Exercise 10: Consensus

Task 1: More Value(s)
In this task, we have a look at *multivalued consensus*, which is like binary consensus, except that inputs and outputs are from a larger domain $[O]$ for some $2 < O \in \mathbb{N}$. Our goal is an efficient reduction to binary consensus.

a) Give a 1-round algorithm with the following property. If all correct nodes have the same input, each node outputs this value. Otherwise, there is a single value $o \in [O]$ such that all nodes output either $o$ or 0.

b) We replace all inputs with the output computed in a). Then we use binary consensus to decide whether to use output 0 or output $o$. What is the problem with this naive approach?

c) Fix the problem from b) in a way that still guarantees that if all correct nodes had opinion $o$ after a), then the binary consensus routine will decide in favor of $o$. Exploit that if this is not the case, it is perfectly fine to output 0!

d) Plug these results together to obtain the desired reduction. Show that it costs $O(1)$ rounds with messages of size $O(\log O)$.

e*) If $f \ll n$, this reduction costs $\omega(fn)$ messages. Can you adapt the solution so that $O(fn)$ messages (and $O(fn \log O)$ bits) are sent by correct nodes in total?

Task 2: As Fast as it Gets
Consider the task of binary consensus with up to $f < n$ crash faults.

a) Describe and prove correct an $(f + 1)$-round algorithm using 1-bit broadcasts for communication.

Task 3*: Timing Issues
In this task, the goal is to transfer synchronous algorithms to the bounded-delay model with faults. Therefore, the setting is the same as in Chapter 4 of the lecture.

a) Use the Srikanth-Toueg algorithm to simulate synchronous execution of some given $R$-round synchronous algorithm in $O(Rd)$ time, assuming that the execution is triggered by events at the individual nodes that are at most $O(Rd)$ time apart (for a known bound).

b) Can you formalize what the term “simulates” here means precisely?

c) Things get a bit messy with randomization, as there typically are additional model assumptions needed for randomization to be useful. Figure out what these might be and whether this poses a problem!

d) Agree on your findings in the TA session!