Homework Sheet 6: Distributed and Sequential LLL

Due Date: 23 June 2019

The latest lecture was based on the following paper: A a constructive proof of the general lovász local lemma [1]. We omitted several proofs in the lecture. Here your task is to complete the missing proof. Of course, they are available in the paper, however, you should write your own understanding.

1. Provide proof of Lemma 3.1. What is a simple description for this lemma?

2. Use lemma 3.1 and lemma 2.1 to prove the theorem 1.2 of the paper.

3. Prove lemma 4.1. Then from its proof conclude the algorithm runs for at most $O(\log n)$ steps w.h.p.

   In each step of the distributed algorithm, we calculate a maximal independent set among bad event, there we employ the algorithm of Luby to obtain a maximal independent set in $O(\log n)$ rounds. Based on this, prove that w.h.p. the whole distributed algorithm ends after $O(\log n)$ communication rounds.

4. (Bonus Exercise) Given a graph G on n vertices, the f-frugal coloring of G is a coloring that satisfies the following two condition.

   1. No two neighboring vertices receive the same color (it is a proper coloring).
   2. For any color c, every vertex v has at most f neighbors of color c.

   The task is to color the graph satisfying the above requirements with at most C colors.

   What should be the relation between f and maximum degree Δ of the graph and C, such that we are sure that the graph has always a frugal coloring with the mentioned attributes? (hint: every vertex encodes two types of bad events: 1. it has the same color as one of its neighbors, this happens with probability $1/C$ (why?) or 2. It has f neighbors of the same color c. This happens with probability X (determine it). Use these two facts and put it in the statement of the symmetric LLL lemma, $epd < 1$, to find a relation between the above parameters).

References

[1] Robin A. Moser, Gábor Tardos"A constructive proof of the general lovsz local lemma," 2010.
Journal of the ACM (JACM),Volume 57 Issue 2, January 2010, Article No. 11