{ii}z_{ii} + \sum_{i\neq j} \frac{c_{ij}+c_{ji}}{2}(z_{ij}+z_{ji}) = b$$

* coefficient splitting holds for symmetric matrices

example:

$$y_{11} = 1 - a_{11} - c_{11}$$

$$\frac{u_{12}+u_{21}}{2} + \frac{u_{45}+u_{54}}{2} + \frac{u_{78}+u_{87}}{2} = 0.$$

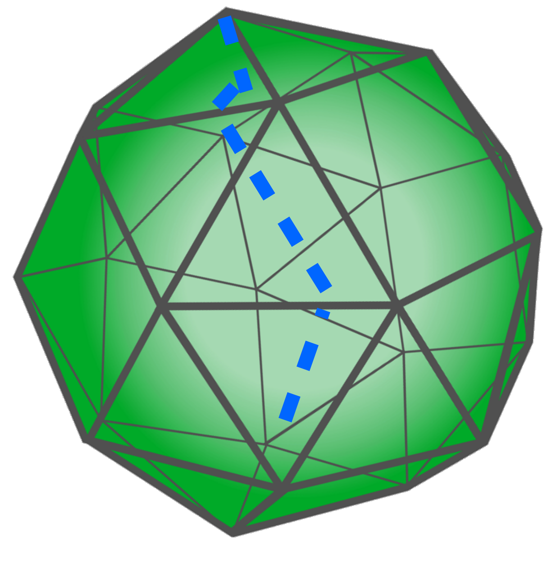
Constraints on traces and elements

$$U = \begin{bmatrix} X_{6\times 6} & 0 \\ 0 & Y_{4\times 4} \end{bmatrix} = \begin{bmatrix} A_{3\times 3} & B & 0 \\ B^\top & C_{3\times 3} & 0 \\ 0 & 0 & Y_{4\times 4} \end{bmatrix} \overset{!}{\succcurlyeq} 0$$

$$\mathrm{tr}(A_1U) = \mathrm{tr}\left(\begin{bmatrix} \mathbf{I}_3 & 0 \\ 0 & \mathbf{0}_{7\times 7} \end{bmatrix} U\right) = 1 \qquad y_{12} = -a_{12} - c_{12}$$

$$\mathrm{tr}(A_2U) = \mathrm{tr}\left(\begin{bmatrix} \mathbf{0}_{3\times 3} & 0 & 0 \\ 0 & \mathbf{I}_3 & 0 \\ 0 & 0 & \mathbf{0}_{4\times 4} \end{bmatrix} U\right) = 1$$

$$\mathrm{tr}(A_3U) = \mathrm{tr}\left(\begin{bmatrix} 0 & \frac{\mathbf{I}_3}{2} & 0 \\ \frac{\mathbf{I}_3}{2} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} U\right) = 0$$



Interior Point Methods converge to an optimal solution through interior of the solution space [5]

Accelerated Metric Projections

Accelerated Metric Projections

Input: measurement matrix $\mathbf{W}$
Output: non-rigid shapes $\mathbf{S}$, camera poses $\mathbf{R}$ (factorisation of $\mathbf{W}$)
1: **Initialisation:** $\mathbf{S}_i$, $\mathbf{R}_i$ and $\mathbf{M}$ from rank 3 rigid factorisation of $\mathbf{W}$
2: **while** not converges **do**
3: project $\mathbf{M}$ framewise onto the motion manifold (a Newton-like algorithm;
4: the first frame is projected by the proposed convex relaxation)
5: $\mathbf{S}^{(t)} = \mathbf{M}^{(t)\dagger}\mathbf{W}$
6: $\mathbf{M}^{(t+1)} = \mathbf{W}\mathbf{S}^{(t)\dagger}$
7: **end while**

