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Homework Sheet 3: Graph Spanners

Due Date: 21. May 2019

Sequential Algorithms

1. Recall the definition of a κ-core of a graph G, which is a maximal connected subgraph H, where each vertex has degree at least κ. Then recall the simple (greedy) algorithm to compute a κ-core: while there is a vertex of degree less than κ in the current graph, delete it from the graph; if all vertices in the current graph have degree at least κ. Prove that this algorithm is correct. What is the running time of this algorithm ?

2. Suppose X is the set of vertices deleted by the above algorithm for computing κ-core in a graph G. Then show that the total number of edges in $E(G)$ incident on vertices of X is upper bounded by $\kappa \cdot |X|$.

3. Recall the definition of (α, β)-spanners. Show that if H is a $(1, 2k)$-spanner of G, then H is also a $(2k+1)$-spanner of G. Then show that, assuming the Erdős Girth Conjecture, a $(1, 2k)$-spanner may have $\Omega(n^{1+\frac{1}{k+1}})$ edges in the worst case.

4. Recall the definition of a dominating set in a graph.

   Fact 1. *Let G be a graph and fix a parameter $\sigma \geq 1$. There exists a set $S \subseteq V(G)$ of $\tilde{O}(\frac{n}{\sigma})$ vertices such that every vertex v with $\delta(v) \geq \sigma$ is dominated by S.*

   The aim of this exercise is to prove Fact 1.

   (a) Prove that if the minimum degree of G is at least σ, then G admits a dominating set of size $\tilde{O}(\frac{n}{\sigma})$.

   (b) Show that a graph G with minimum degree smaller than σ can be transformed into a graph G' having minimum degree σ. The graph G' must satisfy the following property: if S' is a dominating set of G', then $S = S' \cap V(G)$ dominates all vertices v of G such that $\delta(v) \geq \sigma$.

   (c) Explain how (a) and (b) can be used to prove Fact 1. In particular provide a proof of the upper bound of $\tilde{O}(\frac{n}{\sigma})$ on $|S|$.

5. Use Fact 1 to design an algorithm that computes a $(1, 2)$-spanner with $\tilde{O}(n^{\frac{3}{2}})$ edges.