

Exercise 10: Consensus

Task 1: More Value(s)

In this task, we have a look at *multivalued consensus*, which is like binary consensus, except that inputs and outputs are from a larger domain $[O]$ for some $2 < O \in \mathbb{N}$. Our goal is an efficient reduction to binary consensus.

- a) Give a 1-round algorithm with the following property. If all correct nodes have the same input, each node outputs this value. Otherwise, there is a single value $o \in [O]$ such that all nodes output either o or 0.
- b) We replace all inputs with the output computed in a). Then we use binary consensus to decide whether to use output 0 or output o . What is the problem with this naive approach?
- c) Fix the problem from b) in a way that still guarantees that if *all* correct nodes had opinion o after a), then the binary consensus routine will decide in favor of o . Exploit that if this is not the case, it is perfectly fine to output 0!
- d) Plug these results together to obtain the desired reduction. Show that it costs $\mathcal{O}(1)$ rounds with messages of size $\mathcal{O}(\log O)$.
- e*) If $f \ll n$, this reduction costs $\omega(fn)$ messages. Can you adapt the solution so that $\mathcal{O}(fn)$ messages (and $\mathcal{O}(fn \log O)$ bits) are sent by correct nodes in total?

Task 2: As Fast as it Gets

Consider the task of binary consensus with up to $f < n$ crash faults.

- a) Describe and prove correct an $(f + 1)$ -round algorithm using 1-bit broadcasts for communication.

Task 3*: Timing Issues

In this task, the goal is to transfer synchronous algorithms to the bounded-delay model with faults. Therefore, the setting is the same as in Chapter 4 of the lecture.

- a) Use the Srikanth-Toueg algorithm to *simulate* synchronous execution of some given R -round synchronous algorithm in $\mathcal{O}(Rd)$ time, assuming that the execution is triggered by events at the individual nodes that are at most $\mathcal{O}(Rd)$ time apart (for a known bound).
- b) Can you formalize what the term “simulates” here means precisely?
- c) Things get a bit messy with randomization, as there typically are additional model assumptions needed for randomization to be useful. Figure out what these might be and whether this poses a problem!
- d) Agree on your findings in the TA session!