

Advances in Quantum Computer Vision

Workshop on Quantum Information (08.12.2022)



UNIVERSITÄT
DES
SAARLANDES

Vladislav Golyanik

Visual Computing and
AI Department



4D and Quantum
Vision Group $\langle \mathcal{A} | \psi \rangle$

4D and Quantum Vision (4DQV) Group

- Independent research group integrated in the VCAI Department of MPI for Informatics
- Research agenda at the intersection of CV, CG and ML
- We collaborate in Germany and internationally on these topics



Visual Computing and
AI Department

4D and Quantum
Vision Group $\langle \mathcal{A} | \psi \rangle$



MPI-INF, Campus E1 4

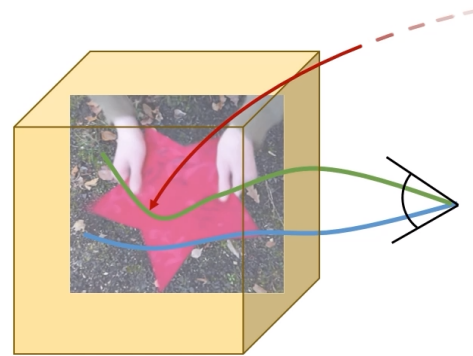


Green-screen Studio

4D and Quantum Vision (4DQV) Group

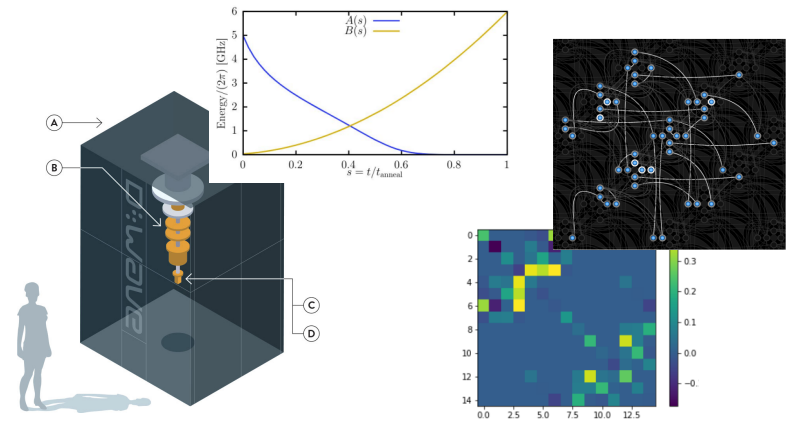
Two Core Research Areas

Non-rigid 3D (=4D) Perception/Reasoning



- 1) explicit and 2) implicit
3D representations

Quantum Computer Vision



- 1) What are the relevant problems?
2) Real quantum hardware is in the foreground.

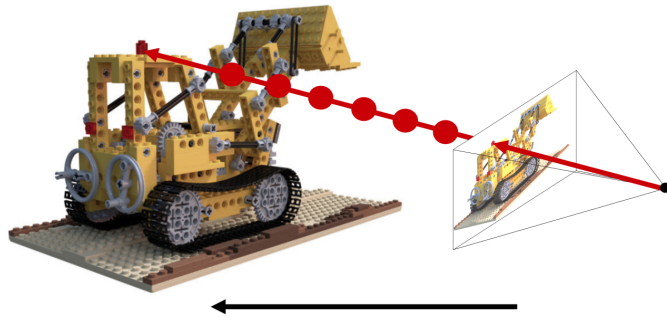
Non-rigid 3D (=4D) Perception/Reasoning



General Objects

Hands

3D Human MoCap (different settings)



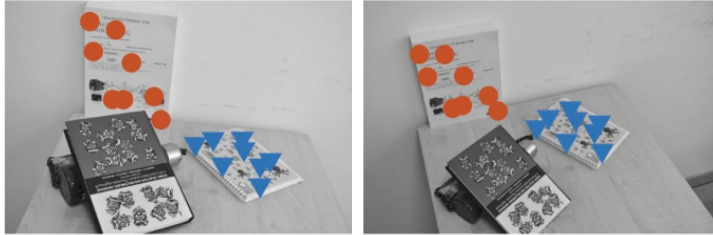
Volumetric 3D Representations



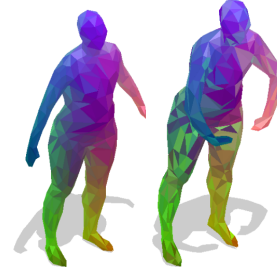
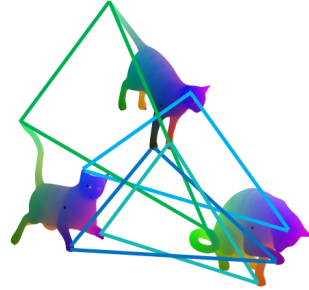
Video/Action Manipulation

Survey: Tretschk and Kairanda et al., arXiv 2022.

Quantum Computer Vision



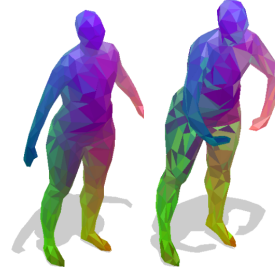
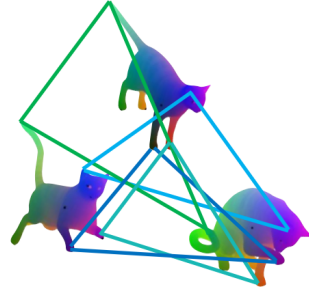
Problems solved by synchronisation



(Iterative) Matching methods

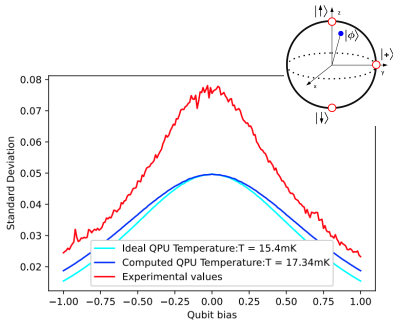


Quantum Computer Vision

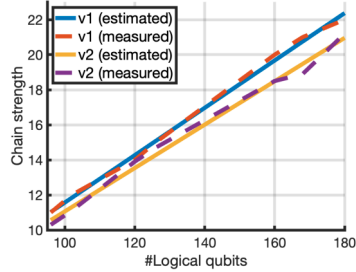


Problems solved by synchronisation

(Iterative) Matching methods



qubit properties



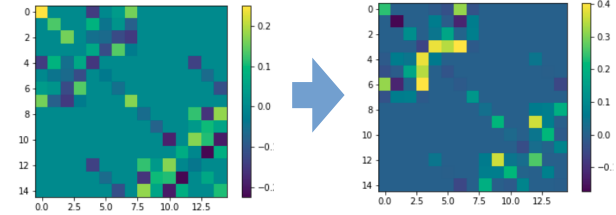
Constrained optimisation

Ⓒ Primal/Dual Update

$$\begin{aligned} \text{Primal } \mathbf{H}_t &:= \mathbf{w}_t \mathbf{w}_t^\top \\ \mathbf{W}_{t+1} &= (1 - \eta_t) \mathbf{W}_t + \eta_t \mathbf{H}_t \\ \text{Dual } \mathbf{g}_t &= \mathcal{A} \mathbf{W}_{t+1} - \mathbf{v} \\ \mathbf{y}_{t+1} &= \mathbf{y}_t + \gamma_t \mathbf{g}_t \end{aligned}$$

\mathbf{W}_t

Gradient



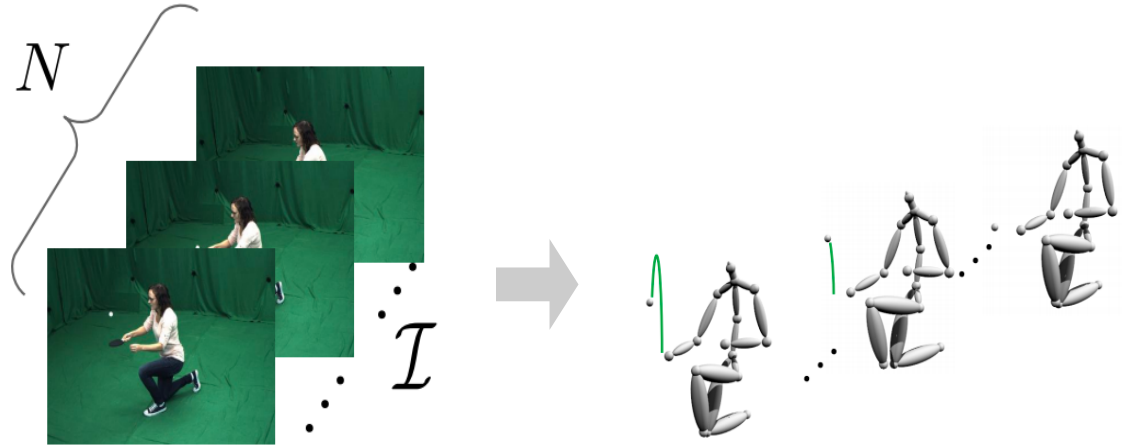
AQC + ML

Methodology of mapping problems to quantum hardware

3D Human MoCap with Physics Priors



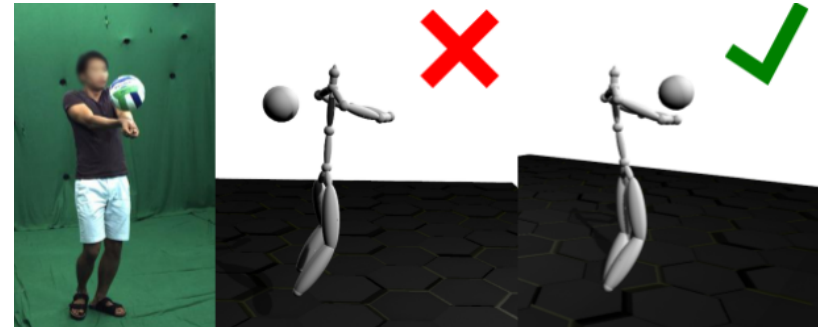
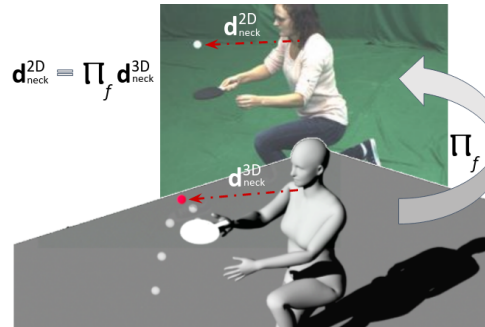
3D Human-Object Motion Capture



$$x_i = f \frac{X_i}{Z_i} + c_x, \quad y_i = f \frac{Y_i}{Z_i} + c_y, \quad \forall i,$$

$$\text{s. t. } \sqrt{g_x^2 + g_y^2 + g_z^2} = 9.81 \text{ m/s}^2,$$

$$\text{where } \begin{cases} X_i = X_0 + u_x t + \frac{1}{2} g_x t^2, \\ Y_i = Y_0 + u_y t + \frac{1}{2} g_y t^2, \text{ and} \\ Z_i = Z_0 + u_z t + \frac{1}{2} g_z t^2. \end{cases}$$



3D Human MoCap with a Scene Prior



(Top View)

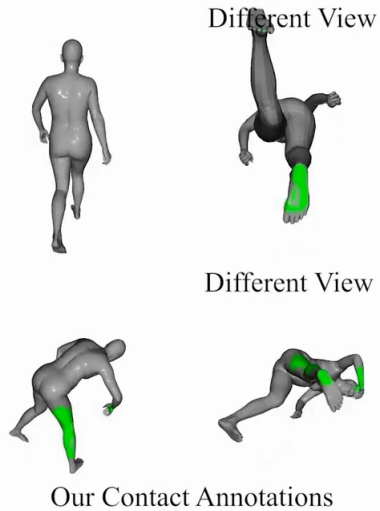


3D Human MoCap with a Scene Prior

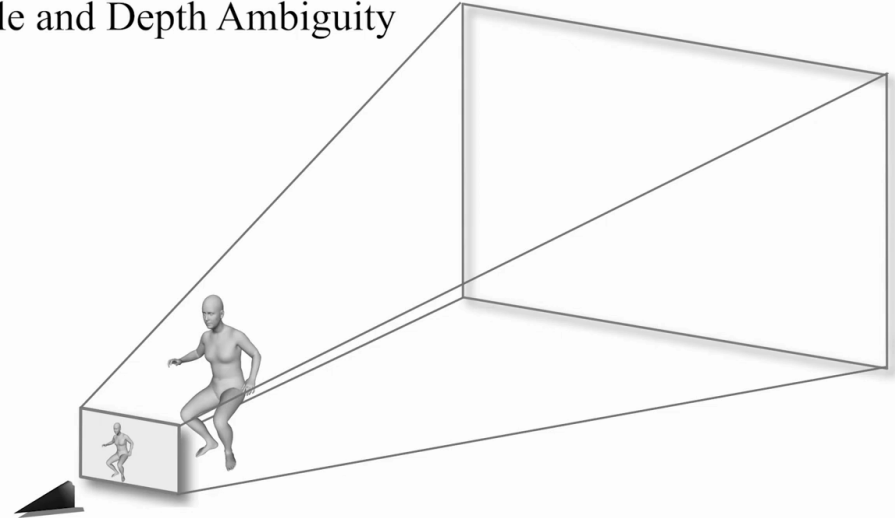
Contact Label Annotations on GTA-IM dataset



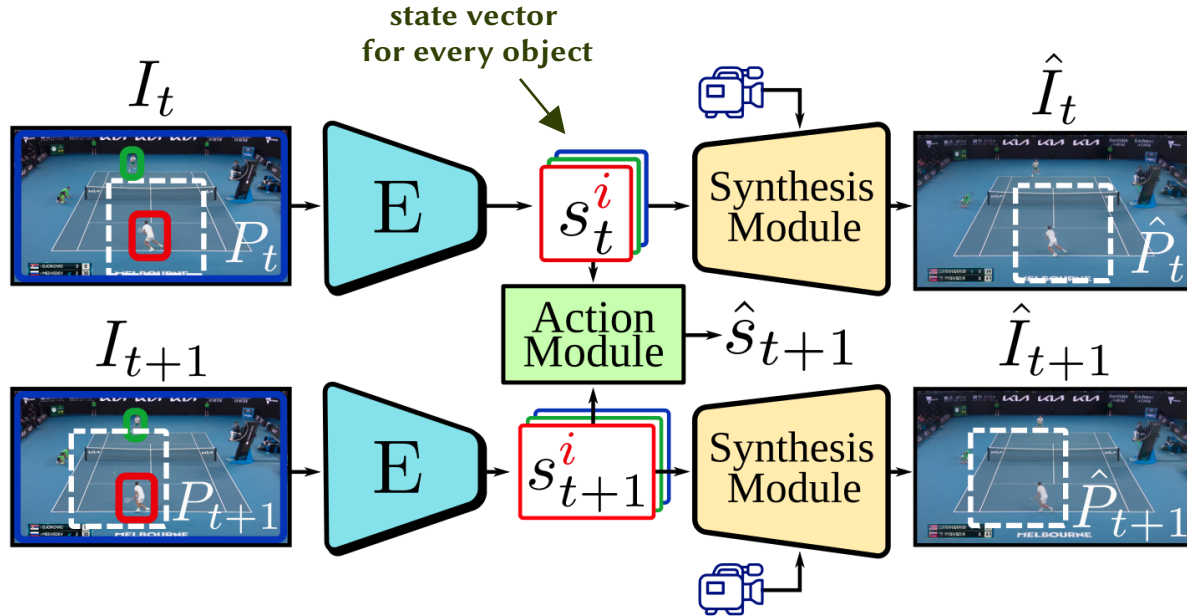
GTA-IM dataset [Cao *et al.*]



