Chernoff Bounds Cheat Sheet

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Weighted Chernoff Bound Let $X_1, \ldots, X_n$ be independent 0/1 random variables, each with a weight $0 \leq w_1, \ldots, w_n \leq 1$, and let X be the weighted sum, i.e., $X = w_1 X_1 + \ldots + w_n X_n$. For every $\mu \geq \mathbf{E}[X]$ and every $\delta > 0$, we have

$$\mathbf{Pr}\left[X \geq (1+\delta)\mu\right] \leq \left(\frac{e^\delta}{(1+\delta)^{1+\delta}}\right)^\mu \quad .$$

Note that the weights have to be at most 1. In every other case, you have to divide X by the maximum weight. (This also gives you better guarantees for very small weights.)

Simpler Bounds For $0 < \delta \leq 1$, we have

$$\left(\frac{e^\delta}{(1+\delta)^{1+\delta}}\right)^\mu \leq \exp\left(\frac{\delta^2 \mu}{3}\right) \quad .$$

For $\delta \geq 1$, we have

$$\left(\frac{e^\delta}{(1+\delta)^{1+\delta}}\right)^\mu \leq \exp\left(\frac{\delta \mu}{3}\right) \quad .$$

Sum of Correlated Random Variables The standard prove only shows that the Chernoff bound holds for independent X_i. This assumption can be relaxed in multiple ways to some stronger form of negative correlation. For example, the bound also holds if we drawn without replacement, which formally means $X_1 + \ldots, X_n = 1$. These two papers contain helpful bounds:

[1] Devdatt P. Dubhashi and Desh Ranjan. Balls and bins: A study in negative dependence. *Random Struct. Algorithms*, 13(2):99–124, 1998.

[2] Alessandro Panconesi and Aravind Srinivasan. Randomized distributed edge coloring via an extension of the Chernoff-Hoeffding bounds. *SIAM J. Comput.*, 26(2):350–368, 1997.