Karl Bringmann and Sebastian Krinninger **Summer 2016**

Exercises for Complexity Theory of Polynomial-Time Problems

https://www.mpi-inf.mpg.de/departments/algorithms-complexity/teaching/summer16/poly-complexity/

Exercise sheet 4 Due: **Monday, June 27, 2016**

Total points: 40

Either email an electronic version of your assignment submission to gjindal@mpi-inf.mpg.de or give it to Gorav in his office at Room 425, Building E1.3. If Gorav is not in his office then you can just slide your submission under the door of his office.

You are allowed to collaborate on the exercise sheets, but you have to write down a solution on your own, using your own words. Please indicate the names of your collaborators for each exercise you solve. Further, cite all external sources that you use (books, websites, research papers, etc.).

You need to collect at least 50% of all points on exercise sheets.

Exercise 1 (*8 points*) Show a sub-quadratic reduction from **Convolution 3SUM** to **3SUM**. More precisely, show that if **3SUM** can be solved in time $O(n^{2-\epsilon})$ for some $\epsilon > 0$, then **Convolution 3SUM** can be solved in time $O(n^{2-\delta})$ for some $\delta > 0$.

Exercise 2 (*10 points*) The problem **3SUM** can be generalized to k-**SUM** as below.

k-**SUM** problem: Given k sets $A_1, A_2, \ldots, A_k$ of n integers, are there $a_1 \in A_1, a_2 \in A_2, \ldots, a_k \in A_k$ such that $a_1 + a_2 + \ldots + a_k = 0$?

Demonstrate a $O(n^{\lceil \frac{k}{2} \rceil} \cdot \mathrm{poly}(\log n))$ algorithm for k-**SUM** for constant k.

Exercise 3 (*11 points*) $\mathbf{X+Y}$ problem is the following problem.

$\mathbf{X+Y}$ problem: Given two sets X and Y of n integers, does the multi-set $X + Y \triangleq \{a + b \mid a \in X, b \in Y\}$ contain n^2 unique integers or are there duplicates?

Show that $\mathbf{X+Y}$ problem is **3SUM**-hard under sub-quadratic reductions. More precisely, show that if the $\mathbf{X+Y}$ problem can be solved in time $O(n^{2-\epsilon})$ for some $\epsilon > 0$, then 3SUM can be solved in time $O(n^{2-\delta})$ for some $\delta > 0$.

Exercise 4 (*11 points*)

Exponential Time Hypothesis (ETH) is the conjecture that states that **3SAT** has no $2^{o(n)}$ time algorithm.

Prove: **ETH** implies that k-**SUM** cannot be solves in $n^{o(k)}$ time (using the Sparsification Lemma).