Scale and Depth Ambiguity

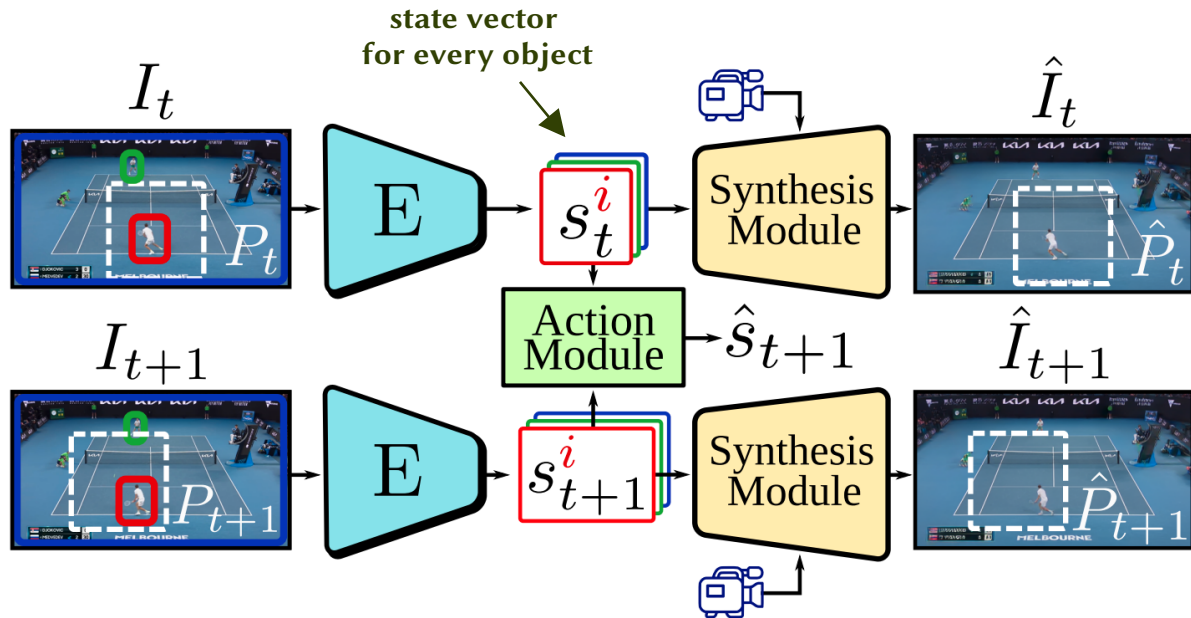


Playable Environments

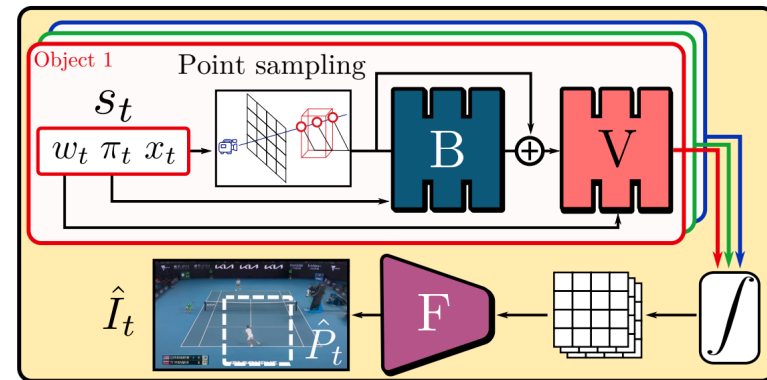


The Approach to Construct Playable Environments (PE)

Playable Environments

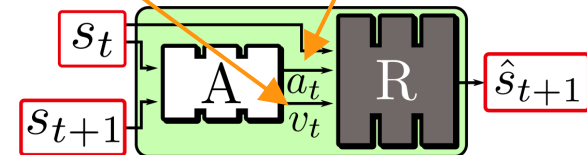


The Approach to Construct Playable Environments (PE)



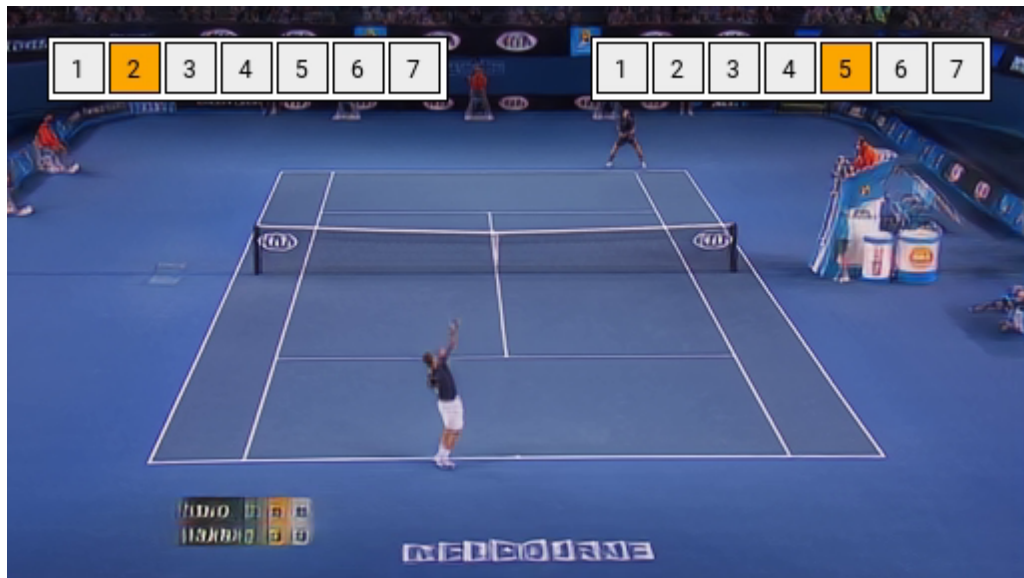
The Synthesis Module

action variability embedding discrete representation of the action



The Action Module

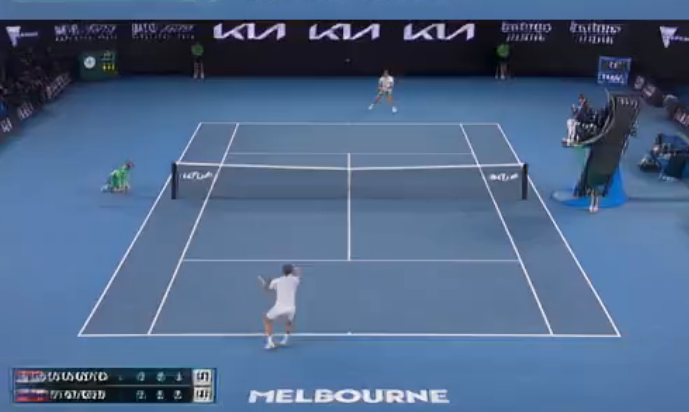
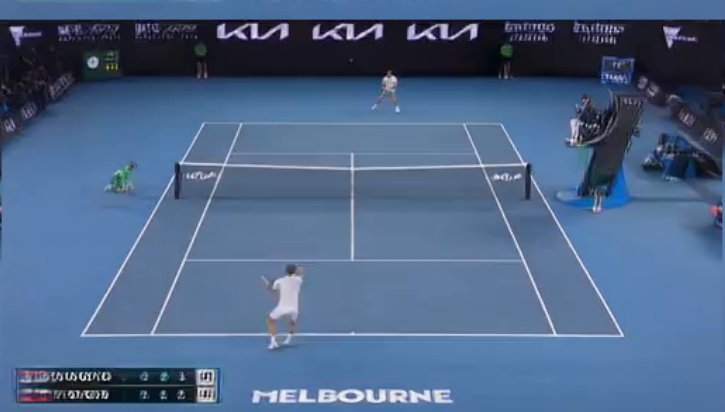
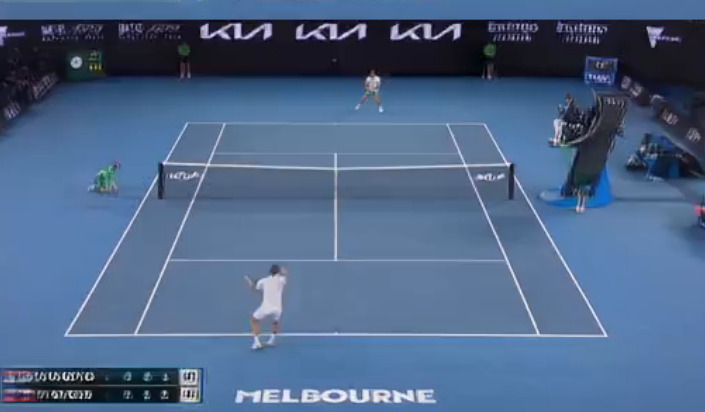
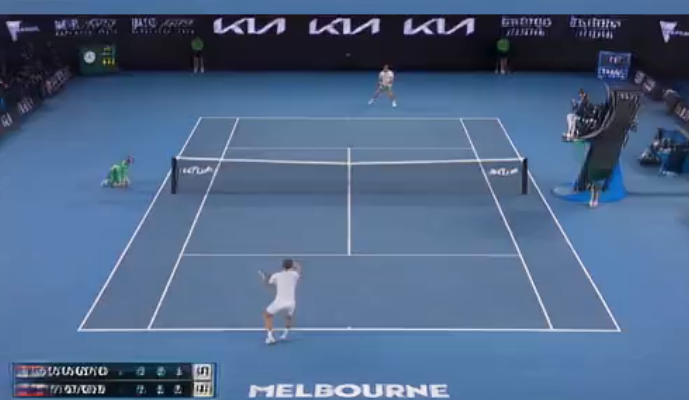
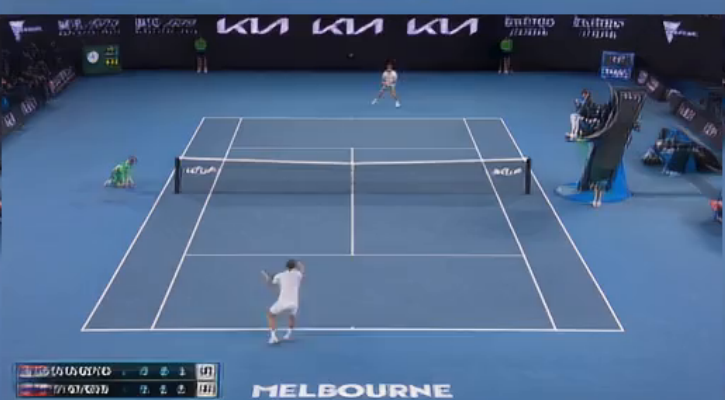
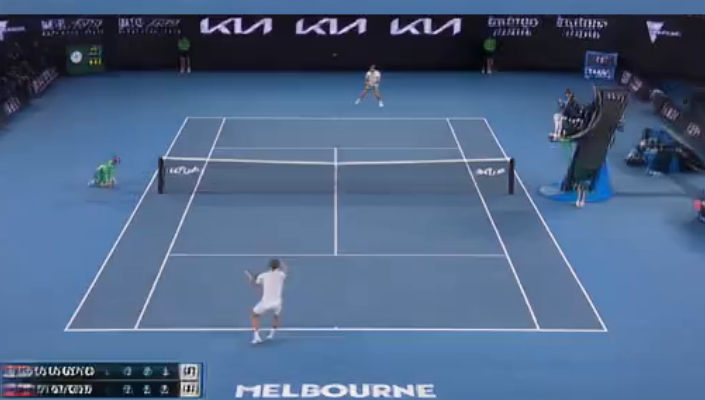
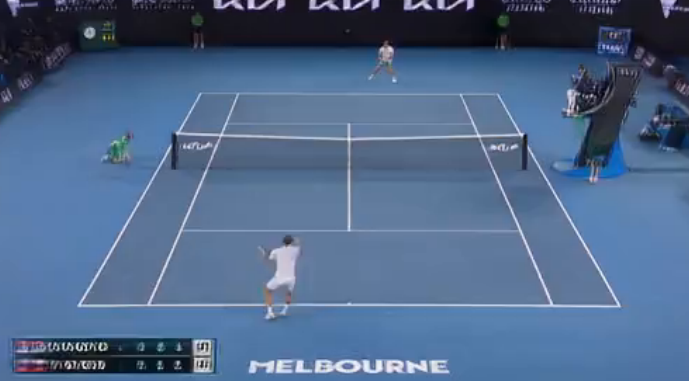
Playable Environments: Players Control



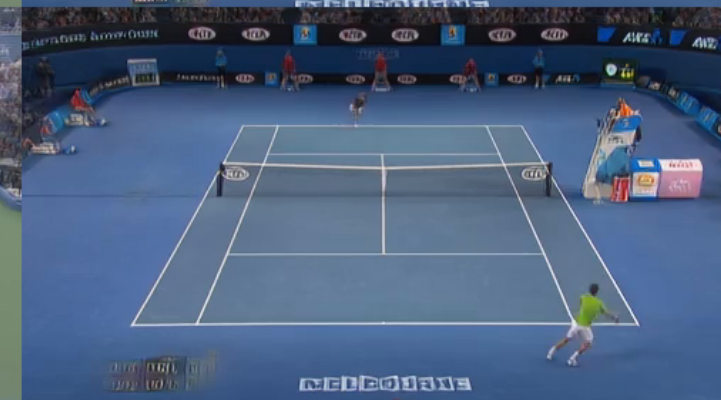
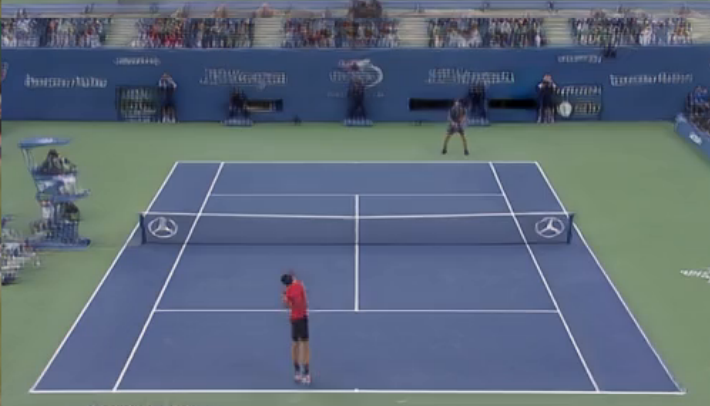
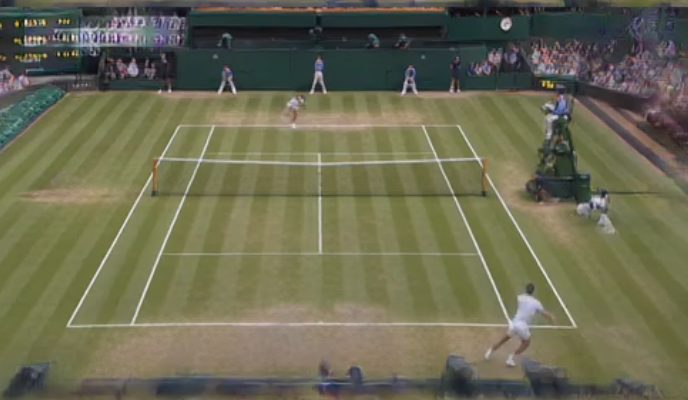
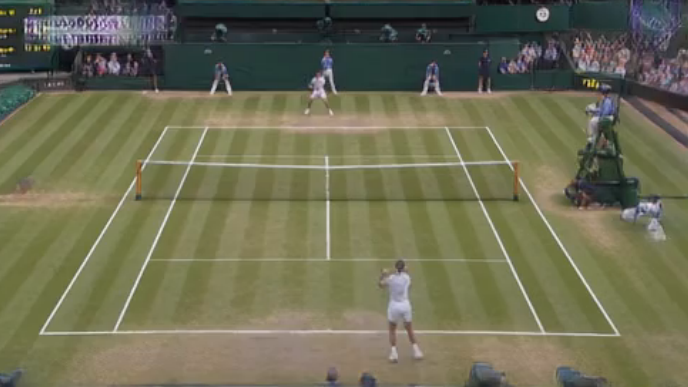
Tennis