* projection of **M** is performed according to <Method Overview>

Semi-Definite Programming

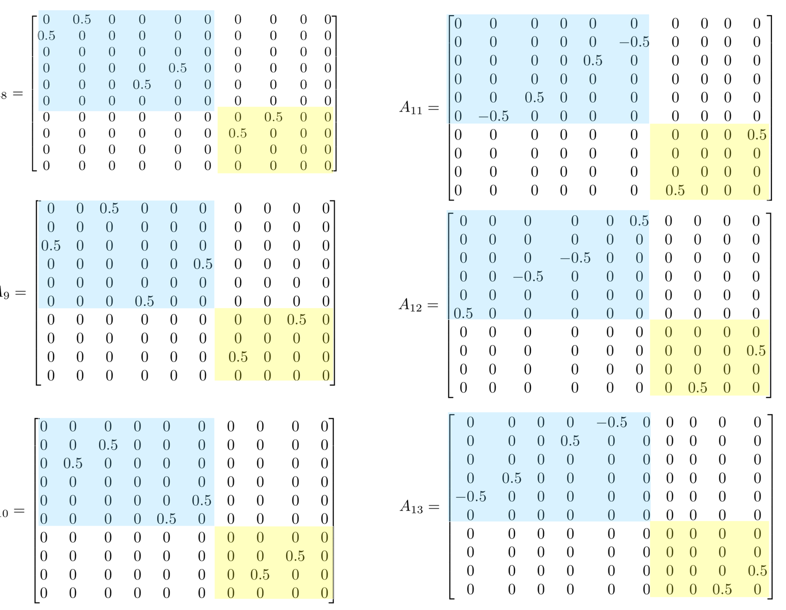
- is a field of convex optimisation
- optimised is a linear objective functon
- a special case of cone programming
- constraints are provided by linear matrix inequalities and linear equalities
- an SDP can be solved by interior point methods

CSDP library

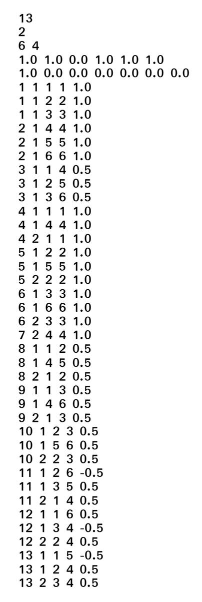
- is build upon *lapack*
- provides an efficient implementation of Primal-Dual Interior Point algorithm [5]
- enables SDP programs to be solved in polynomial time

$$a = (1101111000000)^\top$$

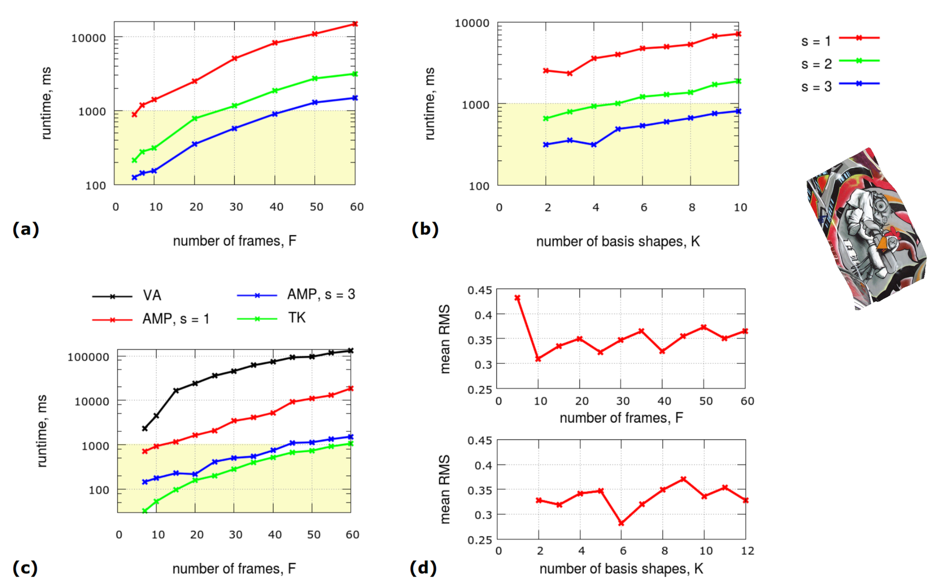
tr(A1 U) =
tr(A2 U) =
...
tr(A13 U) =



Prob.dat-s:



Experimental evaluation

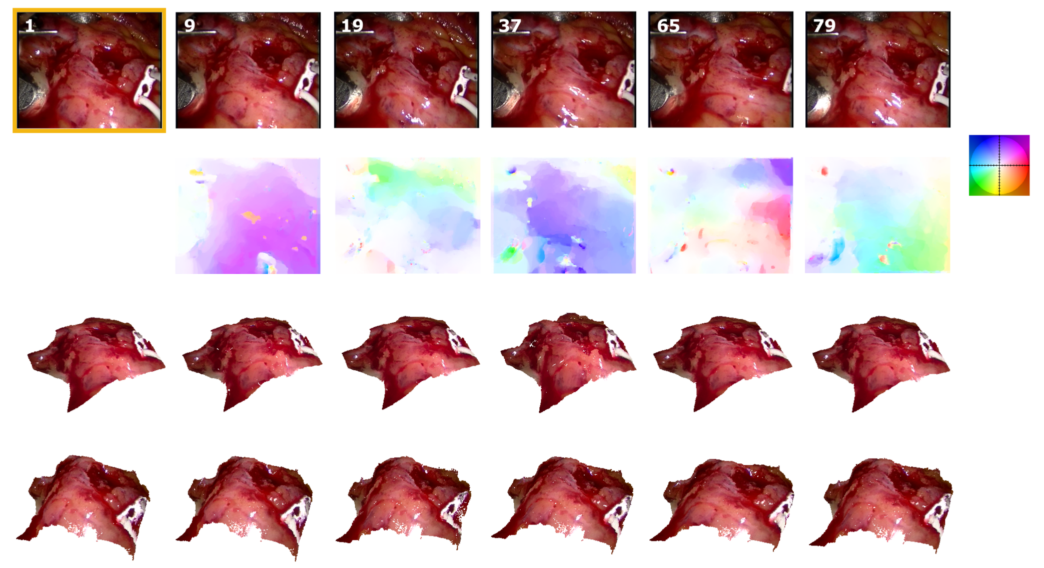


mean RMS or $e_{3D} = \frac{1}{F}\sum_{f=1}^{F} \frac{\|\mathbf{S}_f^{ref} - \mathbf{S}_f\|_{\mathcal{F}}}{\|\mathbf{S}_f^{ref}\|_{\mathcal{F}}}$

AMP runtime measurements for different subsampling factors compared to T&K [1] and VA [3] on Intel Xeon E5-1650

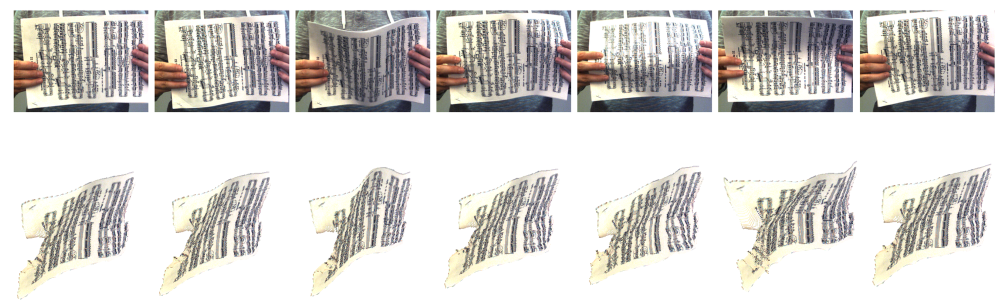
Examples of dense reconstructions (meshed outputs) obtained with AMP

heart bypass surgery



face sequence [3]

music notes



barn owl sequence

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