Minecraft

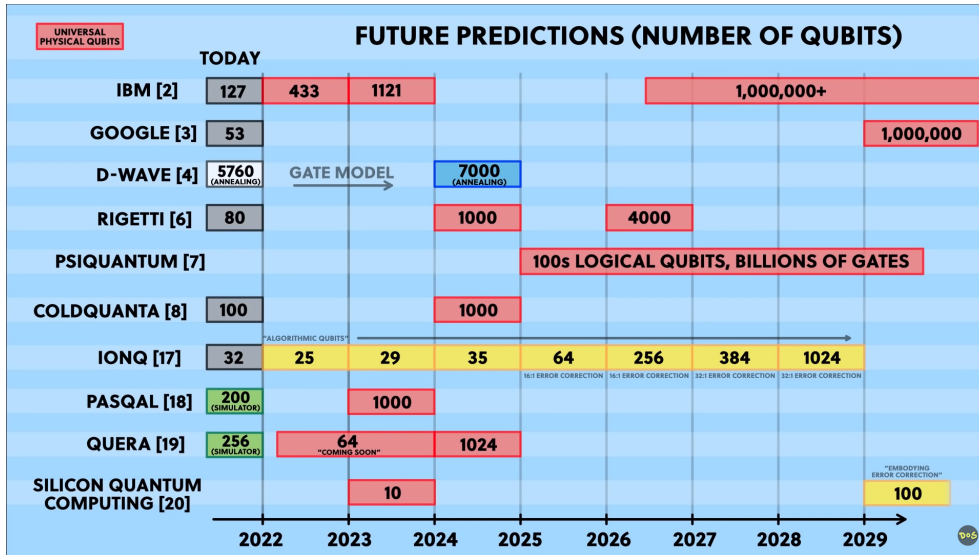






From a Classical to a Quantum Perspective

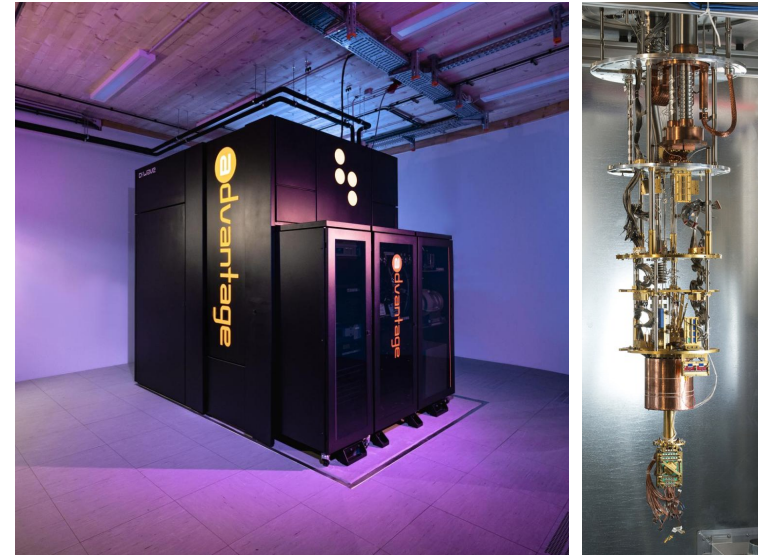
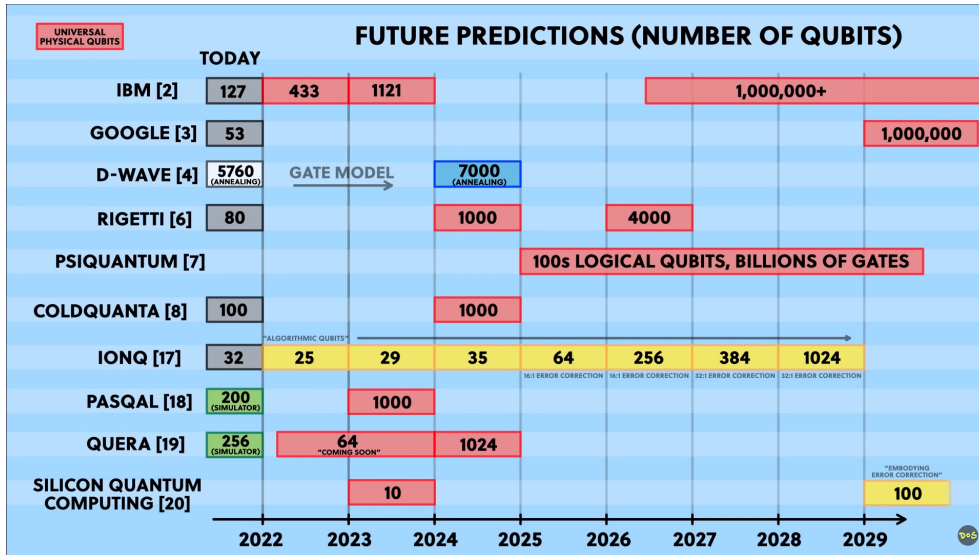
From a Classical to a Quantum Perspective



Copyright: D. Walliman. The Map of Quantum Computing, 2022.

- QC are steadily improving
- How can computer vision benefit from QC?

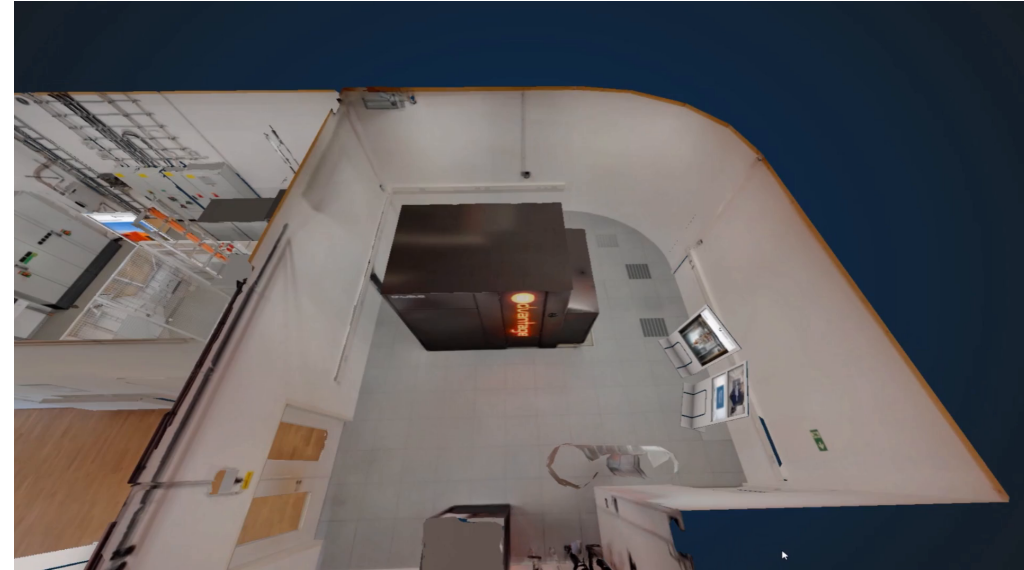
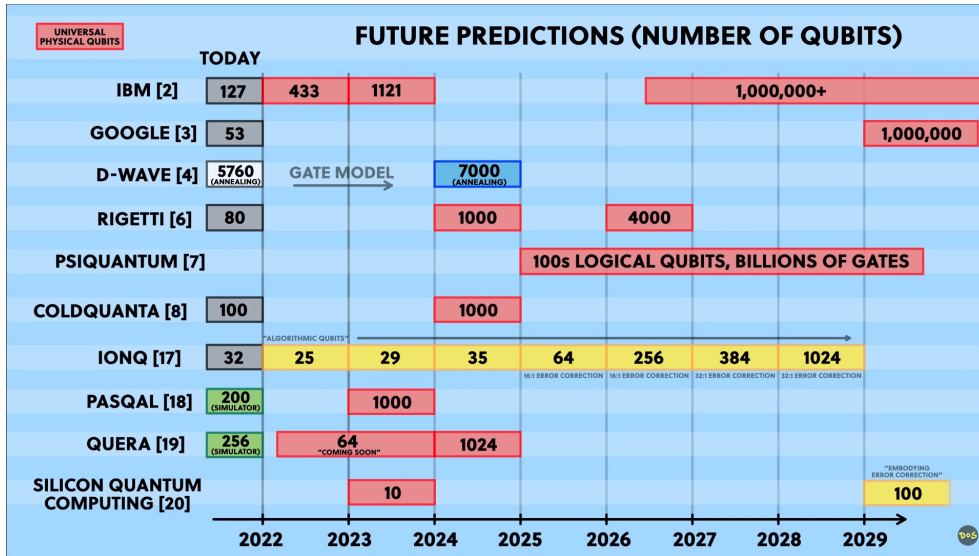
From a Classical to a Quantum Perspective



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- QC are steadily improving
- How can computer vision benefit from QC?

From a Classical to a Quantum Perspective



- QC are steadily improving
- How can computer vision benefit from QC?

Foundations of Adiabatic Quantum Computing (AQC)

Quantum annealing in the transverse Ising model

Tadashi Kadowaki and Hidetoshi Nishimori

Department of Physics, Tokyo Institute of Technology, Oh-okayama, Meguro-ku, Tokyo 152-8551, Japan

(Received 30 April 1998)

We introduce quantum fluctuations into the simulated annealing process of optimization problems, aiming at faster convergence to the optimal state. Quantum fluctuations cause transitions between states and thus play the same role as thermal fluctuations in the conventional approach. The idea is tested by the transverse Ising model, in which the transverse field is a function of time similar to the temperature in the conventional method. The goal is to find the ground state of the diagonal part of the Hamiltonian with high accuracy as quickly as possible. We have solved the time-dependent Schrödinger equation numerically for small size systems with various exchange interactions. Comparison with the results of the corresponding classical (thermal) method reveals that the quantum annealing leads to the ground state with much larger probability in almost all cases if we use the same annealing schedule. [S1063-651X(98)02910-9]

Kadowaki and Nishimori, 1998

A Quantum Adiabatic Evolution Algorithm Applied to Random Instances of an NP-Complete Problem

Edward Farhi,^{1*} Jeffrey Goldstone,¹ Sam Gutmann,²
Joshua Lapan,³ Andrew Lundgren,³ Daniel Preda³

A quantum system will stay near its instantaneous ground state if the Hamiltonian that governs its evolution varies slowly enough. This quantum adiabatic behavior is the basis of a new class of algorithms for quantum computing. We tested one such algorithm by applying it to randomly generated hard instances of an NP-complete problem. For the small examples that we could simulate, the quantum adiabatic algorithm worked well, providing evidence that quantum computers (if large ones can be built) may be able to outperform ordinary computers on hard sets of instances of NP-complete problems.

Although a large quantum computer has yet to be built, the rules for programming such a device, which are derived from the laws of

quantum mechanics, are well established. It is already known that quantum computers could solve problems believed to be intractable on

REPORTS

classical (i.e., nonquantum) computers. An intractable problem is one that necessarily takes too long to solve when the input gets too big. More precisely, a classically intractable problem is one that cannot be solved using any classical algorithm whose running time grows only polynomially as a function of the length of the input. For example, all known classical factoring algorithms require a time that grows faster than any polynomial as a function of the number of digits in the integer to be factored. Shor's quantum algorithm for the factoring problem (f) can factor an integer in a time that grows (roughly) as the square of the number of digits. This raises the question of whether quantum computers could solve other classically difficult prob-

¹Center for Theoretical Physics, Massachusetts Institute of Technology, Cambridge, MA 02139, USA. ²Department of Mathematics, Northeastern University, Boston, MA 02115, USA. ³Massachusetts Institute of Technology, Cambridge, MA 02139, USA.

*To whom correspondence should be addressed. E-mail: farhi@mit.edu

Farhi et al., 2001

Kadowaki and Nishimori. Quantum Annealing in the Transverse Ising Model. Phys. Rev. E, 1998.

Farhi et al. Quantum Adiabatic Evolution Algorithm Applied to Random Instances of an NP-Complete Problem. Science, 2001.

Foundations of AQC (D-Wave)

Initial state:
$$|\psi(t=0)\rangle = \bigotimes_{i=1}^n \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle)$$

Transition (simplified):
$$H(t) = \left(1 - \frac{t}{\tau}\right) H_I + \frac{t}{\tau} H_P$$

Quadratic Unconstrained Binary Optimisation (QUBO) problem:

$$\arg \min_{\mathbf{x} \in \mathcal{B}^n} \mathbf{x}^\top \mathbf{Q} \mathbf{x} + \mathbf{s}^\top \mathbf{x}$$

Final state encoding the problem and the data:

$$\min_{\mathbf{s} \in \{-1,1\}^n} \mathbf{s}^\top \mathbf{J} \mathbf{s} + \mathbf{b}^\top \mathbf{s}$$

binary vector
(qubits during optimisation)

qubit couplings
(interaction weights)

qubit biases
(individual weights)

Exemplary Q (QUBO, 21 qubits):

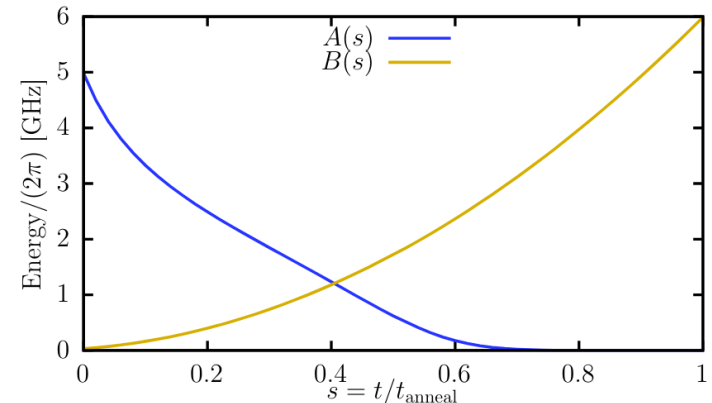
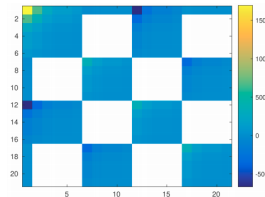


Image: Willsch et al. Computer Physics Communications, 2022.

Annealing functions (schedules)

Five Steps of Every AQC Algorithm

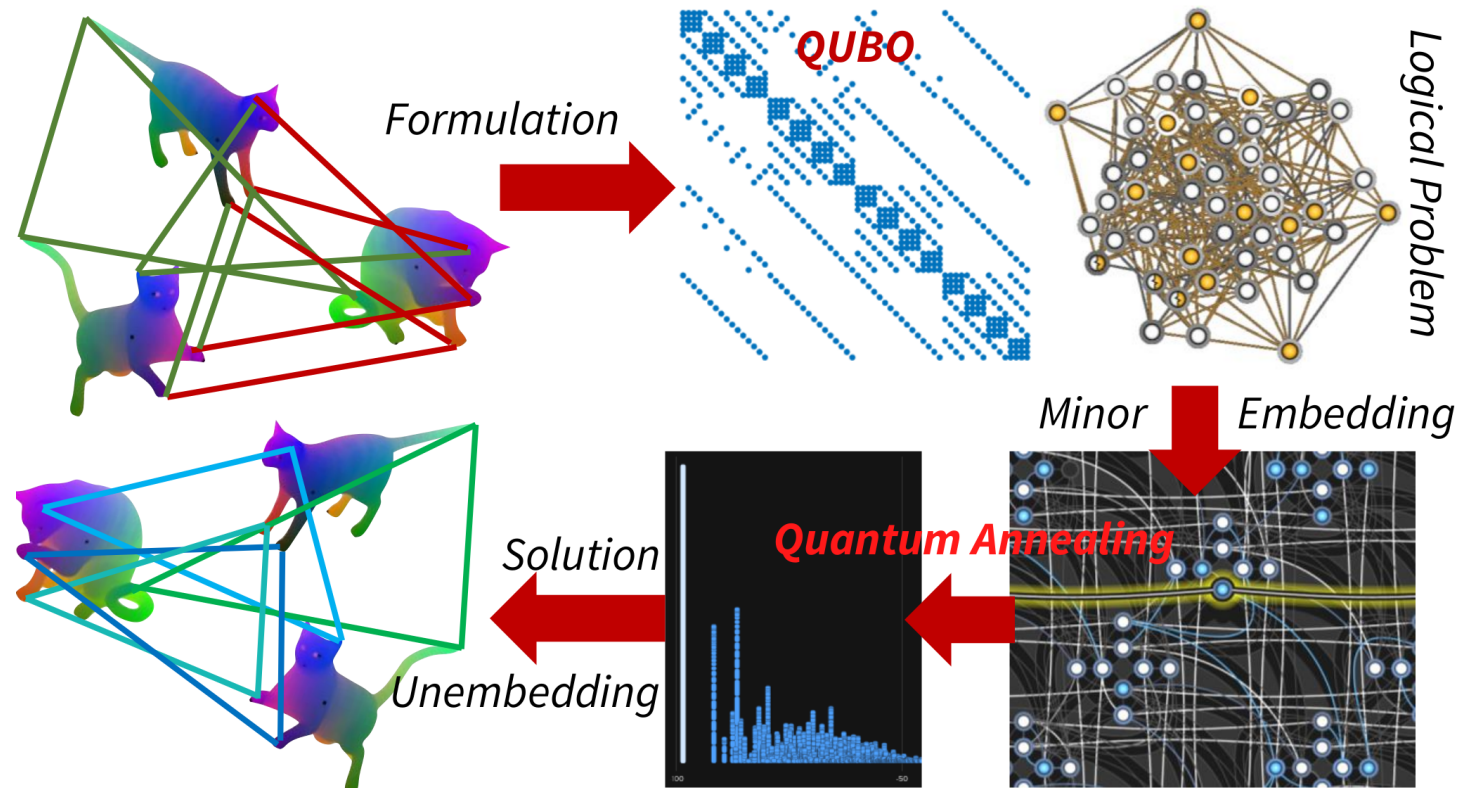


Image: Birdal and Golyanik et al., CVPR 2021.

Five Steps of Every AQC Algorithm

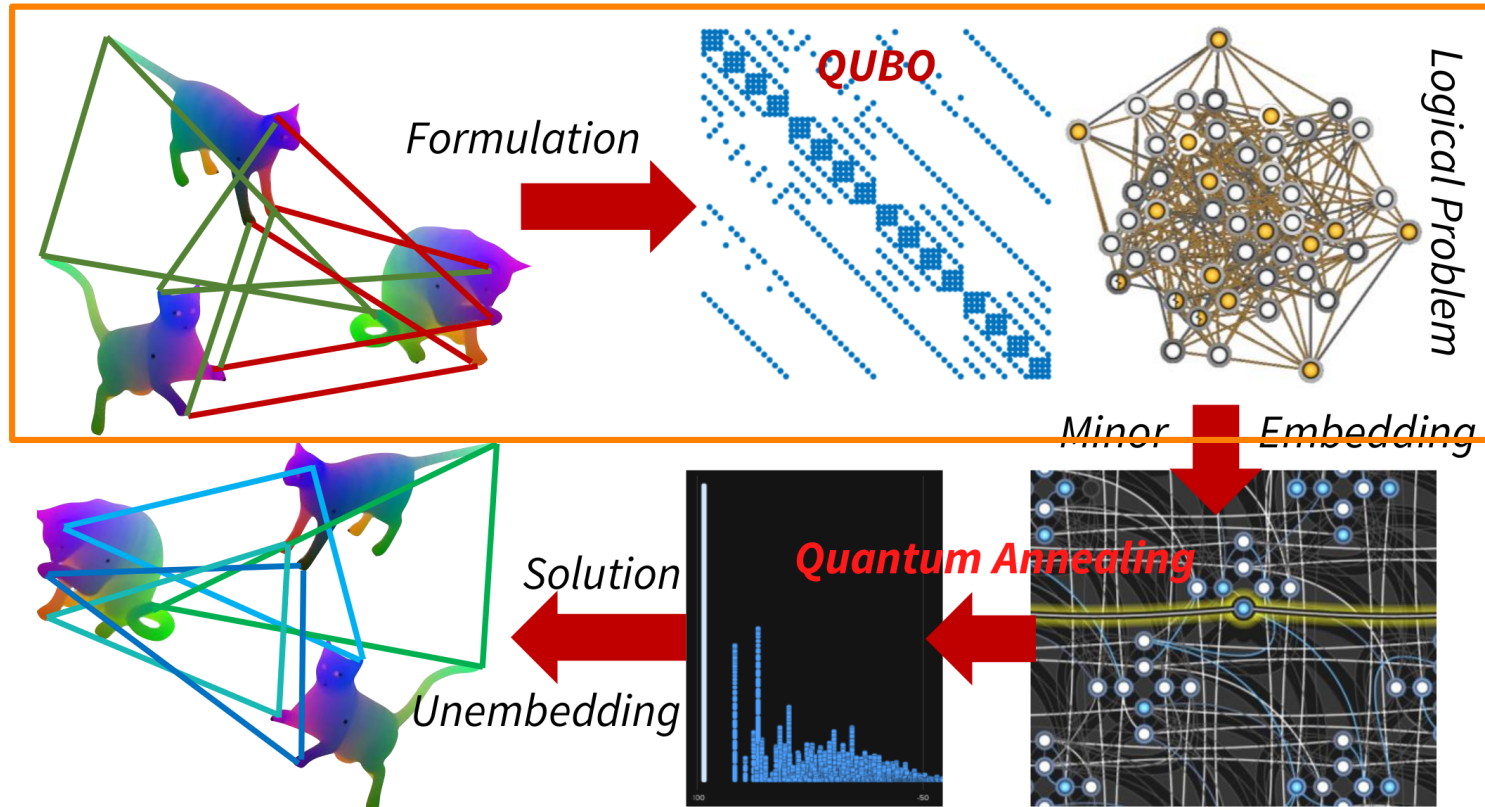
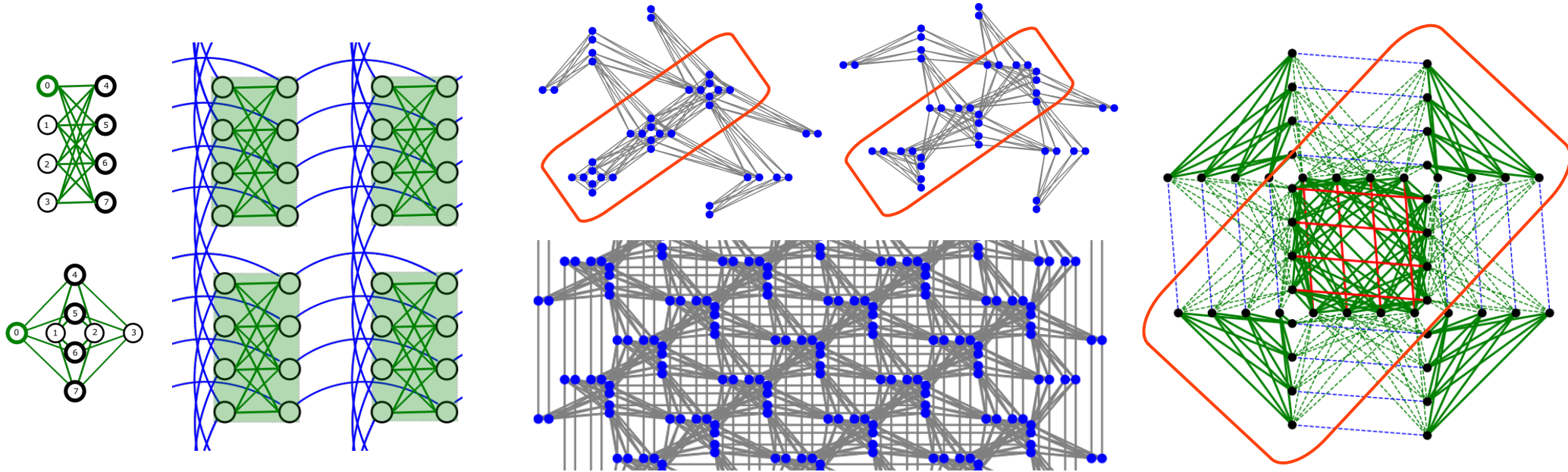


Image: Birdal and Golyanik et al., CVPR 2021.

D-Wave Quantum Annealers



- 2048 qubits (16x16x8)
- Nominal length 4 (internal couplers)
- Degree 6 (+2 external qubits)
- Internal and external couplers

2000Q

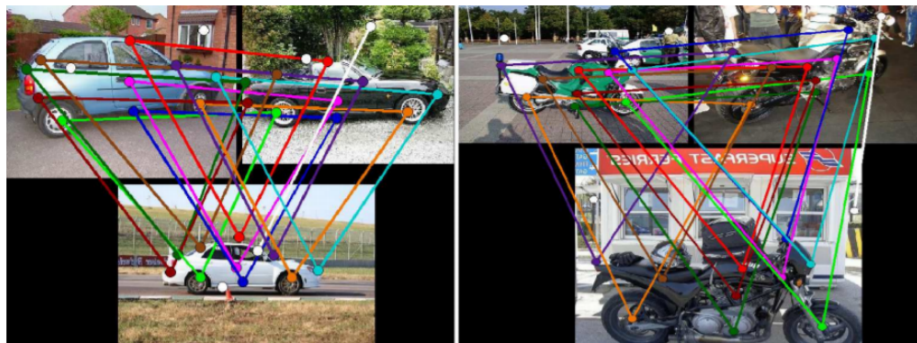
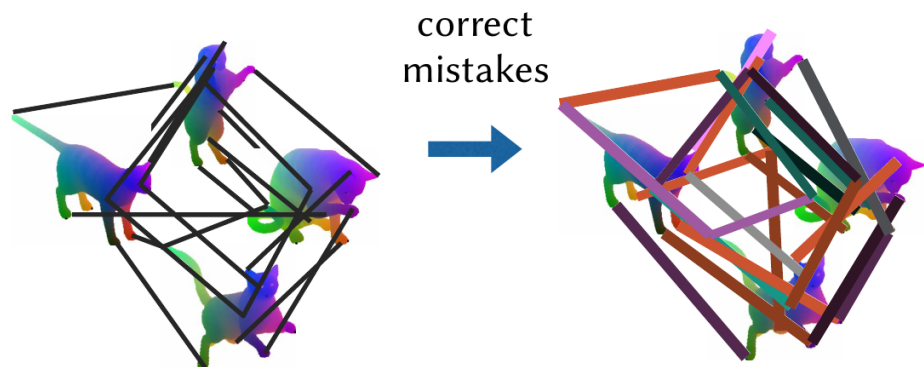
- 5640 qubits (~16x16x24)
- Nominal length 12 (internal couplers)
- Degree 15 (+3 external qubits)
- Internal, external and odd couplers

Advantage

- 7440 qubits (~15x15x32)
- Nominal length 16 (internal couplers)
- Degree 20 (+4 external qubits)
- Internal, external and odd couplers

Advantage 2

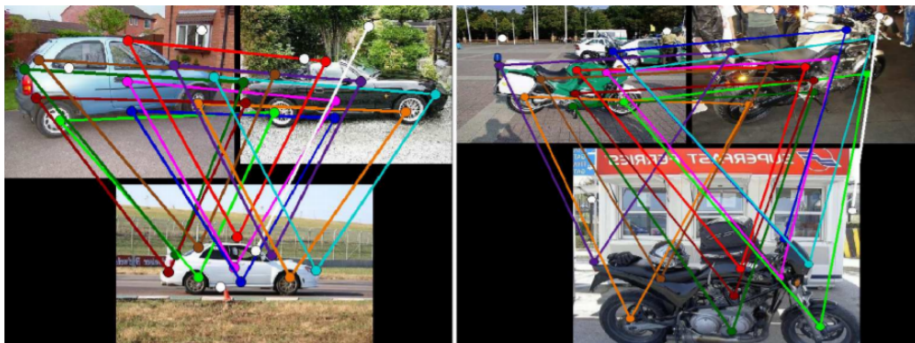
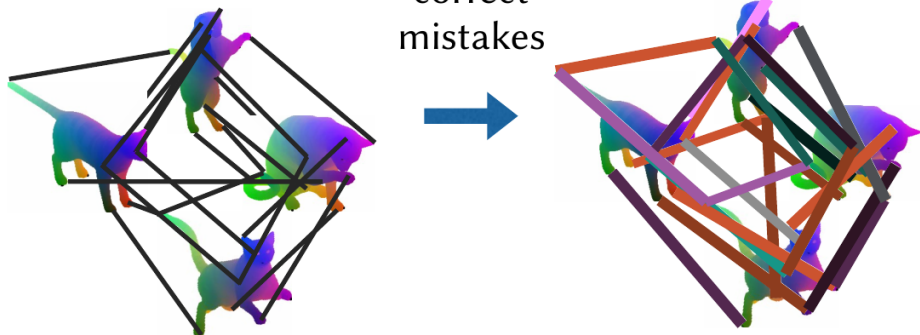
Permutation Synchronisation



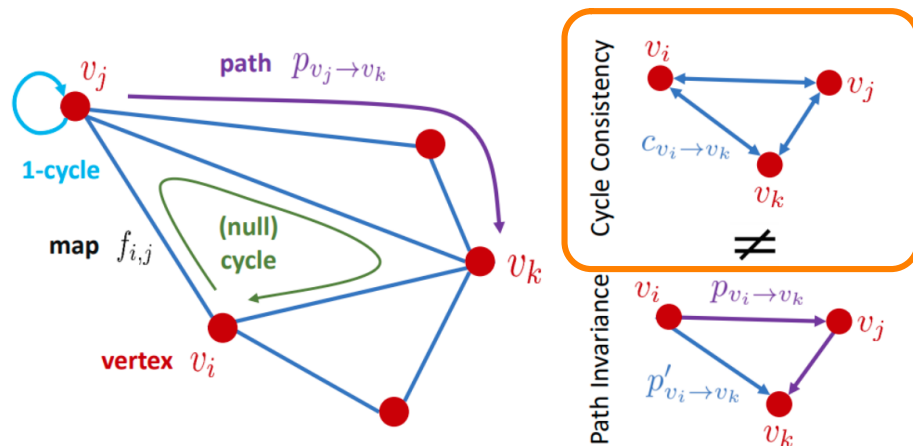
2D or 3D inputs

Permutation Synchronisation

correct
mistakes



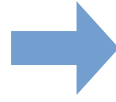
2D or 3D inputs



Permutation Synchronisation

$$\arg \min_{\{\mathbf{X}_i \in \mathcal{P}_n\}} \sum_{(i,j) \in \mathcal{E}} \|\mathbf{P}_{ij} - \mathbf{X}_i \mathbf{X}_j^\top\|_F^2 = \arg \min_{\{\mathbf{X}_i \in \mathcal{P}_n\}} \mathbf{x}^\top \mathbf{Q}' \mathbf{x},$$

$$\mathbf{Q}' = - \begin{bmatrix} \mathbf{I} \otimes \mathbf{P}_{11} & \mathbf{I} \otimes \mathbf{P}_{12} & \cdots & \mathbf{I} \otimes \mathbf{P}_{1m} \\ \mathbf{I} \otimes \mathbf{P}_{21} & \mathbf{I} \otimes \mathbf{P}_{22} & \cdots & \mathbf{I} \otimes \mathbf{P}_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{I} \otimes \mathbf{P}_{m1} & \mathbf{I} \otimes \mathbf{P}_{m2} & \cdots & \mathbf{I} \otimes \mathbf{P}_{mm} \end{bmatrix}.$$

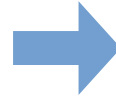


Permutation Synchronisation

$$\arg \min_{\{\mathbf{X}_i \in \mathcal{P}_n\}} \sum_{(i,j) \in \mathcal{E}} \|\mathbf{P}_{ij} - \mathbf{X}_i \mathbf{X}_j^\top\|_F^2 = \arg \min_{\{\mathbf{X}_i \in \mathcal{P}_n\}} \mathbf{x}^\top \mathbf{Q}' \mathbf{x},$$

$$\arg \min_{\mathbf{x} \in \mathcal{B}} \mathbf{x}^\top \mathbf{Q}' \mathbf{x} \quad \text{s.t.} \quad \mathbf{A} \mathbf{x} = \mathbf{b}$$

$$\mathbf{Q}' = - \begin{bmatrix} \mathbf{I} \otimes \mathbf{P}_{11} & \mathbf{I} \otimes \mathbf{P}_{12} & \cdots & \mathbf{I} \otimes \mathbf{P}_{1m} \\ \mathbf{I} \otimes \mathbf{P}_{21} & \mathbf{I} \otimes \mathbf{P}_{22} & \cdots & \mathbf{I} \otimes \mathbf{P}_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{I} \otimes \mathbf{P}_{m1} & \mathbf{I} \otimes \mathbf{P}_{m2} & \cdots & \mathbf{I} \otimes \mathbf{P}_{mm} \end{bmatrix}.$$



is turned into

$$\arg \min_{\mathbf{x} \in \mathcal{B}} \mathbf{x}^\top \mathbf{Q} \mathbf{x} + \mathbf{s}^\top \mathbf{x},$$

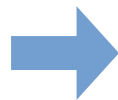
where $\mathbf{Q} = \mathbf{Q}' + \lambda \mathbf{A}^\top \mathbf{A}$ and $\mathbf{s} = -2\lambda \mathbf{A}^\top \mathbf{b}$.

Permutation Synchronisation

$$\arg \min_{\{\mathbf{X}_i \in \mathcal{P}_n\}} \sum_{(i,j) \in \mathcal{E}} \|\mathbf{P}_{ij} - \mathbf{X}_i \mathbf{X}_j^\top\|_F^2 = \arg \min_{\{\mathbf{X}_i \in \mathcal{P}_n\}} \mathbf{x}^\top \mathbf{Q}' \mathbf{x},$$

$$\arg \min_{\mathbf{x} \in \mathcal{B}} \mathbf{x}^\top \mathbf{Q}' \mathbf{x} \quad \text{s.t.} \quad \mathbf{A} \mathbf{x} = \mathbf{b}$$

$$\mathbf{Q}' = - \begin{bmatrix} \mathbf{I} \otimes \mathbf{P}_{11} & \mathbf{I} \otimes \mathbf{P}_{12} & \cdots & \mathbf{I} \otimes \mathbf{P}_{1m} \\ \mathbf{I} \otimes \mathbf{P}_{21} & \mathbf{I} \otimes \mathbf{P}_{22} & \cdots & \mathbf{I} \otimes \mathbf{P}_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{I} \otimes \mathbf{P}_{m1} & \mathbf{I} \otimes \mathbf{P}_{m2} & \cdots & \mathbf{I} \otimes \mathbf{P}_{mm} \end{bmatrix}.$$



is turned into

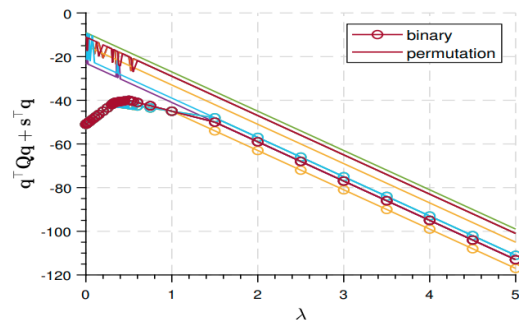
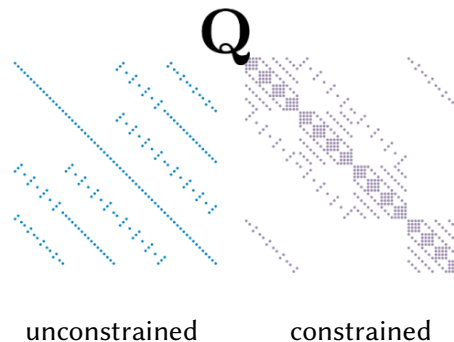
$$\arg \min_{\mathbf{x} \in \mathcal{B}} \mathbf{x}^\top \mathbf{Q} \mathbf{x} + \mathbf{s}^\top \mathbf{x},$$

where $\mathbf{Q} = \mathbf{Q}' + \lambda \mathbf{A}^\top \mathbf{A}$ and $\mathbf{s} = -2\lambda \mathbf{A}^\top \mathbf{b}$.

$$\mathcal{P}_n := \{ \mathbf{P} \in \{0, 1\}^{n \times n} : \underbrace{\mathbf{P} \mathbf{1}_n = \mathbf{1}_n}_{\text{Binary}}, \underbrace{\mathbf{1}_n^\top \mathbf{P} = \mathbf{1}_n^\top}_{\text{Rows sum to 1}}, \underbrace{\mathbf{1}_n^\top \mathbf{P} = \mathbf{1}_n^\top}_{\text{Cols sum to 1}} \}$$

$$\mathbf{A}_i = \begin{bmatrix} \mathbf{I} \otimes \mathbf{1}^\top \\ \mathbf{1}^\top \otimes \mathbf{I} \end{bmatrix} \quad \mathbf{A} = \begin{bmatrix} \mathbf{A}_1 & & & \\ & \mathbf{A}_2 & & \\ & & \ddots & \\ & & & \mathbf{A}_n \end{bmatrix}$$

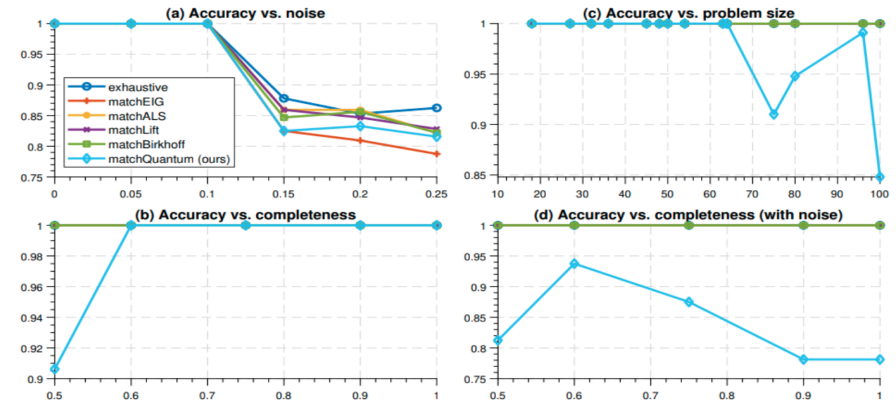
$$\mathbf{b}_i = \mathbf{1} \quad \mathbf{b} = \mathbf{1}$$



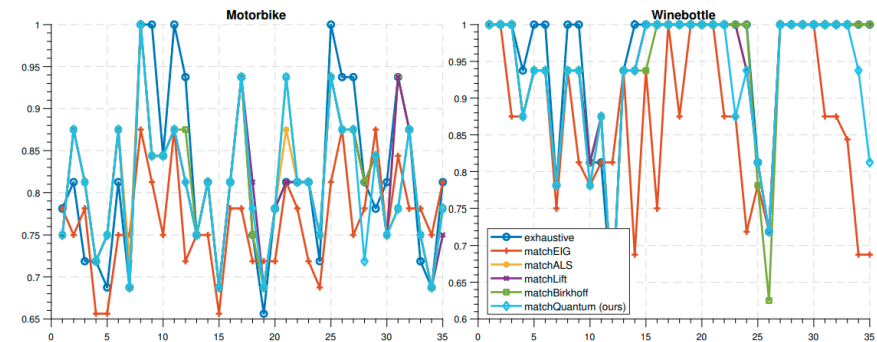
Permutation Synchronisation

	Car	Duck	Motorbike	Winebottle	Average
Exhaustive	0.84 ± 0.104	0.91 ± 0.115	0.82 ± 0.10	0.95 ± 0.096	0.88 ± 0.104
EIG	0.81 ± 0.083	0.86 ± 0.102	0.77 ± 0.059	0.87 ± 0.107	0.83 ± 0.088
ALS	0.84 ± 0.095	0.90 ± 0.102	0.81 ± 0.078	0.94 ± 0.092	0.87 ± 0.092
LIFT	0.84 ± 0.102	0.90 ± 0.103	0.81 ± 0.078	0.94 ± 0.092	0.87 ± 0.094
Birkhoff	0.84 ± 0.094	0.90 ± 0.107	0.81 ± 0.079	0.94 ± 0.093	0.87 ± 0.093
D-Wave(Ours)	0.84 ± 0.104	0.90 ± 0.104	0.81 ± 0.080	0.93 ± 0.095	0.87 ± 0.096

Average Errors



Evaluations on the synthetic dataset (4 views and 4 points)

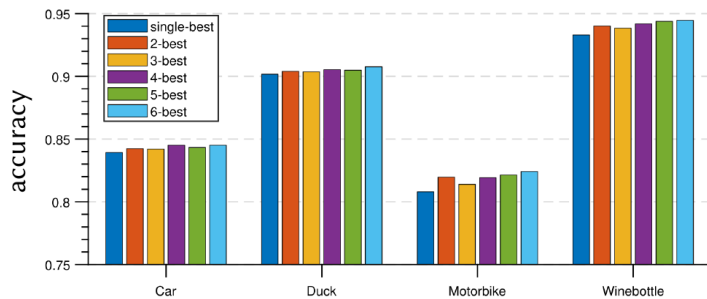


Detailed evaluations over all the subsets

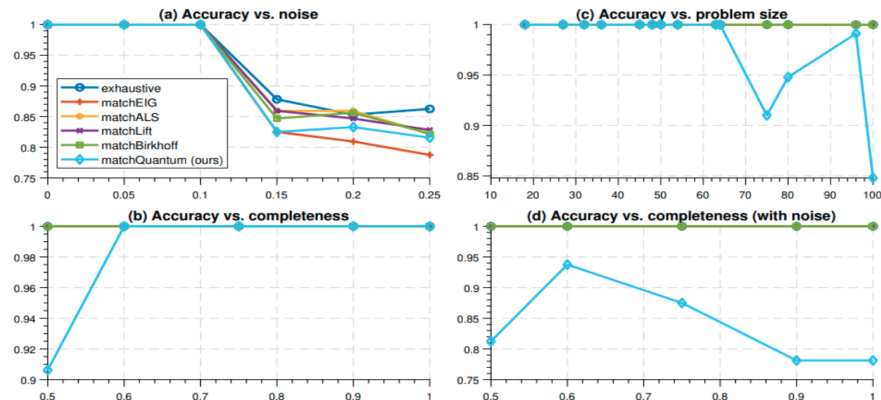
Permutation Synchronisation

	Car	Duck	Motorbike	Winebottle	Average
Exhaustive	0.84 ± 0.104	0.91 ± 0.115	0.82 ± 0.10	0.95 ± 0.096	0.88 ± 0.104
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Birkhoff	0.84 ± 0.094	0.90 ± 0.107	0.81 ± 0.079	0.94 ± 0.093	0.87 ± 0.093
D-Wave(Ours)	0.84 ± 0.104	0.90 ± 0.104	0.81 ± 0.080	0.93 ± 0.095	0.87 ± 0.096

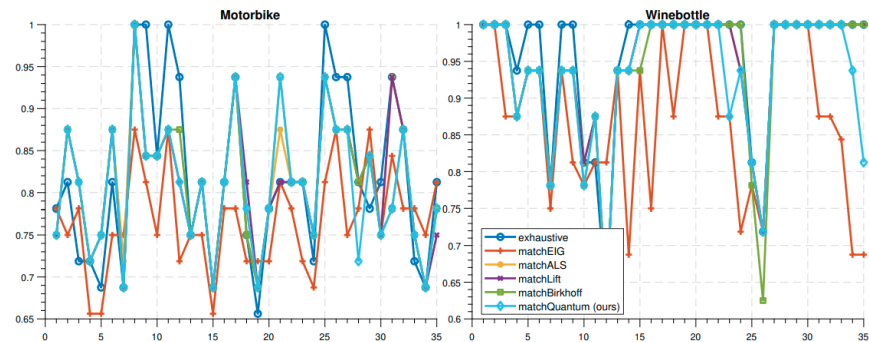
Average Errors



Bit corrections using multiple measurements of different energies

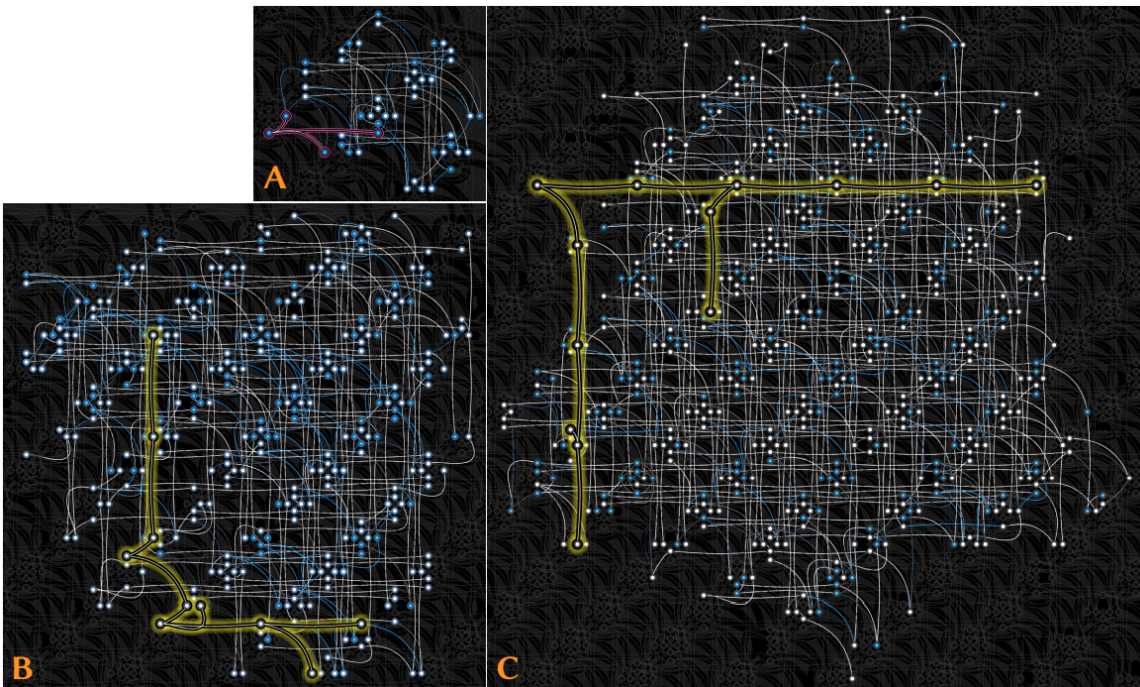


Evaluations on the synthetic dataset (4 views and 4 points)



Detailed evaluations over all the subsets

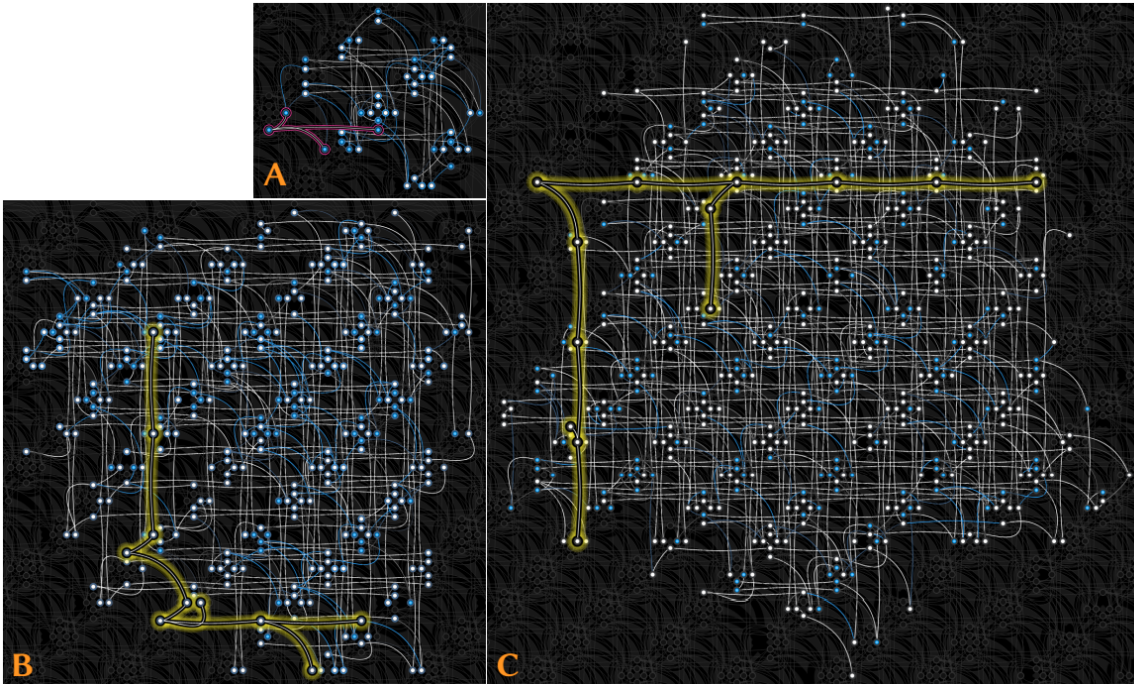
Permutation Synchronisation



Exemplary minor embeddings in the experiments with

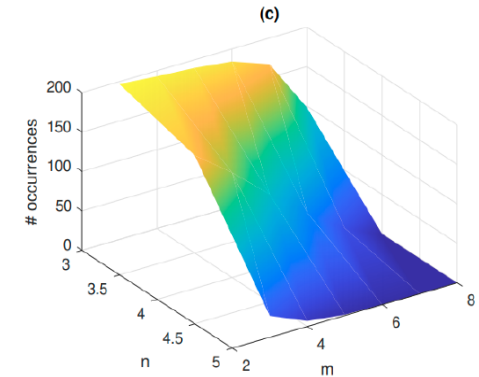
- $n = 3, m = 3$ (A, $N = 49$),
- $n = 4, m = 4$ (B, $N = 341$), and
- $n = 3, m = 8$ (C, $N = 550$).

Permutation Synchronisation

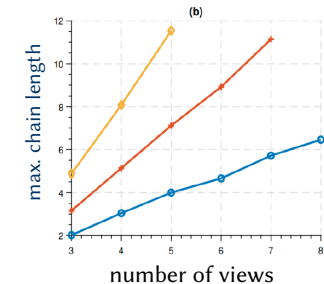
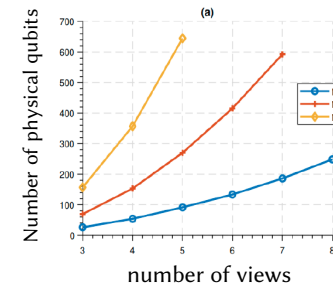


Exemplary minor embeddings in the experiments with

- $n = 3, m = 3$ (A, $N = 49$),
- $n = 4, m = 4$ (B, $N = 341$), and
- $n = 3, m = 8$ (C, $N = 550$).



Average number of measured optimal solutions



Minor embedding functions

Motion Segmentation

$$Z_{ij} = X_i X_j^T$$



Goal: Classify points in multiple images into different motions

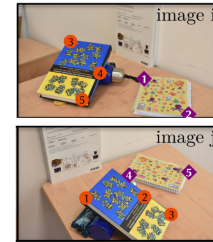


	image j				
image i	1	2	3	4	5
1	0	0	0	1	1
2	0	0	0	1	1
3	1	1	1	0	0
4	1	1	1	0	0
5	1	1	1	0	0

Z_{ij}

two-frame matches
(known)

	motions	
image i	orange	purple
1	0	1
2	0	1
3	1	0
4	1	0
5	1	0

X_i

absolute segmentations
(unknown)

	motions	
image j	orange	purple
1	1	0
2	1	0
3	1	0
4	0	1
5	0	1

X_j

Motion Segmentation

$$Z_{ij} = X_i X_j^T$$



Goal: Classify points in multiple images into different motions

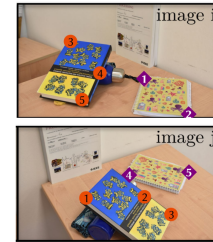


image j

	1	2	3	4	5
image i	1	2	3	4	5
1	0	0	0	1	1
2	0	0	0	1	1
3	1	1	1	0	0
4	1	1	1	0	0
5	1	1	1	0	0

Z_{ij}
two-frame matches
(known)

motions

image i	1	2
1	0	1
2	0	1
3	1	0
4	1	0
5	1	0

motions

image j	1	2
1	1	0
2	1	0
3	1	0
4	0	1
5	0	1

X_i X_j
absolute segmentations
(unknown)

$$\min_{X_1, \dots, X_n} \sum_{(i,j) \in \mathcal{E}} \|Z_{ij} - X_i X_j^T\|_F^2,$$

$$\text{s.t. } \text{vec}(X_i) \in \mathcal{B}^{p_i}, \quad X_i \mathbf{1}_d = \mathbf{1}_{p_i} \quad \forall i = 1, \dots, n$$

$$X = \begin{bmatrix} X_1 \\ X_2 \\ \dots \\ X_n \end{bmatrix}, \quad Z = \begin{bmatrix} 0 & Z_{12} & \dots & Z_{1n} \\ Z_{21} & 0 & \dots & Z_{2n} \\ \dots & \dots & \dots & \dots \\ Z_{n1} & Z_{n2} & \dots & 0 \end{bmatrix}$$

Motion Segmentation

$$Z_{ij} = X_i X_j^T$$



Goal: Classify points in multiple images into different motions

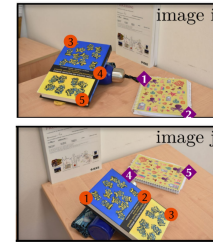


	image j				
image i	1	2	3	4	5
1	0	0	0	1	1
2	0	0	0	1	1
3	1	1	1	0	0
4	1	1	1	0	0
5	1	1	1	0	0

Z_{ij}

two-frame matches
(known)

	motions	
image i	orange	purple
1	0	1
2	0	1
3	1	0
4	1	0
5	1	0

X_i

	motions	
image j	orange	purple
1	1	0
2	1	0
3	1	0
4	0	1
5	0	1

X_j

absolute segmentations
(unknown)

$$\min_{X_1, \dots, X_n} \sum_{(i,j) \in \mathcal{E}} \|Z_{ij} - X_i X_j^T\|_F^2,$$

$$\text{s.t. } \text{vec}(X_i) \in \mathcal{B}^{p_i}, \quad X_i \mathbf{1}_d = \mathbf{1}_{p_i} \quad \forall i = 1, \dots, n$$

$$X = \begin{bmatrix} X_1 \\ X_2 \\ \dots \\ X_n \end{bmatrix}, \quad Z = \begin{bmatrix} 0 & Z_{12} & \dots & Z_{1n} \\ Z_{21} & 0 & \dots & Z_{2n} \\ \dots & \dots & \dots & \dots \\ Z_{n1} & Z_{n2} & \dots & 0 \end{bmatrix}$$



$$\min_{\mathbf{y} \in \mathcal{B}^k} \mathbf{y}^T Q \mathbf{y} + \mathbf{s}^T \mathbf{y} + \sum_i \lambda_i \|A_i \mathbf{y} - \mathbf{b}_i\|^2$$

Permutation Synchronisation

$$\min_{X_1, \dots, X_n} \sum_{(i,j) \in \mathcal{E}} \|Z_{ij} - X_i X_j^T\|_F^2,$$

$$\text{s.t. } \text{vec}(X_i) \in \mathcal{B}^{p_i}, \quad X_i \mathbf{1}_d = \mathbf{1}_{p_i} \quad \forall i = 1, \dots, n$$

binary variables each point belongs to one motion

$$X = \begin{bmatrix} X_1 \\ X_2 \\ \dots \\ X_n \end{bmatrix}, \quad Z = \begin{bmatrix} 0 & Z_{12} & \dots & Z_{1n} \\ Z_{21} & 0 & \dots & Z_{2n} \\ \dots & \dots & \dots & \dots \\ Z_{n1} & Z_{n2} & \dots & 0 \end{bmatrix}$$



Permutation Synchronisation

$$\min_{X_1, \dots, X_n} \sum_{(i,j) \in \mathcal{E}} \|Z_{ij} - X_i X_j^\top\|_F^2,$$

$$\text{s.t. } \text{vec}(X_i) \in \mathcal{B}^{p_i}, \quad X_i \mathbf{1}_d = \mathbf{1}_{p_i} \quad \forall i = 1, \dots, n$$

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QuMoSeg-v1, dense Q

$$\max_{X_1, \dots, X_n} \sum_{(i,j) \in \mathcal{E}} \text{trace}(X_i^\top (2Z_{ij} - \mathbf{1}_{p_i \times p_j}) X_j)$$

$$\text{s.t. } \text{vec}(X_i) \in \mathcal{B}^{p_i}, \quad X_i \mathbf{1}_d = \mathbf{1}_{p_i} \quad \forall i = 1, \dots, n$$



$$\max_X \text{vec}(X)^\top (I_{d \times d} \otimes (2Z - \mathbf{1}_{p \times p})) \text{vec}(X),$$

$$\text{s.t. } \text{vec}(X) \in \mathcal{B}^{dp}, \quad X \mathbf{1}_d = \mathbf{1}_p.$$



Permutation Synchronisation

$$\min_{X_1, \dots, X_n} \sum_{(i,j) \in \mathcal{E}} \|Z_{ij} - X_i X_j^\top\|_F^2,$$

$$\text{s.t. } \text{vec}(X_i) \in \mathcal{B}^{p_i}, \quad X_i \mathbf{1}_d = \mathbf{1}_{p_i} \quad \forall i = 1, \dots, n$$

binary variables each point belongs to one motion

$$X = \begin{bmatrix} X_1 \\ X_2 \\ \dots \\ X_n \end{bmatrix}, \quad Z = \begin{bmatrix} 0 & Z_{12} & \dots & Z_{1n} \\ Z_{21} & 0 & \dots & Z_{2n} \\ \dots & \dots & \dots & \dots \\ Z_{n1} & Z_{n2} & \dots & 0 \end{bmatrix}$$



QuMoSeg-v1, dense Q

$$\max_{X_1, \dots, X_n} \sum_{(i,j) \in \mathcal{E}} \text{trace}(X_i^\top (2Z_{ij} - \mathbf{1}_{p_i \times p_j}) X_j)$$

$$\text{s.t. } \text{vec}(X_i) \in \mathcal{B}^{p_i}, \quad X_i \mathbf{1}_d = \mathbf{1}_{p_i} \quad \forall i = 1, \dots, n$$



$$\max_X \text{vec}(X)^\top (I_{d \times d} \otimes (2Z - \mathbf{1}_{p \times p})) \text{vec}(X),$$

$$\text{s.t. } \text{vec}(X) \in \mathcal{B}^{dp}, \quad X \mathbf{1}_d = \mathbf{1}_p.$$



QuMoSeg-v2, sparse Q (additional assumptions)

$$\max_{X_1, \dots, X_n} \sum_{(i,j) \in \mathcal{E}} \text{trace}(X_i^\top Z_{ij} X_j),$$

$$\text{s.t. } \text{vec}(X_i) \in \mathcal{B}^{p_i}, \quad X_i \mathbf{1}_d = \mathbf{1}_{p_i}, \quad \mathbf{1}_{p_i}^\top X_i = \mathbf{m}_i^\top \quad \forall i = 1, \dots, n.$$

extra constraints



$$\max_X \text{vec}(X)^\top (I_{d \times d} \otimes Z) \text{vec}(X),$$

$$\text{s.t. } \text{vec}(X) \in \mathcal{B}^{dp}, \quad X \mathbf{1}_d = \mathbf{1}_p, \quad KX = M$$

Motion Segmentation

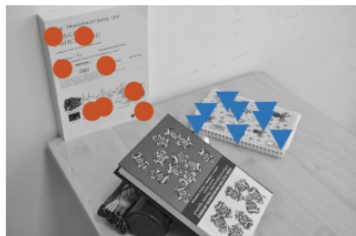
 $\mu =$

0.97

0.93

0.93

0.89



ground truth

QuMoSeg

Mode, ICCV'19

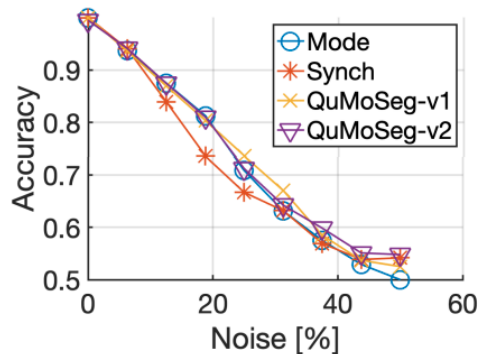
Synch, ICCVW'19

Xu et al., TPAMI'19

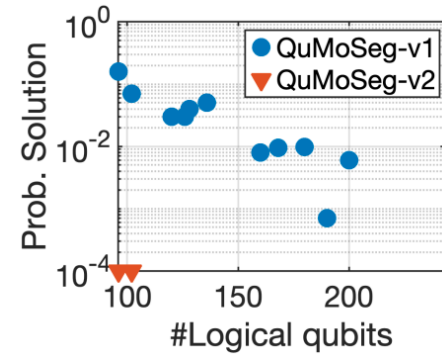
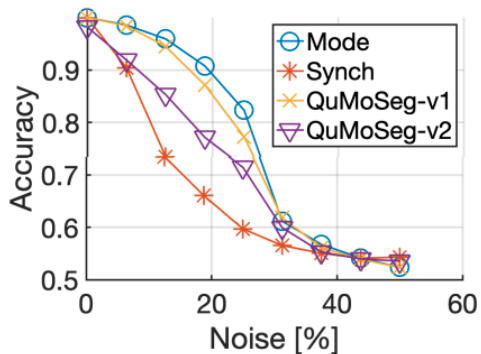
Motion Segmentation

	# Qubits:	96	102	120	126	128	136	160	168	180	190	200	216	220	243						
Q-MSEG	Xu et al. [63]	0.89	0.89	0.94	0.75	0.96	0.97	0.86	0.86	0.97	0.88	0.96	0.77	0.83	0.74						
	MODE [5]	0.93	0.93	0.96	0.93	0.97	0.97	0.98	0.99	0.98	0.99	0.99	0.93	1	0.94						
	SYNCH [4]	0.93	0.94	0.95	0.95	0.84	0.92	0.97	1	0.89	0.95	0.90	0.94	0.99	0.92						
	QUMoSEG-v1	0.97	0.97	0.97	0.96	0.95	0.98	0.98	0.99	0.98	0.99	0.99	0.99	0.64	–	–					
	QUMoSEG-v2	0.96	0.97	0.95	0.94	0.89	0.89	0.88	0.85	0.74	0.75	0.79	0.59	0.75	0.58						
	QUMoSEG-v1, SA	0.97	0.97	0.97	0.96	0.95	0.98	0.98	1	0.98	0.99	0.99	0.99	0.68	0.98	0.72					
	QUMoSEG-v2, SA	0.98	0.99	0.99	1	0.96	0.98	0.98	1	0.94	0.97	0.99	0.80	1	0.59						
	# Qubits:	120	126	132	138	144	156	162	168	174	180	186	192	198	204	210	216	222	228	234	240
Hopkins	Xu et al. [63]	0.80	0.78	0.81	0.79	0.83	0.81	0.84	0.81	0.85	0.89	0.88	0.94	0.96	0.96	0.97	1	0.98	1	0.99	1
	MODE [5]	0.89	0.91	0.90	0.93	0.92	0.94	0.95	0.95	0.96	0.95	0.97	0.98	0.98	0.98	0.98	0.99	0.98	0.98	0.99	0.99
	SYNCH [4]	0.87	0.93	0.95	0.96	0.99	0.96	0.99	0.99	0.96	0.99	1	1	0.99	1	1	0.70	0.97	1	0.99	0.65
	QUMoSEG-v1	0.92	0.89	0.93	0.93	0.93	0.93	0.95	0.94	0.96	0.95	0.96	0.97	0.96	-	-	-	-	-	-	-
	QUMoSEG-v2	0.91	0.92	0.91	0.92	0.94	0.89	0.91	0.89	0.90	0.88	0.88	0.89	0.88	0.89	0.88	-	-	-	-	-
	QUMoSEG-v1, SA	0.93	0.90	0.92	0.94	0.93	0.94	0.95	0.96	0.96	0.96	0.98	0.98	0.98	0.98	0.99	0.99	0.98	0.98	0.99	0.99
	QUMoSEG-v2, SA	0.96	0.97	0.98	0.98	0.99	0.99	0.99	0.97	0.99	1	1	1	1	1	1	1	1	0.99	1	0.99

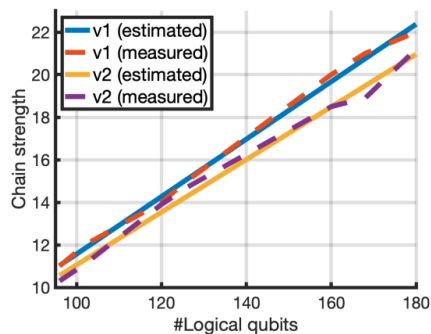
Motion Segmentation



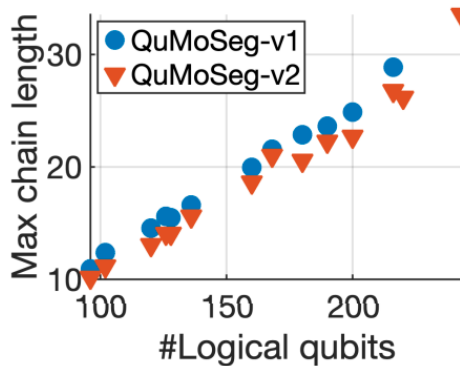
Accuracy under noisy inputs



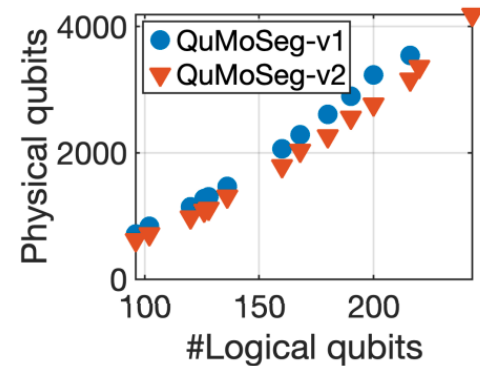
Optimal solution probabilities



Chain strengths

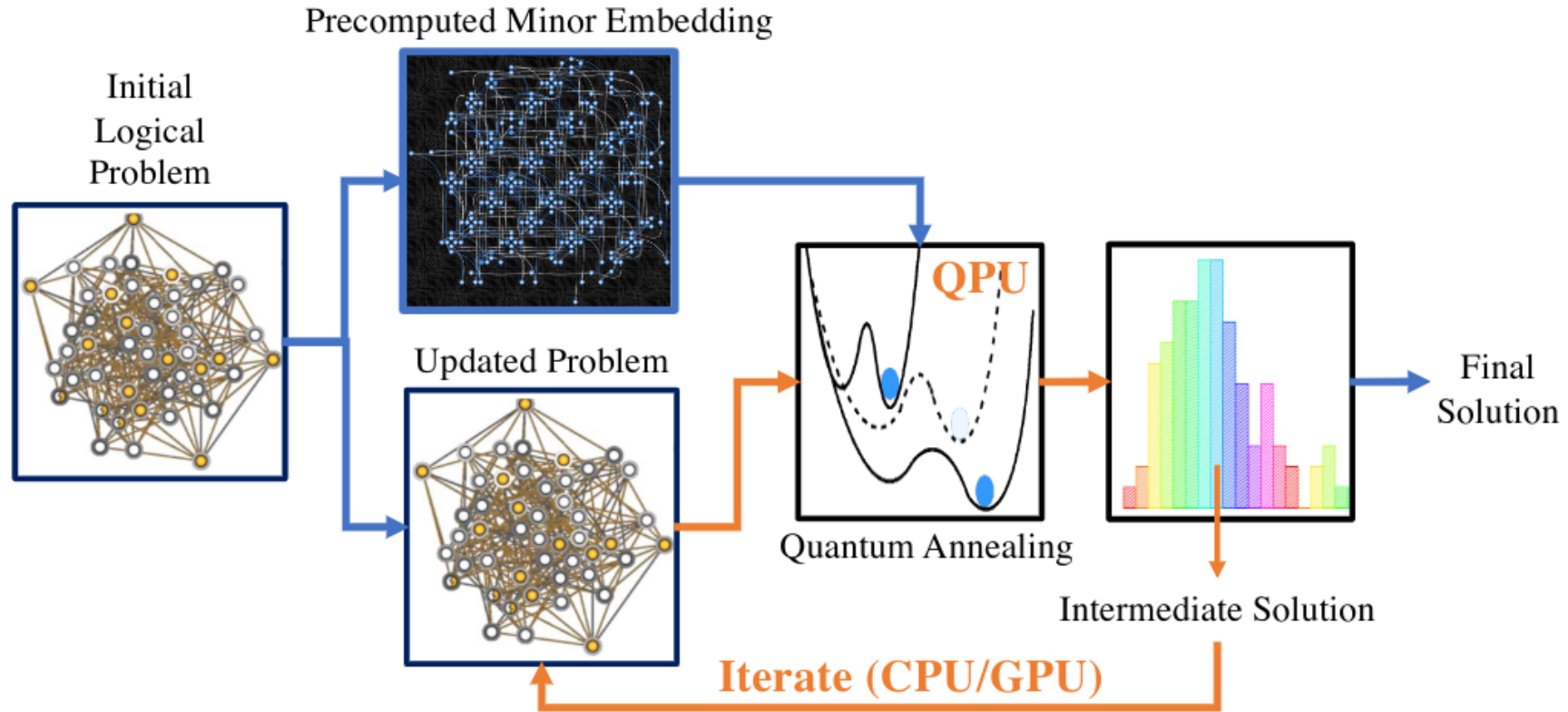


Minor embedding functions

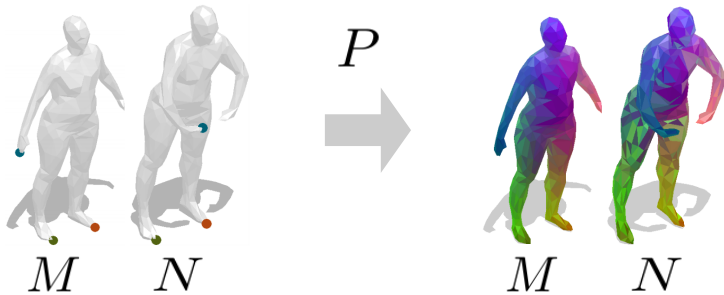


Iterative AQC Algorithms

Iterative AQC Algorithms



Shape Matching with AQC



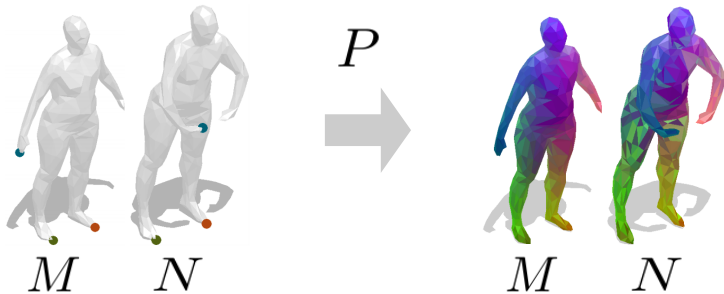
$$\min_{X \in \mathbb{P}_n} E(X) := \mathbf{x}^T W \mathbf{x}$$

$$\mathbf{x} = \text{vec}(X) \quad W \in \mathbb{R}^{n^2 \times n^2}$$

$$\mathbb{P} \subset \{0, 1\}^{n \times n} \text{ (permutation matrix)}$$

$$\mathbb{P}_n = \{X \in \{0, 1\}^{n \times n} \mid \sum_i X_{ij} = 1, \sum_j X_{ij} = 1 \forall i, j\}.$$

Shape Matching with AQC



$$\min_{X \in \mathbb{P}_n} E(X) := \mathbf{x}^T W \mathbf{x}$$

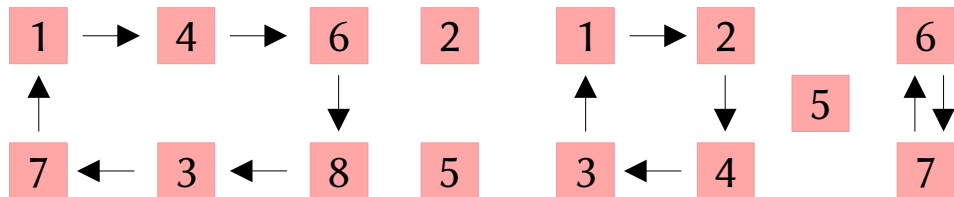
$$\mathbf{x} = \text{vec}(X) \quad W \in \mathbb{R}^{n^2 \times n^2}$$

$$\mathbb{P} \subset \{0, 1\}^{n \times n} \text{ (permutation matrix)}$$

$$\mathbb{P}_n = \{X \in \{0, 1\}^{n \times n} \mid \sum_i X_{ij} = 1, \sum_j X_{ij} = 1 \forall i, j\}.$$

k-cycles:

Disjoint permutations commute:



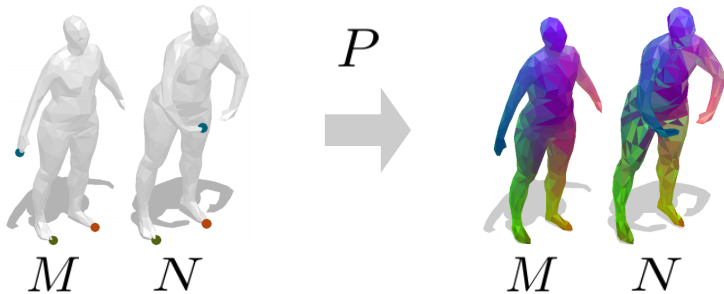
six-cycle

fixed points

four-cycle

two-cycle

Shape Matching with AQC



$$\min_{X \in \mathbb{P}_n} E(X) := \mathbf{x}^T W \mathbf{x}$$

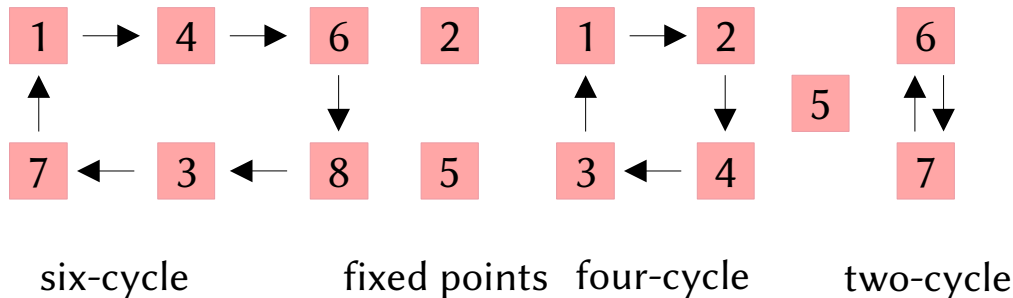
$$\mathbf{x} = \text{vec}(X) \quad W \in \mathbb{R}^{n^2 \times n^2}$$

$$\mathbb{P} \subset \{0, 1\}^{n \times n} \text{ (permutation matrix)}$$

$$\mathbb{P}_n = \{X \in \{0, 1\}^{n \times n} \mid \sum_i X_{ij} = 1, \sum_j X_{ij} = 1 \forall i, j\}.$$

k-cycles:

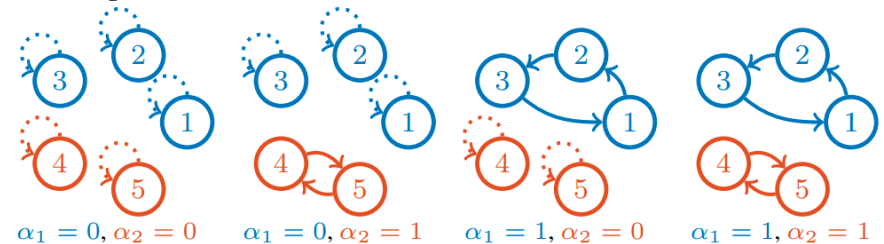
Disjoint permutations commute:



Given two cycles C_1, C_2 we parametrize all combinations with two binary variables α_1, α_2

$$\begin{pmatrix} 1-\alpha_1 & 0 & \alpha_1 & 0 & 0 \\ \alpha_1 & 1-\alpha_1 & 0 & 0 & 0 \\ 0 & \alpha_1 & 1-\alpha_1 & 0 & 0 \\ 0 & 0 & 0 & 1-\alpha_2 & \alpha_2 \\ 0 & 0 & 0 & \alpha_2 & 1-\alpha_2 \end{pmatrix}$$

Possible permutations for all choices of α_1, α_2 :

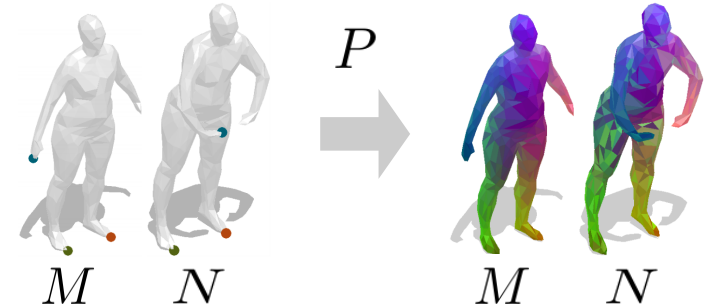


Shape Matching with AQC

Given: 3D shapes M and N , both discretised with n vertices.

$$W_{i \cdot n+k, j \cdot n+l} = |d_M^g(i, j) - d_N^g(k, l)|$$

Find: optimal P

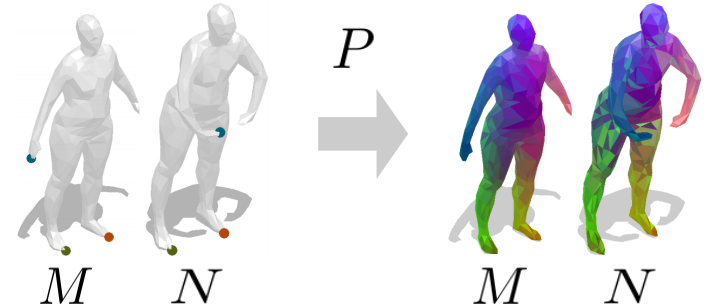


Shape Matching with AQC

Given: 3D shapes M and N , both discretised with n vertices.

$$W_{i \cdot n + k, j \cdot n + l} = |d_M^g(i, j) - d_N^g(k, l)|$$

Find: optimal P



Want to solve but cannot:

$$\min_{X \in \mathbb{P}_n} E(X) := \mathbf{x}^T W \mathbf{x}$$

$$W_{i \cdot n + k, j \cdot n + l} = |d_M^g(i, j) - d_N^g(k, l)|$$

... leading to

$$\min_{\alpha \in \{0,1\}^m} \alpha^T \tilde{W} \alpha \quad \tilde{W}_{ij} = \begin{cases} E(C_i, C_j) & \text{if } i \neq j, \\ E(C_i, C_i) + E(C_i, P_0) + E(P_0, C_j) & \text{otherwise.} \end{cases}$$

not submodular

Instead solve

$$\arg \min_{\{P \in \mathbb{P}_n \mid \exists \alpha \in \{0,1\}^m : P = (\prod_i c_i^{\alpha_i}) P_0\}} E(P)$$

$$C = \{c_1, \dots, c_m\}$$

$$E(Q, R) = \text{vec}(Q)^T W \text{vec}(R)$$

$$P(\alpha) = P_0 + \sum_{i=1}^m \alpha_i \frac{(c_i - I)P_0}{C_i}$$

Shape Matching with AQC

Initialise P_0 via descriptor-based similarity

repeat until converged

obtain I_M and I_N and choose from them a set of k random and disjoint 2-cycles

construct a submatrix of worst matches W_s

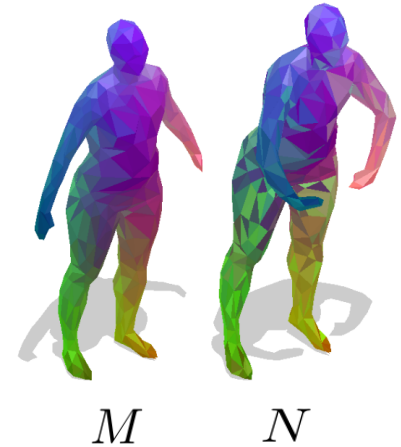
repeat until every 2-cycle occurred

choose a random set of 2-cycles

calculate \tilde{W} and solve $\min_{\alpha \in \{0,1\}^m} \alpha^\top \tilde{W} \alpha$ on QPU

$$P_i = \left(\prod_j c_j^{\alpha_j} \right) P_{i-1}$$

apply the obtained permutation to worst matches



Shape Matching with AQC

Initialise P_0 via descriptor-based similarity

repeat until converged

obtain I_M and I_N and choose from them a set of k random and disjoint 2-cycles

construct a submatrix of worst matches W_s

repeat until every 2-cycle occurred

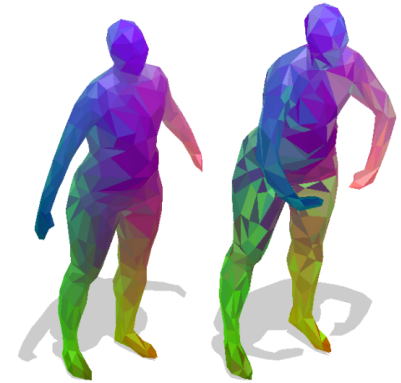
choose a random set of 2-cycles

calculate \tilde{W} and solve $\min_{\alpha \in \{0,1\}^m} \alpha^\top \tilde{W} \alpha$ on QPU

$$P_i = \left(\prod_j c_j^{\alpha_j} \right) P_{i-1}$$

apply the obtained permutation to worst matches

NP-hard; decides
to apply c_i or not

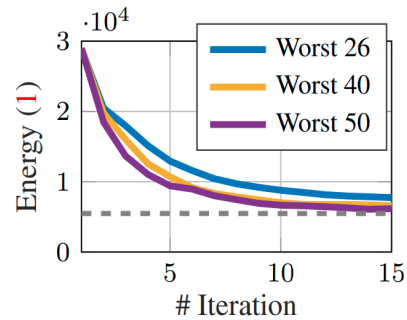
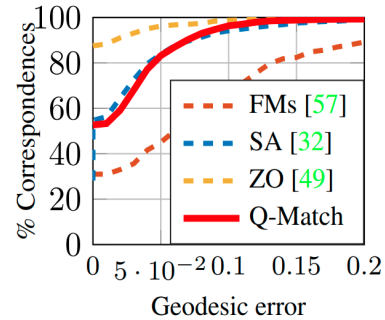


M

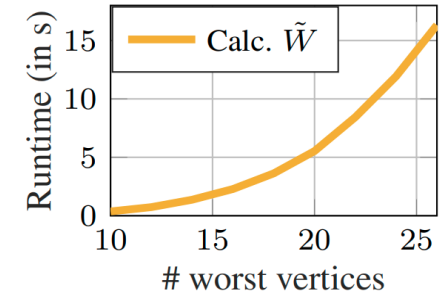
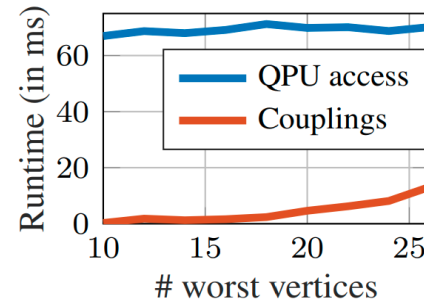
N

$$\begin{pmatrix} 1-\alpha_1 & 0 & \alpha_1 & 0 & 0 \\ \alpha_1 & 1-\alpha_1 & 0 & 0 & 0 \\ 0 & \alpha_1 & 1-\alpha_1 & 0 & 0 \\ 0 & 0 & 0 & 1-\alpha_2 & \alpha_2 \\ 0 & 0 & 0 & \alpha_2 & 1-\alpha_2 \end{pmatrix}$$

Shape Matching with AQC

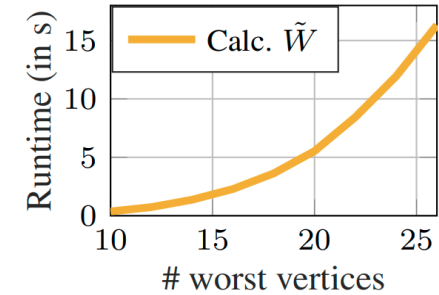
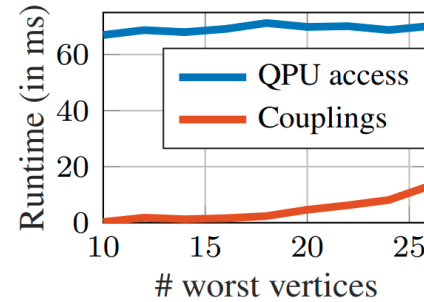
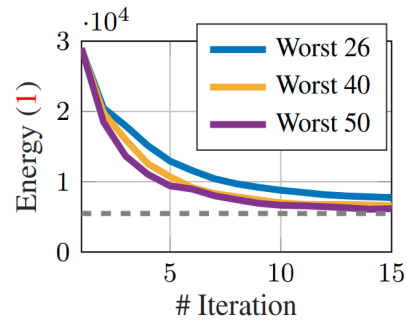
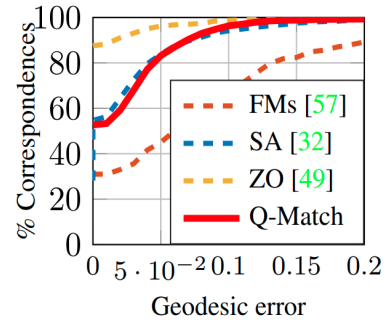


Accuracy and convergence (FAUST)



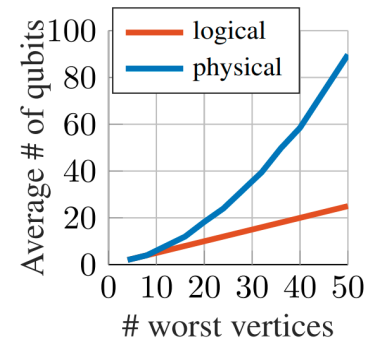
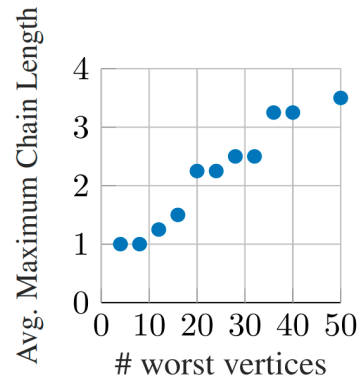
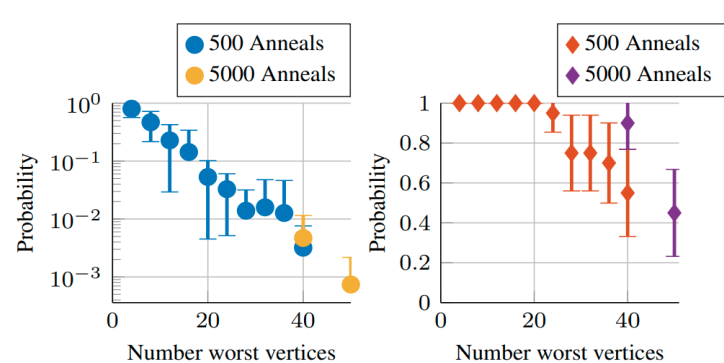
Runtime

Shape Matching with AQC



Accuracy and convergence (FAUST)

Runtime



optimal solution probabilities

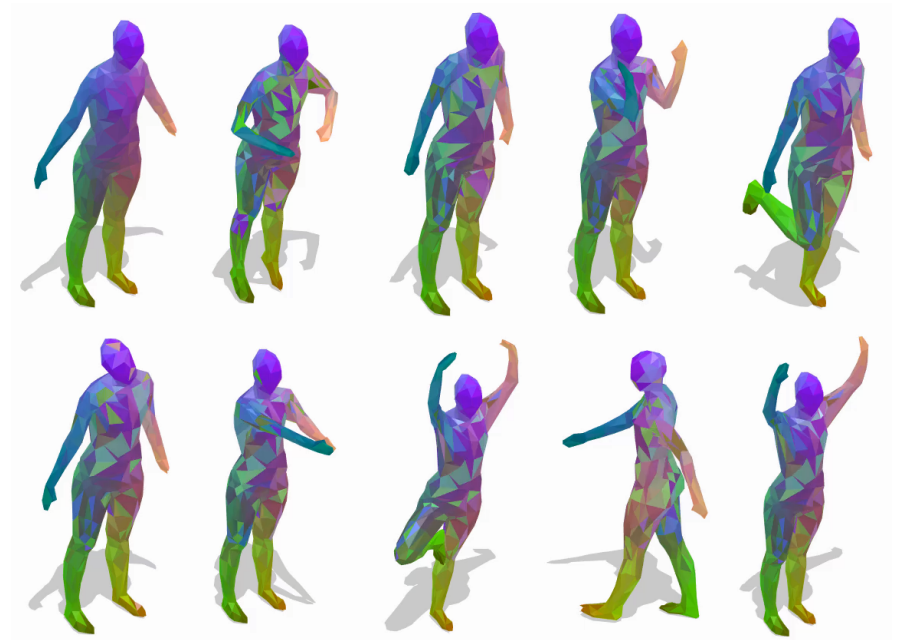
minor embedding statistics

Minor embeddings
(40 and 50 vertices)

WIP: Matching Multiple Shapes with AQC

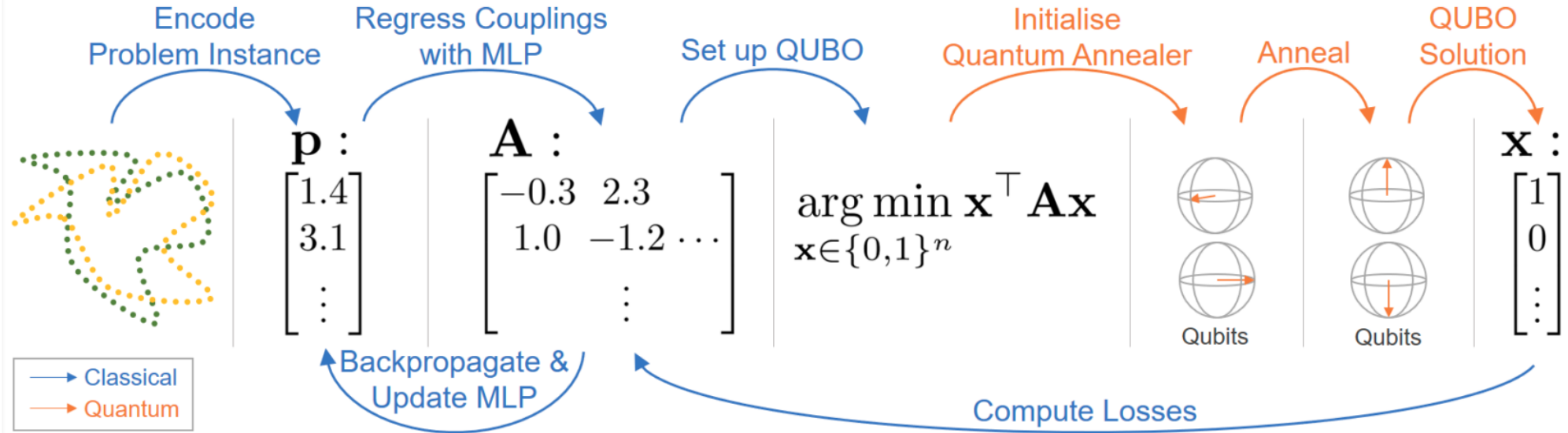


cycle consistency

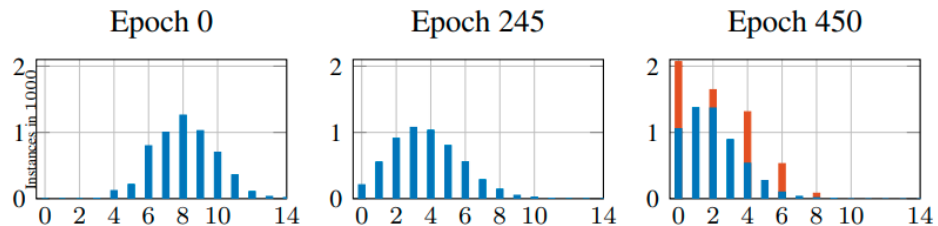
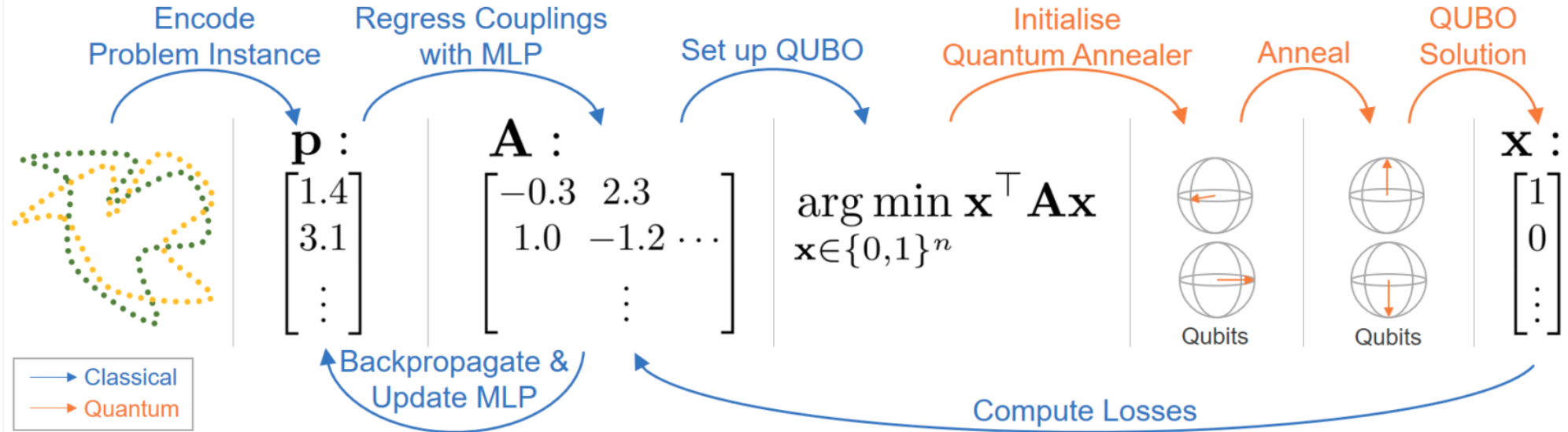


evolution of the matches

WIP: Quantum Annealing with Learnt Couplings

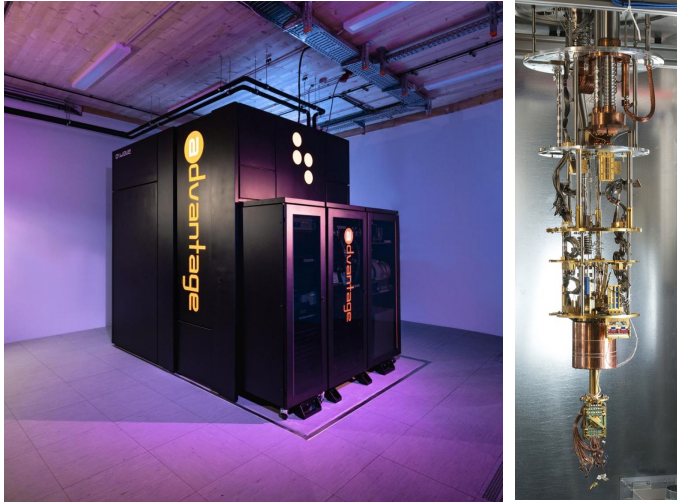


WIP: Quantum Annealing with Learnt Couplings

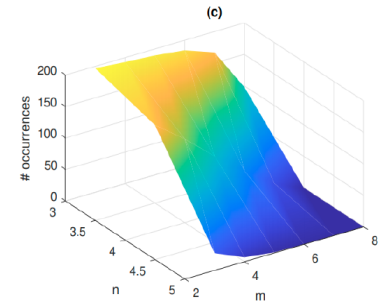
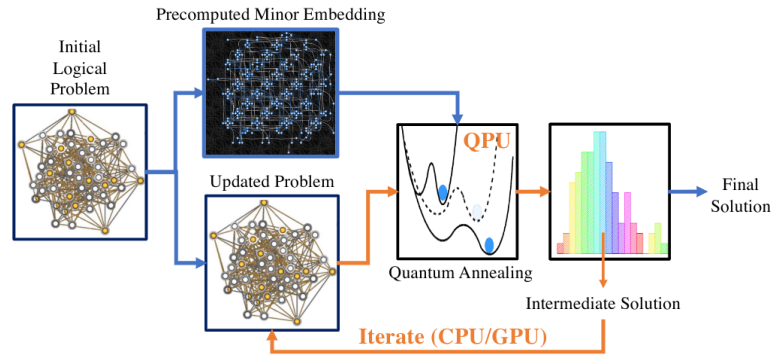
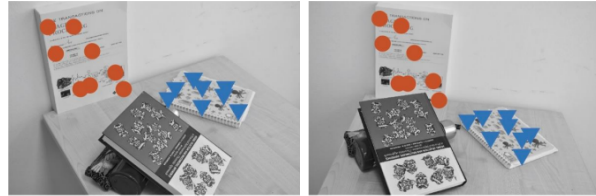


Hamming distance evolution over training epochs

Conclusion



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- QCV gains momentum
- QPUs as accelerators for CV, CG and ML
- Our mission: To enhance the visibility of QCV in the communities

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Thank You!